How would our analysis be affected if marginal cost were not constant? The same basic principles apply with one important change. If marginal production costs are not constant, we cannot treat the two markets independently since whatever output the monopolist chooses to supply to the United States, for example, affects the marginal cost of supplying Europe. So the different markets have to be looked at together. Nevertheless, we still have simple rules that guide the monopolist's pricing decisions in these markets.

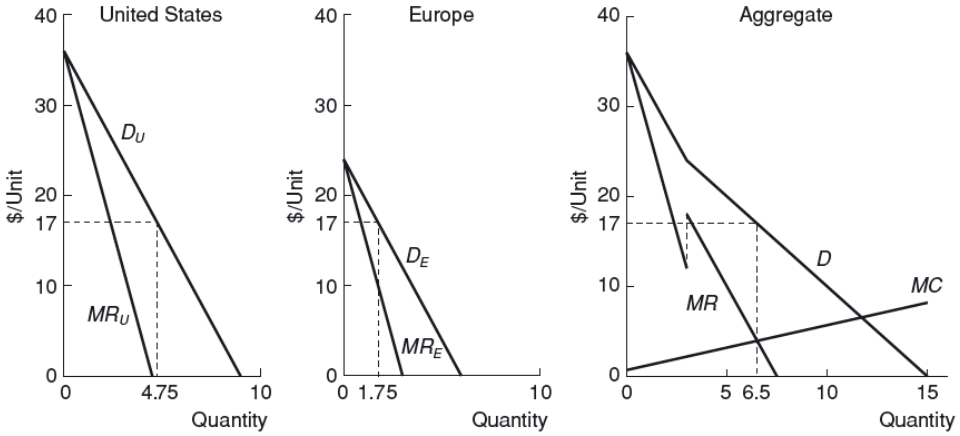

Figure 1 Non-discriminatory pricing with non-constant marginal cost
The firm identifies aggregate demand and the associated marginal revenue. It chooses total output where marginal revenue equals marginal cost and the non-discriminatory price from the aggregate demand function. Output in each market is the market clearing output.

To illustrate this point, suppose that the publisher of *Harry Potter and the Deathly Hallows* has a single printing facility that produces books for both the United States and European markets and that marginal cost is given by $MC = 0.75 + Q/2$, where Q is the total number of books printed.

Figure 5.3 illustrates the profit-maximizing behavior if the monopolist chooses not to price discriminate. The basic analytical steps in this process are as follows:

1. Calculate aggregate market demand as above.
2. Identify the marginal revenue function for this aggregate demand function. From our example, if $Q > 3$ so that both markets are active, this is $MR = 30 - 4Q$.
3. Equate marginal revenue with marginal cost to determine aggregate output. So we have $0.75 + Q/2 = 30 - 4Q$ giving $Q^* = 6.5$ million books.
4. Identify the equilibrium price from the aggregate demand function. Since both markets are active, the relevant part of the aggregate demand function is $P = 30 - 2Q$, giving an equilibrium price of $P^* = \$17$.
5. Calculate demand in each market at this price: 4.75 million books in the United States and 1.75 million books in Europe.

Now suppose that the monopolist chooses to price discriminate. This outcome is illustrated in Figure **2. The** underlying process is clearly different, and the steps in implementing profit maximizing price discrimination are as follows:

1. Derive marginal revenue in each market and add these *horizontally* to yield an allocation of output across the two markets with the same marginal revenue. Marginal revenue in the United States is $MR = 36 - 8Q_U$ for any marginal revenue less than \$36 and in Europe is $MR = 24 - 8Q_E$ for any marginal revenue below \$24. Inverting these gives $Q_U = 4.5 - MR/8$ and $Q_E = 3 - MR/8$. Summing these gives an aggregated marginal revenue:

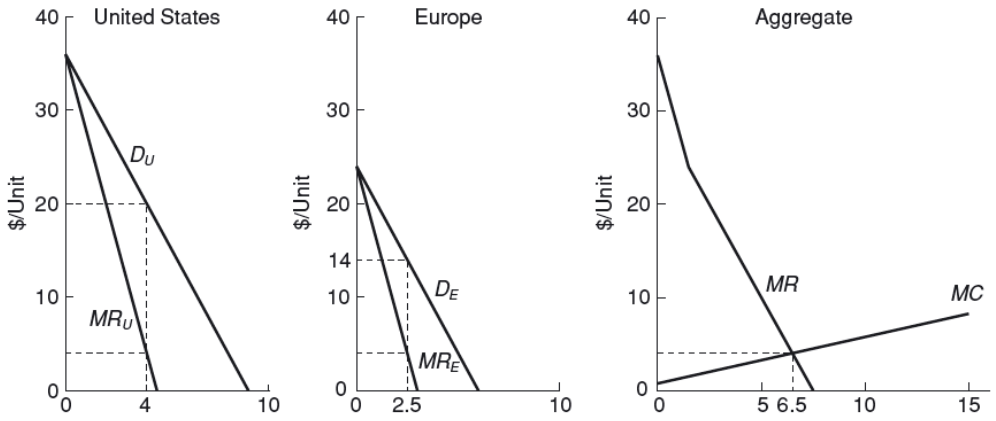

Figure 2 Third-degree price discrimination or group pricing with non-constant marginal cost
The firm calculates aggregate marginal revenue and equates this with marginal cost. Output in each market equates marginal revenue with aggregate marginal cost. Price in each market is the market-clearing price.

$$\begin{aligned} Q = Q_U + Q_E &= 4.5 - MR/8 \quad \text{for } Q \le 1.5 \\ Q = Q_U + Q_E &= 7.5 - MR/4 \quad \text{for } Q > 1.5 \end{aligned} \tag{1}$$

This can be inverted to give aggregate marginal revenue its more usual form

$$\begin{aligned} MR &= 36 - 8Q \quad \text{for } Q \le 1.5 \\ MR &= 30 - 4Q \quad \text{for } Q > 1.5 \end{aligned} \tag{2}$$

Note how this step differs from the non-discriminatory case. In the latter both markets are treated as one, so we start with aggregate demand and derive its associated marginal revenue. In the discriminatory pricing case, by contrast, the markets are supplied separately, with the profit maximizing condition that $MC = MR$ in both markets so we need aggregate marginal revenue, not aggregate demand.

2. Equate aggregate marginal revenue with marginal cost to identify the equilibrium aggregate quantity *and* marginal revenue. So we have $30 - 4Q = 0.75 + 2/Q$ giving $Q^* = 6.5$. As a result, the equilibrium marginal revenue is \$4, which is equal to the marginal cost of the last unit produced.
3. Identify the equilibrium quantities in each market by equating individual market marginal revenue with the equilibrium marginal revenue and marginal cost. In the United States this gives $36 - 8Q_U = 4$ or $Q_U^* = 4$ million books and in Europe $24 - 8Q_U = 4$ or $Q_E^* = 2.5$ million books.
4. Identify the equilibrium price in each market from the individual market demand functions, giving a price of \$20 in the United States and \$14 in Europe.

The foregoing procedure is again derived from two simple rules that guide the monopolist's pricing decisions with third-degree price discrimination. These rules apply no matter the shape of the monopolist's marginal cost function. The rules are:

1. Marginal revenue must be equalized in each market.
2. Marginal revenue must equal marginal cost, where marginal cost is measured at the *aggregate* output level.

There is one further interesting point that is worth noting regarding the contrast between uniform pricing (no price discrimination) and third-degree price discrimination. When demand is linear and both markets are active under both pricing schemes *aggregate demand is identical with the two pricing policies*. This is proved formally in the Derivation Checkpoint: Discriminatory and Nondiscriminatory Pricing. The intuition is simple to see. When both markets are active aggregate marginal revenue is identical with the two pricing policies (we are below the discontinuity in *MR* in Figure 1). So equating aggregate marginal revenue with aggregate marginal cost must give the same aggregate output. The reason that third-degree price discrimination is more profitable in this case is because the aggregate output is allocated more profitably across the two markets—to ensure that marginal revenue on the last unit sold in each market is equal.

Derivation Checkpoint

Discriminatory and Non-discriminatory Pricing

Suppose that a monopolist supplies two groups of consumers with inverse demand for each group given by:

$$P_1 = A_1 - B_1Q_1;\ P_2 = A_2 - B_2Q_2$$

In these demand functions we assume that $A_1 > A_2$ so that group 1 is the "high demand" group whose demand is the less elastic at any given price. Inverting the inverse demands gives the direct demands at some price P:

$$Q_1 = (A_1 - P)/B_1;\ Q_2 = (A_2 - P)/B_2$$

and so aggregate demand is:

$$Q = Q_1 + Q_2 = \frac{A_1B_2 + A_2B_1}{B_1B_2} - \frac{B_1 + B_2}{B_1B_2}P$$

Of course, this holds only for any price less that A_2. Invert this to get the aggregate inverse demand for the two groups, again for any price less than A_2 yielding:

$$P = \frac{A_1B_2 + A_2B_1}{B_1 + B_2} - \frac{B_1B_2}{B_1 + B_2}Q$$

The marginal revenue associated with this aggregate demand is:

$$MR = \frac{A_1B_2 + A_2B_1}{B_1 + B_2} - 2\frac{B_1B_2}{B_1 + B_2}Q$$

We can simplify matters a bit by assuming, without loss of generality, that marginal cost is zero. So solving $MR = 0$ for Q gives the equilibrium aggregate output with uniform pricing:

$$Q^U = \frac{A_1B_2 + A_2B_1}{2B_1B_2}$$

Substituting Q^U into the price equation gives the equilibrium uniform price

$$P^U = \frac{A_1B_2 + A_2B_1}{2(B_1 + B_2)}$$

Substituting this price into the individual demands then gives equilibrium output in each market

$$Q_1^U = \frac{(2A_1 - A_2)B_1 + A_1B_2}{2B_1(B_1 + B_2)};\ Q_2^U = \frac{(2A_2 - A_1)B_2 + A_2B_1}{2B_2(B_1 + B_2)}$$

With third-degree price discrimination the firm sets marginal revenue equal to marginal cost for each group. From the demand curves, we know that the marginal revenues are:

$$MR_1 = A_1 - 2B_1Q_1;\ MR_2 = A_1 - 2B_2Q_2$$

It follows immediately that the equilibrium outputs for each group are

$$Q_1^D = \frac{A_1}{2B_1};\ Q_2^D = \frac{A_2}{2B_2}$$

Comparison then confirms that $Q_1^D \leq Q_1^U$ and $Q_2^D \geq Q_2^U$. In other words, third-degree price discrimination diverts output from the high-demand market to the low-demand market increasing price in the former and lowering price in the latter. You can also confirm that $Q_1^D + Q_2^D = Q^U$. In other words, when demands are linear *aggregate output is identical with uniform pricing and with third-degree price discrimination or group pricing.*