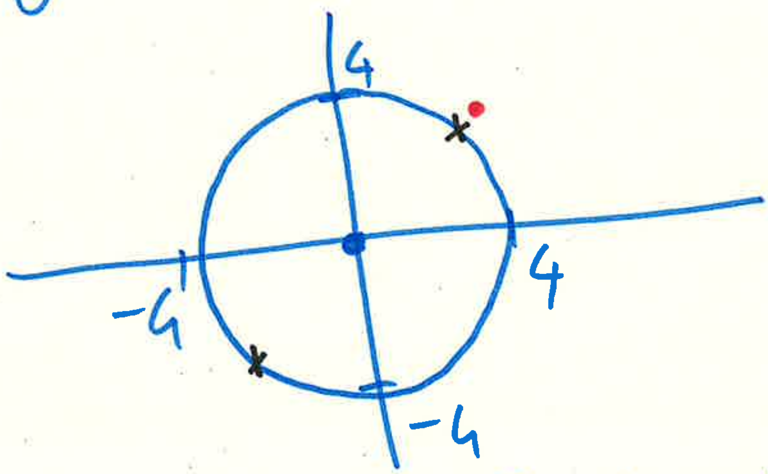
MAT. PER L'EC. TUTORATO 4°

1. OTTIMIZ. VINC. CON VINCOLI DI $=$ 28/10/25

2. OTTIMIZ. VINC. CON VINCOLI DI $\geq$ $\leq$

ES. $f(x,y) = (x-3)^2 + (y-3)^2 - 1$

$$g(x,y) = x^2 + y^2 - 16 = 0$$



$$L(x,y,\lambda) = (x-3)^2 + (y-3)^2 - 1 + \lambda(x^2+y^2-16)$$

$$\begin{cases} 2(x-3) + \lambda 2x = 0 \\ 2(y-3) + \lambda 2y = 0 \\ x^2 + y^2 - 16 = 0 \end{cases}$$

$$\begin{cases} 2x-6+2\lambda x=0 \\ 2y-6+2\lambda y=0 \\ x^2+y^2-16=0 \end{cases}$$

$\lambda \neq -1$

$$\begin{cases} x(2+2\lambda)=6 \rightarrow x=\frac{3}{1+\lambda} \\ y(2+2\lambda)=6 \rightarrow y=\frac{3}{1+\lambda} \end{cases}$$

$$\frac{9}{(1+\lambda)^2}+\frac{9}{(1+\lambda)^2}=16$$

$$18=16(1+\lambda)^2$$

$$\frac{9}{8}=(1+\lambda)^2$$

$$1+\lambda=\pm\sqrt{\frac{9}{8}}$$

$$1+\lambda=\pm\frac{3}{2\sqrt{2}}$$

$$\lambda_1=\frac{3}{2\sqrt{2}}-1$$

$$\lambda_2=-\frac{3}{2\sqrt{2}}-1$$

$$\lambda_1 = \frac{3}{2\sqrt{2}} - 1$$

$$x = \frac{3}{\frac{3}{2\sqrt{2}}} = \frac{\cancel{3} \cdot 2\sqrt{2}}{\cancel{3}} = 2\sqrt{2}$$

$$y = 2\sqrt{2} \qquad (2\sqrt{2}, 2\sqrt{2})$$

$$\lambda_2 = -\frac{3}{2\sqrt{2}} - 1$$

$$x = \frac{3}{-\frac{3}{2\sqrt{2}}} = -2\sqrt{2}$$

$$y = \frac{3}{-\frac{3}{2\sqrt{2}}} = -2\sqrt{2} \qquad (-2\sqrt{2}, -2\sqrt{2})$$

$$f(2\sqrt{2}, 2\sqrt{2}) = (2\sqrt{2} - 3)^2 + (2\sqrt{2} - 3)^2 - 1 = m^\circ = 33 - 24\sqrt{2}$$

$$f(-2\sqrt{2}, -2\sqrt{2}) = m_1^\circ > m^\circ$$

$$33 + 24\sqrt{2}$$

$(2\sqrt{2}, 2\sqrt{2})$ MIN ASS

$(-2\sqrt{2}, -2\sqrt{2})$ MAX ASS.

ES.

$$f(x,y) = x^2 + \frac{y^2}{2}$$

$$g(x,y) = x^2 + y^2 - 6y = 0$$

$$L(x,y,\lambda) = x^2 + \frac{y^2}{2} + \lambda(x^2+y^2-6y)$$

$$\begin{cases} 2x + 2x\lambda = 0 \\ y + 2\lambda y - 6\lambda = 0 \\ x^2 + y^2 - 6y = 0 \end{cases}$$

$$A \begin{cases} x = 0 \\ \dots\dots \\ y^2 - 6y = 0 \end{cases}$$

$$y(y-6) = 0$$

$$B \begin{cases} \lambda = -1 \\ 2x - 2x = 0 \\ y - 2y + 6 = 0 \end{cases}$$

$$B \begin{cases} \lambda = -1 \\ y = 6 \\ x^2 + 36 - 36 = 0 \end{cases}$$

$$\begin{cases} \lambda = -1 \\ y = 6 \\ x = 0 \end{cases}$$

$$A1 \begin{cases} x = 0 \\ y = 0 \\ \lambda = 0 \end{cases} \qquad A2 \begin{cases} x = 0 \\ y = 6 \\ 6 + 6\lambda = 0 \end{cases}$$

$$\lambda = -1$$

$$\overline{H}(x,y,\lambda) = \begin{bmatrix} 0 & 2x & 2y-6 \\ 2x & 2+2\lambda & 0 \\ 2y-6 & 0 & 1+2\lambda \end{bmatrix}$$

$$\overline{H}(0,0,0) = \begin{bmatrix} 0 & 0 & -6 \\ 0 & 2 & 0 \\ -6 & 0 & 1 \end{bmatrix}$$

$$\det \overline{H} = -6(12) < 0 \quad (0,0) \text{ MIN. R.}$$

$$\overline{H}(0,6,-1) = \begin{bmatrix} 0 & 0 & 6 \\ 0 & 0 & 0 \\ 6 & 0 & -1 \end{bmatrix}$$

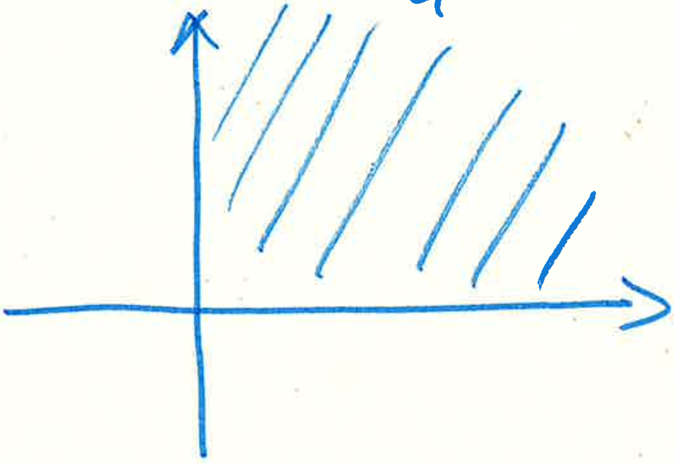
$$\det \overline{H} = 0$$

$(0,0)$ MIN A.

$(0,6)$ MAX A. T. di W.

ES. MAX $f(x,y) = -2x - \frac{1}{4}y^4 + 27y$

$$x \geq 0$$
$$y \geq 0$$



$$\lambda_1 \quad -x \leq 0$$
$$\lambda_2 \quad -y \leq 0$$

$$L(x,y,\lambda_1,\lambda_2) = -2x - \frac{1}{4}y^4 + 27y + \lambda_1 x + \lambda_2 y$$

$$\begin{cases} -2 + \lambda_1 = 0 \\ -y^3 + 27 + \lambda_2 = 0 \\ x \geq 0 \qquad \lambda_1 \cdot x = 0 \\ y \geq 0 \qquad \lambda_2 \cdot y = 0 \\ \lambda_1 \geq 0, \ \lambda_2 \geq 0 \end{cases}$$

$$\begin{cases} \lambda_1 = 2 \\ -y^3 + 27 + \lambda_2 = 0 \\ \lambda_1 \cdot x = 0 \\ \lambda_2 \cdot y = 0 \end{cases}$$

$$A_1 \begin{cases} \lambda_1 = 2 \\ x = 0 \\ \lambda_2 = 0 \\ -y^3 + 27 = 0 \end{cases}$$

$$A_2 \begin{cases} \lambda_1 = 2 \\ x = 0 \\ y = 0 \end{cases}$$

$\lambda_2 = -27$

no

$y^3 = 27$

$y = 3$

$(0, 3, 2, 0)$ ~~$(0, 0, 2, 0)$~~

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$$\bar{H}(x, y, \lambda_1, \lambda_2) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3y^3 \end{bmatrix}$$

$$\det \bar{H} = 1 \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -3y^3 \end{bmatrix}$$

$$= 1 > 0 \Longrightarrow \text{MAX R.}$$

$(0, 3)$ MAX R.

è ASSOL. $f(x,y)$ CONCAVA

$$H(x,y) = \begin{bmatrix} 0 & 0 \\ 0 & -3y^2 \end{bmatrix}$$

SDN