TUTOR. MAT. PER L'ECON. 6°

MA 11/11

1. EQ. DIFF. I ORDINE A VAR. SEPARAB.
2. EQ. DIFF. LINEARI DEL I E II ORDINE A COEFF. COSTANTI
3. EQ. DIFF. LOGISTICA

① VERIFICARE SE LA FUNZ.

$$y(x) = c_1 e^{2x} + c_2$$ È SOLUZ.

DELLA E.D.O. $y'' + 2y' = 0$

$$y'(x) = 2c_1 e^{2x} + 0$$
$$y''(x) = 4c_1 e^{2x}$$

SOSTITUISCO

$$4c_1 e^{2x} + 2 \cdot 2c_1 e^{2x} = 8c_1 e^{2x} \quad \text{NO}$$

RISOLVO $y'' + 2y' = 0$ LIN. II OMOG.

$$\Longrightarrow \lambda^2 + 2\lambda = 0 \quad < \begin{matrix} \lambda = 0 \\ \lambda = -2 \end{matrix}$$

$$y^*_{OM}(x) = c_1 e^{-2x} + c_2 e^{0x} =$$
$$= c_1 e^{-2x} + c_2$$

② $y'(y^2-1)=x$ E.D.O. I A V.S.

$$dy\,(y^2-1)=x\,dx$$

$$\int (y^2-1)\,dy=\int x\,dx$$

$$\frac{y^3}{3}-y=\frac{x^2}{2}+c$$

SOL. ↑ IN FORMA IMPLICITA

③ PB DI CAUCHY E.D.O. I A V.S.

$$\begin{cases} y'=\dfrac{x}{y+1} \\ y(0)=0 \end{cases}$$

$$\int (y+1)\,dy=\int x\,dx$$

$$\frac{y^2}{2}+y=\frac{x^2}{2}+c \quad \text{FORMA IMPLICITA}$$

$$y(0)=0 \Longrightarrow \frac{0^2}{2}+0=\frac{0^2}{2}+c$$

$$\Longrightarrow \boxed{c=0}$$

④ $\begin{cases} y' = \sqrt{y} \\ y(0) = 1 \end{cases}$ E.D.O. I A V.S.

$$2 \cdot \int \frac{1}{2\sqrt{y}}\, dy = \int dx$$

$$2\sqrt{y} = x + c$$

$$y^* = \left(\frac{x+c}{2}\right)^2$$

$$y(0) = 1$$

$$1 = \left(\frac{0+c}{2}\right)^2$$

$$\frac{c^2}{4} = 1 \implies \begin{matrix} c_1 = 2 \\ c_2 = -2 \end{matrix} \quad \underline{\underline{2 \text{ cost.}}}$$

⑤ $y' = \frac{1}{x\sqrt{y}}$ E.D.O. I V.S.

$$\int \sqrt{y}\, dy = \int \frac{1}{x}\, dx$$

$$\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \ln|x| + c$$

$$y^{\frac{3}{2}} = \frac{3}{2} \ln|x| + c$$

$$y^* = \left(\frac{3}{2} \ln|x| + c\right)^{\frac{2}{3}}$$

⑥ $$\begin{cases} y' + 2xy = x \\ y(0) = 1 \end{cases}$$

E.D.O. I LIN.
A COEF. NON
COSTANTI
NON OMOG.

OMOG. ASSOCIATA

V.S.

$$y' + 2xy = 0$$

$$y' = -2xy$$

$$\int \frac{dy}{y} = \int -2x\, dx$$

$$\ln|y| = -\cancel{2}\frac{x^2}{\cancel{2}} + c$$

$$y^*_{OM} = e^{-x^2} \cdot K$$

$$\bar{y} = ax + b \qquad \bar{y}' = a$$

$$a + 2x(ax+b) = x$$

$$a + 2ax^2 + 2bx = x$$

$$\begin{cases} a = 0 \\ 2b = 1 \end{cases} \Rightarrow \bar{y} = \frac{1}{2}$$

$$y^*(x) = K e^{-x^2} + \frac{1}{2}$$

$$y(0) = 1 \qquad 1 = K + \frac{1}{2} \Rightarrow K = \frac{1}{2}$$

⑦ $\begin{cases} y' = e^{x+y} \\ y(0) = 0 \end{cases}$ E.D.O. I A V.S.

⑧ $\begin{cases} y' + \dfrac{y}{x} = e^{x} \\ y(-1) = 0 \end{cases}$

⑨ $\begin{cases} y' = \dfrac{x(y^2-1)}{y(x^2-1)} \\ y(0) = -2 \end{cases}$

LINEARI I ORDINE

$$\begin{cases} 2y' - y = -4 \\ y(1) = 1 \end{cases}$$

OMOG. $2y' - y = 0$

$$2\lambda - 1 = 0$$

$$\lambda = \frac{1}{2}$$

$$y^*_{OM}(x) = c\, e^{\frac{1}{2}x}$$

$\bar{y} = K$ $\quad \bar{y}' = 0 \quad 2\cdot 0 - K = -4$

$\Longrightarrow K = 4 \quad \Longrightarrow y^*(x) = c\,e^{\frac{1}{2}x} + 4$

$y(1) = 1 \quad 1 = c\,e^{1/2} + 4 \Longrightarrow$

$$c\, e^{1/2} = -3 \qquad c = -3\, e^{-1/2} = = -\frac{3}{\sqrt{e}}$$

$$y^*(x) = -\frac{3}{\sqrt{e}}\, e^{\frac{1}{2}x} + 4$$

EQ. LIN. II ORDINE A COEFF. COST.

(10) $u'' - u = 0$

$$\lambda^2 - 1 = 0 \quad \begin{cases} \lambda_1 = 1 \\ \lambda_2 = -1 \end{cases}$$

$$u^*(t) = c_1 e^{t} + c_2 e^{-t}$$

(11) $\begin{cases} y'' - 10y = 0 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

$$\lambda^2 - 10 = 0 \quad \begin{cases} \lambda_1 = \sqrt{10} \\ \lambda_2 = -\sqrt{10} \end{cases}$$

$$y^*(x) = c_1 e^{\sqrt{10}x} + c_2 e^{-\sqrt{10}x}$$

$$y(0) = 1 \implies 1 = c_1 + c_2$$

$$y'(x) = \sqrt{10}\, c_1 e^{\sqrt{10}x} - \sqrt{10}\, c_2 e^{-\sqrt{10}x}$$

$$y'(0) = \sqrt{10}\, c_1 - \sqrt{10}\, c_2 = 1$$

$$\begin{cases} c_1 + c_2 = 1 \\ \sqrt{10}\, c_1 - \sqrt{10}\, c_2 = 1 \end{cases} \qquad \begin{cases} c_2 = 1 - c_1 \\ \sqrt{10}\, c_1 - \sqrt{10} + \sqrt{10}\, c_1 = 1 \end{cases}$$

$$\begin{cases} c_2 = 1 - c_1 \\ 2\sqrt{10}\, c_1 = 1 + \sqrt{10} \end{cases}$$

$$\begin{cases} c_1 = \dfrac{1+\sqrt{10}}{2\sqrt{10}} \\ c_2 = 1 - \dfrac{1+\sqrt{10}}{2\sqrt{10}} = \dfrac{2\sqrt{10} - 1 - \sqrt{10}}{2\sqrt{10}} = \dfrac{\sqrt{10}-1}{2\sqrt{10}} \end{cases}$$

(12) $y'' - 10y = -10x^2$

$$\lambda^2 - 10 = 0 \quad \begin{cases} \lambda_1 = \sqrt{10} \\ \lambda_2 = -\sqrt{10} \end{cases}$$

$$y^*_{om}(x) = c_1 e^{\sqrt{10}x} + c_2 e^{-\sqrt{10}x}$$

$$\bar{y}(x) = ax^2 + bx + c$$

$$\bar{y}' = 2ax + b$$

$$\bar{y}'' = 2a$$

$$2a - 10(ax^2 + bx + c) = -10x^2$$

$$2a - 10ax^2 - 10bx - 10c = -10x^2$$

$$2a - 10ax^2$$

$$\begin{cases} -10a = -10 \\ -10b = 0 \\ 2a - 10c = 0 \end{cases} \qquad \begin{cases} a = 1 \\ b = 0 \\ 2 - 10c = 0 \end{cases}$$

$$\downarrow c = \frac{1}{5}$$

$$\bar{y} = x^2 + \frac{1}{5}$$

(13) $y'' + y' - 3y = e^x$

$\lambda^2 + \lambda - 3 = 0$

$\lambda_{1,2} = \frac{-1 \pm \sqrt{1+12}}{2} = \frac{-1 \pm \sqrt{13}}{2}$

$y^*_{OH}(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

$$\left.\begin{aligned} \bar{y}(x) &= K e^x \\ \bar{y}' &= K e^x \\ \bar{y}'' &= K e^x \end{aligned}\right\} \Rightarrow K e^x + K e^x - 3K e^x = e^x$$

$K + K - 3K = 1$

$-K = 1$

$K = -1$

$y^*(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} - e^x$

(9) $\begin{cases} y' = \dfrac{x(y^2-1)}{y(x^2-1)} \\ y(0) = -2 \end{cases}$

$\frac{1}{2}\int \frac{2y}{y^2-1}\,dy = \frac{1}{2}\int \frac{2x}{x^2-1}\,dx$

$$\int \frac{2y}{y^2-1}\,dy = \int \frac{2x}{x^2-1}\,dx$$

$$\ln|y^2-1| = \ln|x^2-1| + C$$

$$|y^2-1| = |x^2-1| \cdot K$$

$$y^2-1 = (x^2-1)K$$

$$y^2 = (x^2-1)K+1$$

$$y = -\sqrt{K(x^2-1)+1}$$

$$+2 = +\sqrt{-K+1}$$

$$4 = -K+1 \Rightarrow \boxed{K=-3}$$