TUTOR. M.E. 2° 7/10/25

1. AUTOVALORI E AUTOVETTORI
2. MATRICI D.P., D.N., IND., S.D.P., S.D.N
3. FORME QUADRATICHE

DEF. $\lambda \in \mathbb{R}$ e $\underline{x} \in \mathbb{R}^n$ SONO AUTOVAL. e AUTOVET. SE

$$A\underline{x} = \lambda \underline{x} \quad \text{CON } \underline{x} \neq \underline{0}$$

ES. 1

$$A = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \quad 2\times 2$$

$$A - \lambda I = \begin{bmatrix} -1-\lambda & -1 \\ -1 & -1-\lambda \end{bmatrix}$$

PONGO

$$\det(A - \lambda I) = (-1-\lambda)^2 - 1 = 0$$

$$1 + 2\lambda + \lambda^2 - 1 = 0 \quad \leftarrow \text{EQ. CARAT.}$$

POLIN. CAR. DI 2° GRADO

$$\lambda^2 + 2\lambda = 0$$

$$\lambda(\lambda+2) = 0 \quad \begin{matrix} \lambda_1 = 0 \\ \lambda_2 = -2 \end{matrix}$$

AUTOVAL.

I CASO: $\lambda_1 = 0$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -x_1 - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases}$$ SONO UGUALI

$$\Longrightarrow -x_1 - x_2 = 0 \Longrightarrow x_2 = -x_1$$

AUTOVETTORE: $\begin{bmatrix} x_1 \\ -x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

II CASO: $\lambda_2 = -2$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x_1 - x_2 = 0 \\ -x_1 + x_2 = 0 \end{cases} \Longrightarrow x_2 = x_1$$

AUTOVETT. $\begin{bmatrix} x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_1$

ES. 2

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad 3 \times 3$$

$$A - \lambda \underline{I} = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & 1 & 1-\lambda \end{bmatrix}$$

MATRICE TRIANGOLARE INFERIORE

$$\Rightarrow \det(A - \lambda I) = (1-\lambda)(1-\lambda)(1-\lambda)$$

$$\Rightarrow (1-\lambda)^3 = 0 \Rightarrow \begin{matrix} \lambda_1 = 1 \\ \lambda_2 = 1 \\ \lambda_3 = 1 \end{matrix}$$

I CASO: $\lambda_1 = 1$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 0 = 0 \\ x_1 = 0 \\ x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases} \quad e \; x_3 = ? \text{ LIBERA}$$

$$\underline{v} = \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

ES. 3

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$$

MINORI PRINCIPALI DI GUIDA

$1°$ $[3] \longrightarrow \det = 3 > 0$

$2°$ $\begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix} \longrightarrow \det = 6 - 4 = 2 > 0$

$\Longrightarrow$ A È DEF. POS.

METODO DEGLI AUTOVALORI

$$\begin{bmatrix} 3-\lambda & 2 \\ 2 & 2-\lambda \end{bmatrix} \longrightarrow \det = (3-\lambda)(2-\lambda) - 4$$

$$6 - 3\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 5\lambda + 2 = 0$$

$$\lambda_{1,2} = \frac{5 \pm \sqrt{25-8}}{2} = \frac{5 \pm \sqrt{17}}{2} = \begin{cases} > 0 \\ > 0 \end{cases}$$

$\lambda_1 > 0$, $\lambda_2 > 0 \Longrightarrow$ A È D.P.

ES. 4

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \quad 3\times 3$$

1° $[3] \longrightarrow \det = 3 > 0$

2° $\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \longrightarrow \det = 6-1 = 5 > 0$

3° $A \longrightarrow \det = 3(6-1) + 1(-3) =$

$= 15 - 3 = 12 > 0$

È D.P.

ES. 5

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

1° $1 > 0$

2° $1 - 4 = -3 < 0 \implies$ È INDEF.

3° non serve

D.P.	D.N.	IND.	S.D.P.	S.D.N.
+	−	A	+0	−0
+	+	L T	+0	+0
+	−	R I	+0	−0

EQ. QUADRATICHE E FORME Q.

ES. 1 $53x^2 - 72xy + 32y^2 = 80$

EQ. QUAD.

F.Q. $53x^2 - 72xy + 32y^2$

MATRICE ASSOC. $\begin{bmatrix} 53 & -36 \\ -36 & 32 \end{bmatrix}$

ES. 2 $xy = 3$ EQ. Q.

F.Q. xy

MATR. $\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

STUDIO COME SONO DEF.

$\begin{bmatrix} 53 & -36 \\ -36 & 32 \end{bmatrix}$ $53 > 0$

$53 \cdot 32 - 36^2 = 1696 - 1296$
$\Rightarrow > 0$

D.P.

$\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ $1° = 0$

$2° = 0 - \frac{1}{4} \Rightarrow < 0$

INDEF.

ES. 3 $9x^2+4y^2-36x-24y+36=$

F.Q. $9x^2+4y^2$

MATRICE $\begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix}$ $1^\circ > 0$, $2^\circ > 0$ D.P.

F.Q. IN 3 VARIABILI

ES. 4 $x^2-y^2-z^2=16$

F.Q. $x^2-y^2-z^2$

MATRICE $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (DIAGONALE)

$1^\circ > 0$

$2^\circ < 0$ INDEF.

$3^\circ > 0$

METODO DEGLI AUTOVALORI

$\lambda_1 = 1 \quad \lambda_2 = -1 \quad \lambda_3 = -1$

1 NEG. 2 POS. $\Rightarrow$ INDEF.

ESERCIZIARIO p. 227
p. 236

$$A = \begin{bmatrix} 1 & -5 \\ 0 & 2 \end{bmatrix}$$

$\lambda_1 = 1 \qquad \lambda_2 = 2$

$$\begin{bmatrix} 0 & -5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -5x_2 = 0 \\ x_2 = 0 \end{cases} \qquad \boxed{x_2 = 0}$$

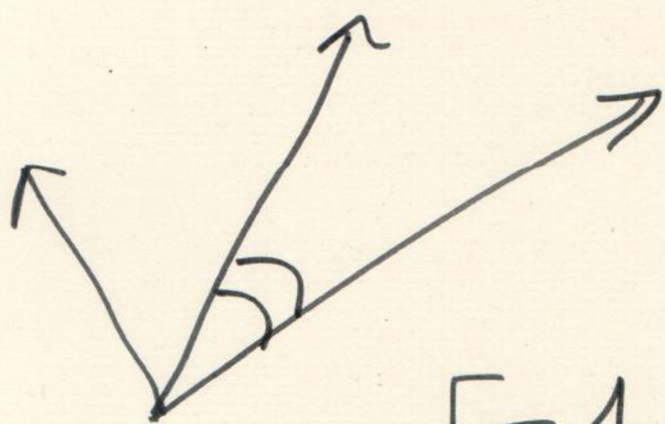
$$\underline{v} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad x_1 \neq 0$$

$$\begin{bmatrix} -1 & -5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$+x_1 + 5x_2 = 0 \qquad x_1 = -5x_2$

$$\underline{v} = \begin{bmatrix} -5x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -5 \\ 1 \end{bmatrix} \quad x_2 \neq 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -5 \\ 1 \end{bmatrix} \quad \text{lin. ind.}$$



ES.
$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\lambda_1 = -1 \quad \lambda_2 = -1 \quad \lambda_3 = 0$$

1° CASO $\lambda_1 = -1$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + x_3 = 0$$

$$x_3 = -x_1 - x_2$$

$$\begin{bmatrix} x_1 \\ x_2 \\ -x_1 - x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

2 vett. l. ind.

2°

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -x_1 = 0 \\ -x_2 = 0 \\ x_1 + x_2 = 0 \end{cases}$$

x_3 LIBERA

$$\underline{v} = \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$f(x,y) = 2x^2 + y^2 - xy$$

$$\begin{bmatrix} 2 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$1^\circ > 0$ D.P.

$2^\circ > 0$

$$f(x,y,z) = x^2 - xy - 2yz - y^2$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$1^\circ > 0$

$2^\circ < 0$

3° non serve

INDEF.