M.E. TUTORATO 3°

21/10/2025

1. DOMINIO
2. DERIV. PARZ. 1e E 2e
3. OTTIMIZZAZIONE LIBERA

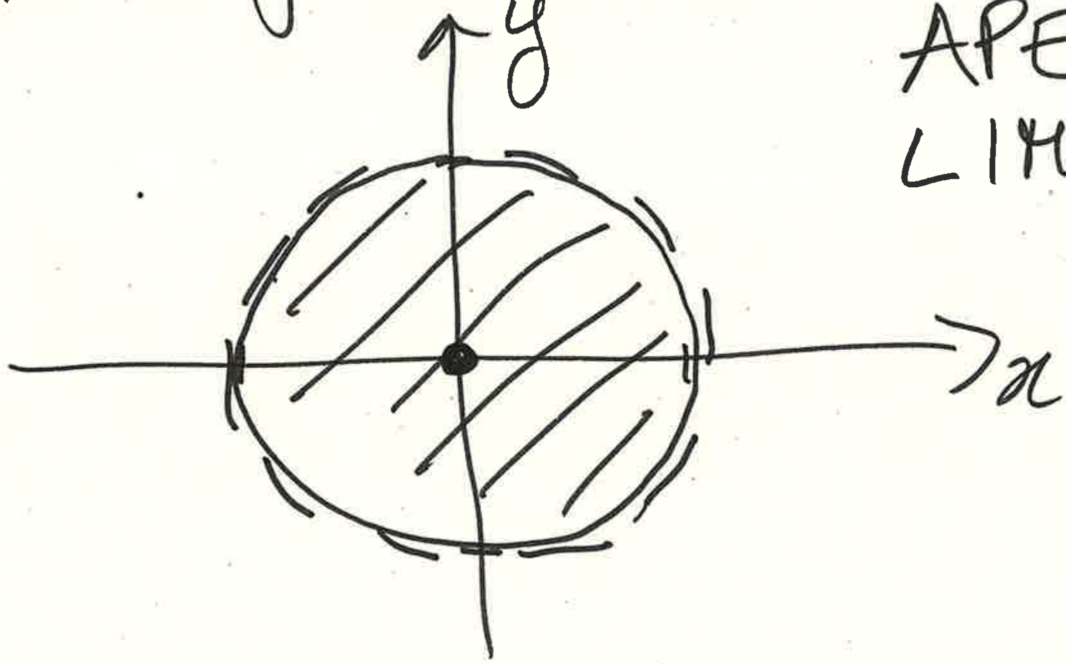
n° 1 DISPENSA DOMINIO

$$f(x,y) = \log(1-x^2-y^2)$$

$$1-x^2-y^2 > 0$$

$$x^2+y^2-1 < 0$$

$$x^2+y^2 < 1$$



APERTO
LIMITATO

$$\begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{1}{1-x^2-y^2}(-2x) = 0 \\ \dfrac{\partial f}{\partial y} = \dfrac{1}{1-x^2-y^2}(-2y) = 0 \end{cases}$$

$$\begin{cases} x = 0 \\ y = 0 \end{cases}$$

1) $\in D$
2) INTERNO
3) $\exists \dfrac{\partial f}{\partial x} \dfrac{\partial f}{\partial y}$

$$\frac{\partial^2 f}{\partial x^2} = \frac{-2(1-x^2-y^2)+2x(-2x)}{(1-x^2-y^2)^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{-2(1-x^2-y^2)+2y(-2y)}{(\quad)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = (-2x)\left(-\frac{1}{(1-x^2-y^2)^2}\right)(-2y)$$

$$= \frac{-4xy}{(1-x^2-y^2)^2}$$

$$H(0,0)=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}$$

1° $-2<0$ D.N. $\Rightarrow (0,0)$

2° $+4>0$ MAX.R.

$$f(x,y)=\sqrt{y^2-x^4}$$

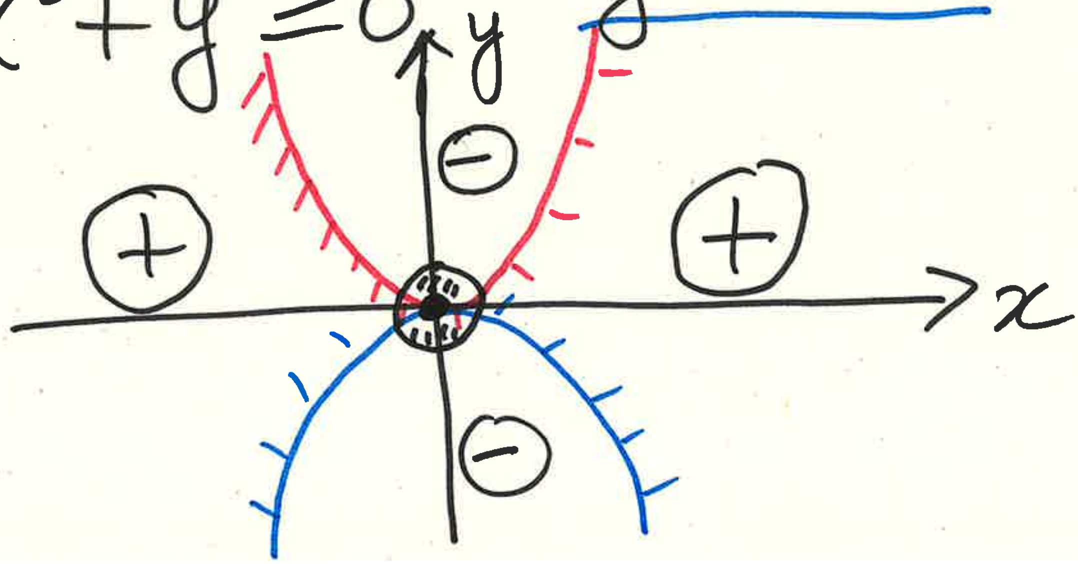
$$D:\ y^2-x^4\geq 0$$

$$x^4-y^2\leq 0$$

$$(x^2-y)(x^2+y)\leq 0$$

$1^\circ)$ $x^2-y\geq 0$ $\quad y\leq x^2$

$2^\circ)$ $x^2+y\geq 0$ $\quad y\geq -x^2$



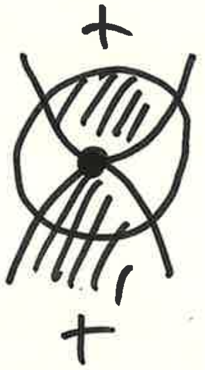
$$\begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{1}{2\sqrt{}}\left(-4x^3\right) = 0 \\[2ex] \dfrac{\partial f}{\partial y} = \dfrac{1}{2\sqrt{}}\left(2y\right) = 0 \end{cases}$$

$(0,0)$

$$\begin{cases} 1) \in D \\ 2) \text{ INTERNO } \cancel{\text{No}} \\ 3) \qquad \text{No} \end{cases}$$

$f(0,0) = 0$

$f(x,y) = +\sqrt{y^2 - x^4}$



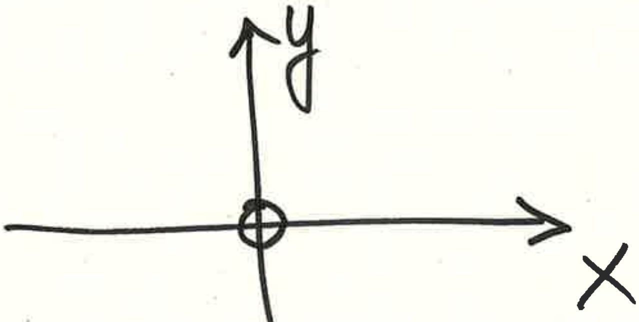
$(0,0)$ MIN ASS.

$y = x^2$ " "

$y = -x^2$ " "

$$f(x,y) = \frac{1}{x^2+y^2}$$

$$D: \ x^2+y^2 \neq 0 \qquad (0,0) \notin D$$



$$\begin{cases} \dfrac{\partial f}{\partial x} = -\dfrac{1}{(x^2+y^2)^2} \cdot 2x = 0 \\ \dfrac{\partial f}{\partial y} = -\dfrac{1}{(\quad)^2} \, 2y = 0 \end{cases}$$

$$(0,0) \notin D$$

$$f(x,y) = |x| + |y|$$

$$f(x,y) = |x+y|$$

$$f(x,y) = \sqrt{|x^2 - xy|}$$

$$z = f(x,y) = \sqrt{x^2+y^2} \quad (2,0,2)$$

$$\frac{\partial f}{\partial x} = \frac{1}{\cancel{2}\sqrt{}} \cdot \cancel{2}x \longrightarrow 1$$

$$\frac{\partial f}{\partial y} = \frac{1}{\cancel{2}\sqrt{}} \cdot \cancel{2}y \longrightarrow 0$$

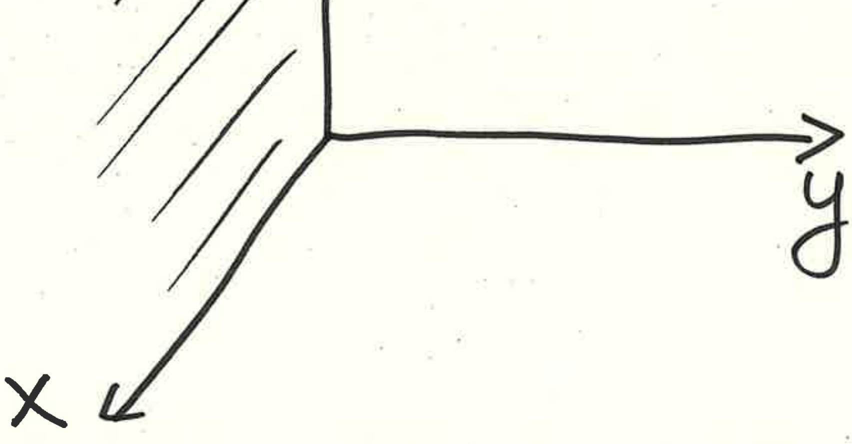
$$\nabla f(2,0) = (1,0) \quad [1 \;\; 0]^T$$

EQ. PIANO TG

$$z - 2 = 1 \cdot (x-2) + 0 \cdot (y-0)$$

$$z = x - 2 + 2$$

$$z = x$$



$$\frac{\partial^2 f}{\partial x^2} = \frac{1 \cdot \sqrt{x^2+y^2} - x \, \frac{1}{2\sqrt{}} \cdot 2x}{\left(\sqrt{x^2+y^2}\right)^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{1 \cdot \sqrt{x^2+y^2} - y \cdot \frac{1}{\cancel{2}\sqrt{}} \cdot \cancel{2}y}{\left(\quad\quad\right)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = x \cdot \left(-\frac{1}{\left(\quad\right)^2}\right) 2y$$

$$H(2,0) = \begin{bmatrix} \boxed{0} & 0 \\ 0 & \boxed{\frac{1}{2}} \end{bmatrix}$$

$1^\circ = 0$ $\quad$ 1° bis. > 0

$2^\circ = 0$ $\quad$ S.D.P.

$$f(x,y)=e^{-(x^2+y^2)}$$

$$D=\mathbb{R}^2$$

$$\begin{cases} \dfrac{\partial f}{\partial x}=e^{-(x^2+y^2)}\cdot(-2x)=0 \\ \dfrac{\partial f}{\partial y}=e^{-(x^2+y^2)}\cdot(-2y)=0 \end{cases}$$

$(0,0)$ P. CRITICO

$$\frac{\partial^2 f}{\partial x^2}=e^{-(x^2+y^2)}(-2x)^2+e^{\sim}\cdot(-2)$$

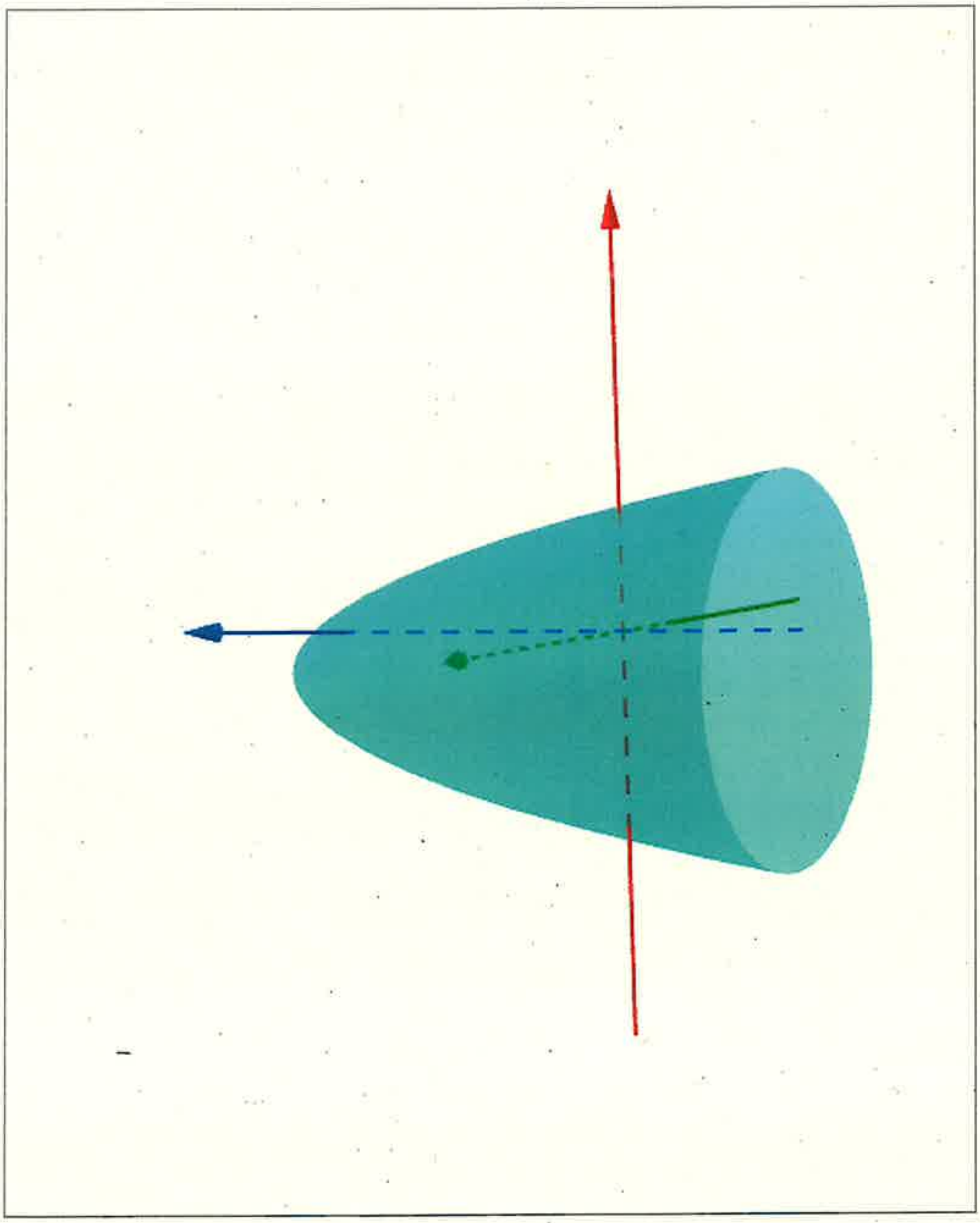
$$\frac{\partial^2 f}{\partial y^2}=e^{\sim}(-2y)^2+e^{\sim}\cdot(-2)$$

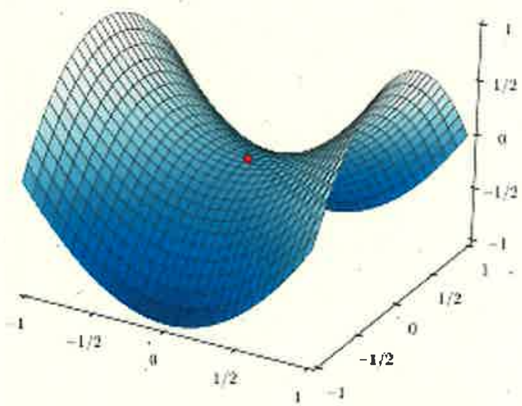
$$\frac{\partial^2 f}{\partial x\partial y}=(-2x)\,e^{\sim}(-2y)$$

$$H(0,0) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

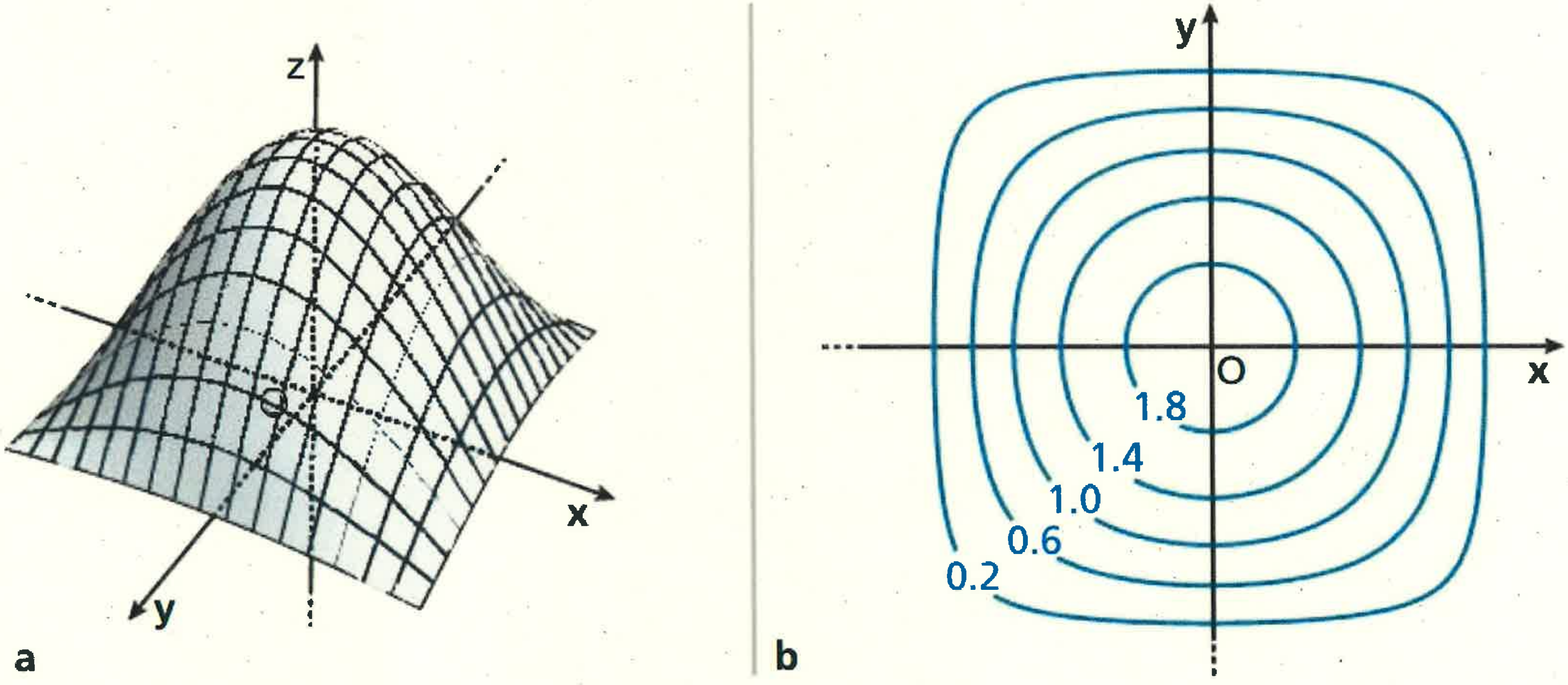
$1^\circ < 0$ D.N. $\Longrightarrow (0,0)$ MAX R.

$2^\circ > 0$





1
1/2
0
−1/2
−1



a b

Un punto stazionario può essere un punto di massimo, di minimo o di sella.

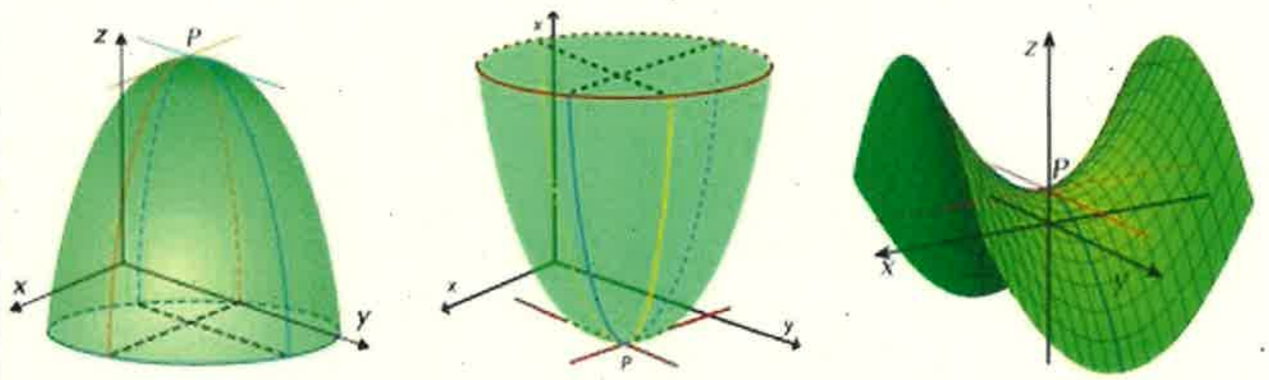


Grafico della funzione $z = xy$.