

MATEMATICA PER L'ECONOMIA

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ESERCIZIARIO
MATRICI E VETTORI

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DET. E MATRICE INVERSA

ES. da pag. 42 a pag 53
SPAZI E SOTTOSPAZI VETT.

n° 62 pag 44 $\pi(A)$

$$a) A = \begin{bmatrix} -3 & -1 & 1 \\ 1 & 0 & 0 \\ -4 & 1 & -10 \end{bmatrix} \quad 3 \times 3$$

$$b) A = \begin{bmatrix} -2 & -1 & 7 & 1 \\ 1 & 0 & 11 & 1 \\ -4 & 1 & 0 & 0 \end{bmatrix} \quad 3 \times 4$$

(TUTOR 1°)

①

$$a) \det A = -1 \cdot \begin{vmatrix} -1 & 1 \\ 1 & -10 \end{vmatrix} =$$

$$= -9 \neq 0$$

$$r(A) = 3$$

$$b) \det \begin{bmatrix} -1 & 7 & 1 \\ 0 & 11 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$1 \cdot (7 - 11) \neq 0$$

$$r(A) = 3$$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 6 & 1 \\ -1 & 2 & 7 \end{bmatrix}$$

$$\begin{aligned} \det A &= 1(42-2) - \\ &- 1(-14+1) + 3(-4+6) \\ &= 40 + 13 + 6 \neq 0 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$\begin{aligned} \det &= 1(45-48) - 4(18-24) \\ &+ 7(12-15) = -3 + \\ &+ 24 - 21 = 0 \end{aligned}$$

$$A = \begin{bmatrix} 0 & 2 & 6 \\ 1 & -1 & -2 \\ -1 & 2 & 5 \end{bmatrix}$$

$$\det A = -1(10 - 12) -$$
$$-1(-4 + 6) = +2 - 2$$

$$= 0$$

$$\{\underline{a}, \underline{b}\}$$

V

$$\dim S = 2$$

$$\{\underline{0}\}$$

n° 66 pag 46 lin. ind. / dip.

a) $\underline{a} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}$ $\underline{c} = \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}$

uio) $\underline{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\underline{w} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $\underline{u} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$

n° 68 pag 48 dim S e base

c) $\underline{a} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ $\underline{c} = \begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix}$

a) $\underline{v} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$ $\underline{w} = \begin{bmatrix} 0 \\ 13 \\ -2 \end{bmatrix}$ $\underline{u} = \begin{bmatrix} -2 \\ -8 \\ 1 \end{bmatrix}$

n° 69 e n° 70 $\mathbb{R}^3$
FACCIAMO IN $\mathbb{R}^2$

$\underline{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\underline{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\underline{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

CALCOLA SE SONO LIN. DIP. / INDIP.
DIM DEL SOTTOSP. GENERATO

(2)

$$\begin{aligned}\underline{x} &= \alpha \underline{w} + \beta \underline{u} = \\ &= \begin{bmatrix} -\alpha \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} = \begin{bmatrix} -\alpha \\ \alpha + \beta \end{bmatrix}\end{aligned}$$

$$\forall \alpha, \beta \in \mathbb{R}$$

BASE CANON.

$$\underline{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \underline{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

DET. UNA BASE DEL SOTTOSP.
SCRIVERE IL GENERICO VETT. DEL
SOTTOSP.

n° 71 pag 51 $\underline{y} = A \underline{x}$

$$a) \underline{y} = \begin{bmatrix} -2x \\ 3x \\ 4x \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix} x$$

$$h) \underline{y} = \begin{bmatrix} 2x_1 + 3x_2 \\ 21x_1 - x_3 \\ x_1 + x_2 + x_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 0 \\ 21 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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$$a) \begin{cases} x_1 + 2x_3 = 1 \\ x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + x_2 = 1 \end{cases}$$

mio "girostra")

$$\begin{cases} x - y = 0 \\ y - z = 0 \\ z - x = 0 \end{cases}$$

CRAMER

$$x_1 = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 1 & 2 & -1 \\ 2 & 1 & 0 \end{vmatrix}} = \frac{1-4}{1-6} = \frac{-3}{-5} = \frac{3}{5}$$

$$x_2 =$$

$$\underline{x}^* = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_3 =$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & -1 \\ 2 & 1 & 0 \end{bmatrix} \quad 3 \times 3$$

$$\underline{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \begin{array}{l} \text{SIST.} \\ \text{NON OHOG.} \end{array}$$

$$\det A = 1(1) + 2(-3) = 1 - 6 = -5$$

$$\left. \begin{array}{l} \kappa(A) = 3 \\ \kappa(A|\underline{b}) = 3 \end{array} \right\} \begin{array}{l} \text{COMP.} \\ \text{DET.} \end{array}$$

$$m = 3 \quad \infty^{m-r}$$

$$\begin{cases} x - y = 0 \\ y - z = 0 \\ z - x = 0 \end{cases}$$

ERRATO

$$\begin{cases} x = y \leftarrow \\ y = z \\ z = x \end{cases}$$

∞^1

$$\begin{cases} x = y \\ y - z = 0 \\ \cancel{z - y = 0} \end{cases}$$

$$\begin{cases} x = y \\ y = y \\ z = y \end{cases}$$

$$\begin{bmatrix} y \\ y \\ y \end{bmatrix} = y \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{mio)} \quad \begin{cases} x + 2y = 0 \\ y + 3x = 1 \\ 2x - y = 2 \end{cases}$$

3x2

$$\text{mio)} \quad \begin{cases} x - y = 0 \\ x + y = 1 \end{cases}$$

2x2

$$\text{mio)} \quad \begin{cases} x + y + z = 1 \\ x - y - z = 2 \end{cases}$$

2x3

TROVE RETE SOTTOSP. VETT.

$$S = \{ \underline{x} \in \mathbb{R}^3 \mid x_1 + x_2 = 0 \wedge x_3 = 1 \}$$

$$S = \{ \underline{x} \in \mathbb{R}^3 \mid x_1 + x_2 = 0 \wedge x_3 = 0 \}$$

$$S = \{ \underline{x} \in \mathbb{R}^2 \mid x_2 = 2x_1 \}$$

$$\begin{cases} x_1 + x_2 = 0 \\ x_3 = 1 \end{cases}$$

$$\begin{cases} x_1 + x_2 = 0 \\ x_3 = 0 \end{cases}$$

$$\begin{cases} x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = -x_2 \end{cases}$$

$$\begin{cases} x_3 = 0 \end{cases}$$

$$\underline{x} = \begin{bmatrix} -x_2 \\ x_2 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\dim S = 1$$