M. E. TUTOR. 7° 25/11/25

1. OTTIMIZ. VINC. CON $\leq$, $\geq$,
2. EQ. DIFF.
3. COMPITI C. M.

1 $f(x,y) = 3x + 3xy - 12y - 3x^4 - 6y^2$

$$E = \left\{ (x,y) \in \mathbb{R}^2 \mid -1 \leq x \leq 0 \wedge -x-1 \leq y \leq 0 \right\}$$

P. CRITICI

$$\begin{cases} 3 + 3y - 12x^3 = 0 \\ 3x - 12 - 12y = 0 \end{cases}$$

$$\begin{cases} 1 + y - 4x^3 = 0 \\ x - 4 - 4y = 0 \end{cases}$$

$$\begin{cases} \cancel{1} + \frac{x}{4} - \cancel{1} - 4x^3 = 0 \\ y = \frac{x}{4} - 1 \end{cases}$$

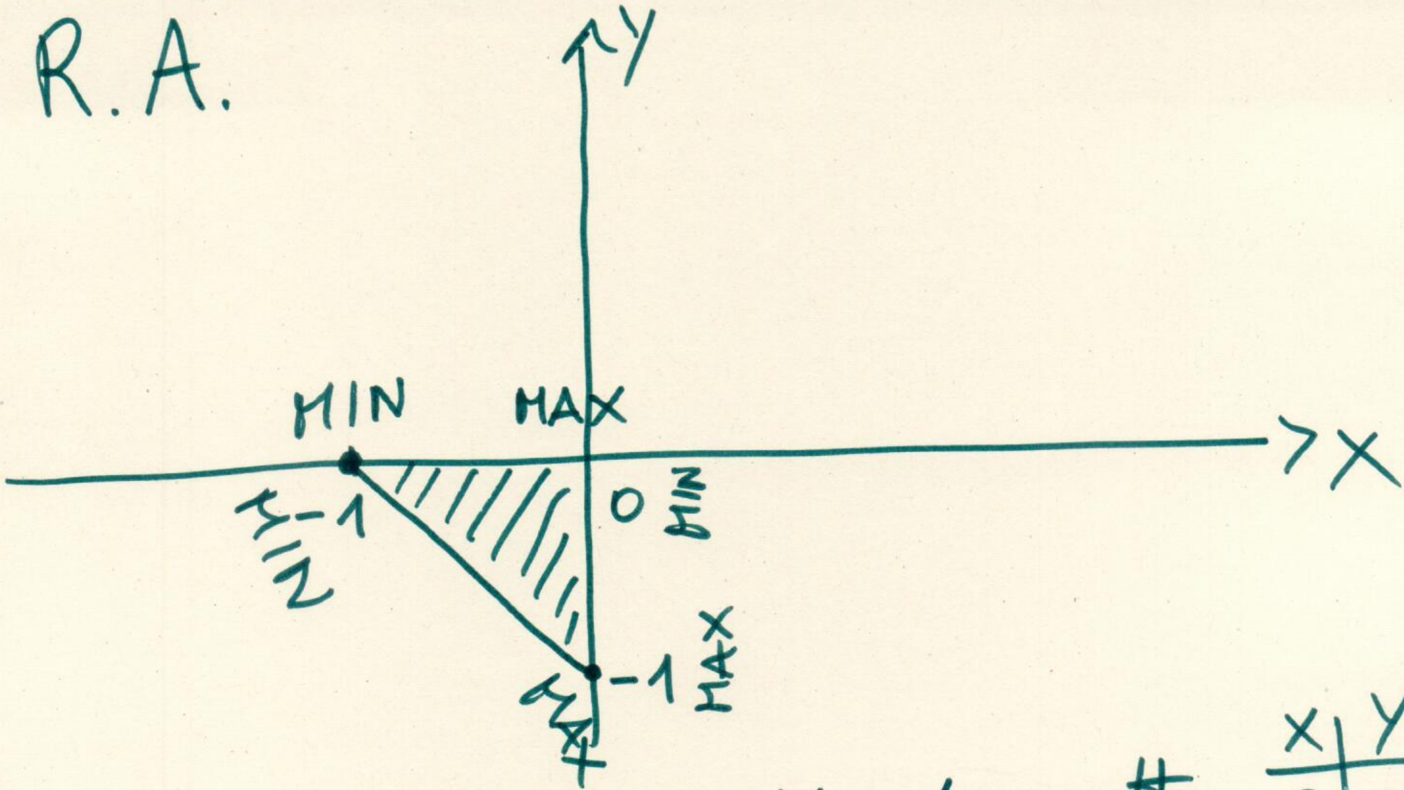
$$\begin{cases} x - 16x^3 = 0 \\ \end{cases}$$

$$x(1 - 16x^2) = 0$$

1) $x = 0 \qquad y = -1$

1 2) $x = \pm \frac{1}{4} \qquad y = \pm \frac{1}{16} - 1$ $\begin{cases} -\frac{15}{16} \\ -\frac{17}{16} \end{cases}$

R.A.



X	Y
0	−1
−1	0

$y \leq 0 \qquad y \geq -x-1$ retta

$(0,-1)$ FRONTIERA

$\left(\frac{1}{4}, -\frac{15}{16}\right) \notin$ R.A.

$\left(-\frac{1}{4}, -\frac{17}{16}\right) \notin$ R.A.

STUDIO LA FRONTIERA

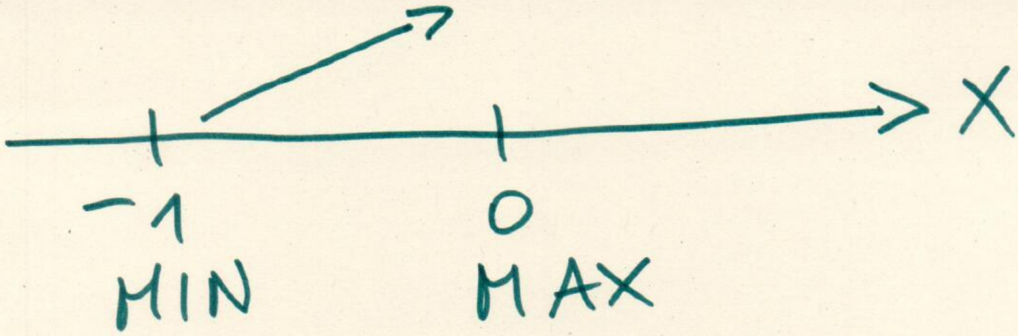
I: $y = 0 \quad -1 \leq x \leq 0$

$f(x,0) = 3x - 3x^4$

$f' = 3 - 12x^3 \geq 0$

$1 - 4x^3 \geq 0$

$x^3 \leq \frac{1}{4} \qquad x \leq \frac{1}{\sqrt[3]{4}}$

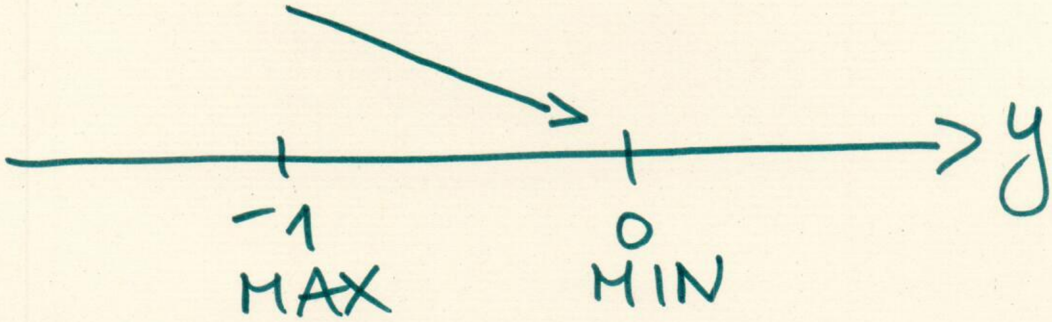


$\underline{II}$: $x = 0 \quad -1 \leq y \leq 0$

$f(0,y) = -12y - 6y^2$

$f' = -12 - 12y \geq 0$

$-1 - y \geq 0 \qquad y \leq -1$



$\underline{III}$: $y = -x - 1 \quad -1 \leq x \leq 0$

$f(x,-x-1) = 3x + 3x(-x-1) -$
$-12(-x-1) - 3x^4 - 6(-x-1)^2 =$
$= \cancel{3x} - 3x^2 - \cancel{3x} + 12x + 12 - 3x^4 -$
$-6(x^2 + 2x + 1) =$
$= \underline{-3x^2} + \cancel{12x} + 12 - 3x^4 \underline{-6x^2} - \cancel{12x}$
$-6 = -3x^4 - 9x^2 + 6$

$$f' = -12x^3 - 18x \geq 0$$

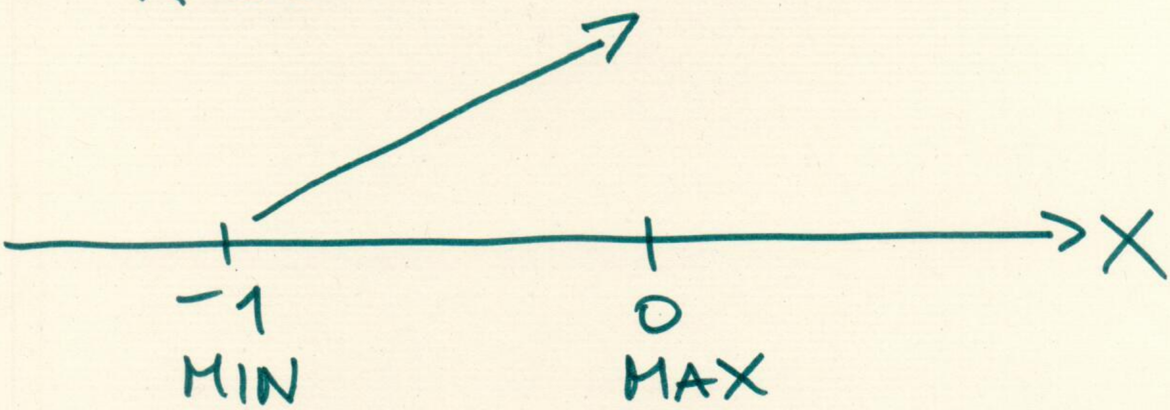
$$-2x^3 - 3x \geq 0$$

$$2x^3 + 3x \leq 0$$

$$x(2x^2+3) \leq 0$$

$+$

$x \leq 0$



$\Longrightarrow$ $(-1,0)$ MIN ASS.

$(0,-1)$ MAX ASS.

$$f(-1,0) = -6$$

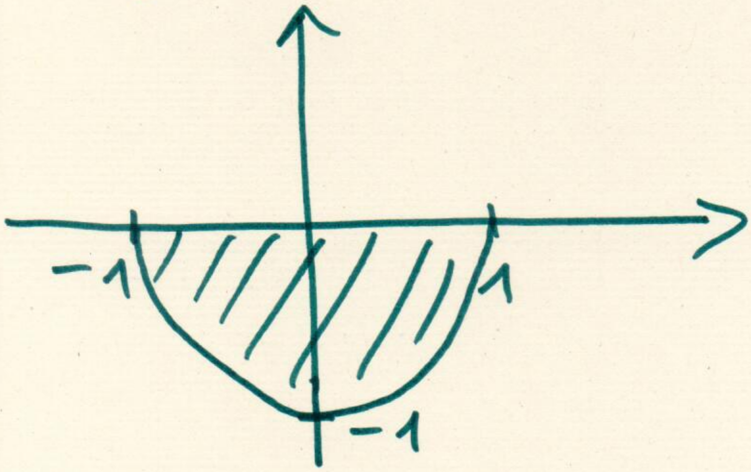
$$f(0,-1) = +6$$

ES. $f(x,y) = 6x^2 + 3y^4 + 12x - 3y - 3xy + 1$

in E di prima

MAX/MIN $f(x,y) = 4x^2y + y^3 - 4y$

$$E = \{(x,y) \in \mathbb{R}^2 \mid y \le 0,\ x^2+y^2 \le 1\}$$



EQ. DIFF. DEL I ORDINE

$y' + e^x y = 3e^x$ NON OMOG.

$y' + e^x y = 0$ OMOG.

$y' = -e^x y$ A VAR. SEP.

$$\int \frac{dy}{y} = \int -e^x\, dx$$

$$\ln|y| = -e^x + c$$

$$y^*_{om} = e^{-e^x + c} = K e^{-e^x}$$

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$$\bar{y} = K e^{x}$$

$$\bar{y}' = K e^{x}$$

$$K e^{x} + e^{x} \cdot K e^{x} = 3 e^{x}$$

$$K e^{x} + K e^{2x} = 3 e^{x} \quad \text{No!}$$

RISCRIVO:

$$e^{-x} y' + y = 3$$

$$\bar{y} = \cancel{K}h \qquad \bar{y}' = 0$$

$$e^{-x} \cdot 0 + \cancel{K}h = 3 \Longrightarrow \boxed{\cancel{K}h = 3}$$

$$y^{*}(x) = e^{-e^{x}} \cdot K + 3$$

$$y'' + y' + \frac{1}{4} y = 0 \qquad \text{LIN. } 2^{\circ} \text{ OMOG.}$$

$$\lambda^{2} + \lambda + \frac{1}{4} = 0$$

$$4\lambda^{2} + 4\lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{-4 \pm \sqrt{16 - 16}}{8} = \frac{-4}{8} = -\frac{1}{2}$$

$$\boxed{\Delta = 0}$$

6

$$y^*_{OH}(x) = c_1 e^{-\frac{1}{2}x} + c_2 x e^{-\frac{1}{2}x}$$

$$\begin{cases} y'' + y' - 6y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} \begin{cases} 2 \\ -3 \end{cases}$$

$$y^*(x) = c_1 e^{2x} + c_2 e^{-3x}$$

$$y(0) = c_1 + c_2 = 1$$

$$y'(x) = 2c_1 e^{2x} - 3c_2 e^{-3x}$$

$$y'(0) = 2c_1 - 3c_2 = 0$$

$$\begin{cases} c_1 + c_2 = 1 \\ 2c_1 - 3c_2 = 0 \end{cases} \qquad \begin{cases} c_2 = 1 - c_1 \\ 2c_1 - 3 + 3c_1 = 0 \end{cases}$$

$$\begin{cases} \\ 5c_1 = 3 \end{cases} \qquad \begin{cases} c_1 = \frac{3}{5} \\ c_2 = 1 - \frac{3}{5} = \frac{2}{5} \end{cases}$$

$$y'' - 4y = 3e^{2x} + 4e^{-x}$$

E.D. 2°
NON OMOG.

$$y'' - 4y = 0$$

$$\lambda^2 - 4 = 0 \Rightarrow \lambda_1 = 2$$
$$\lambda_2 = -2$$

$$y^*_{om}(x) = c_1 e^{2x} + c_2 e^{-2x}$$

$$\bar{y}(x) = Kxe^{2x} + he^{-x}$$

$$\bar{y}' = K(e^{2x} + 2xe^{2x}) + he^{-x}(-1)$$
$$= Ke^{2x} + 2Kxe^{2x} - he^{-x}$$

$$\bar{y}'' = 2Ke^{2x} + 2K(e^{2x} + 2xe^{2x}) - he^{-x}(-1)$$

$$\underline{2Ke^{2x}} + \underline{2Ke^{2x}} + 4Kxe^{2x} + \underline{he^{-x}} = 3e^{2x} + \underline{4e^{-x}}$$

$$\Rightarrow h = 4$$ ma non trovo K

$$\bar{y}_1 = Ke^{2x} \quad \bar{y}_1' = 2Ke^{2x}$$
$$\bar{y}_1'' = 4Ke^{2x}$$

$4K = 3 \quad K = \frac{3}{4}$

$$y^*(x) = c_1 e^{2x} + c_2 e^{-2x} + 4e^{-x} + \frac{3}{4} e^{2x}$$

1

$$A = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 10 & -6 \\ -9 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$A^2 + 4 \cdot A^{-1} = \begin{bmatrix} 10 & -6 \\ -9 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -8 \\ -12 & 5 \end{bmatrix}$$ c

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2

$$B = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 2 & 4 \\ 0 & -h & -K \end{bmatrix}$$

$$\det B = 1(-2K+4h) + 1 \cdot (-3K+h) =$$

$$= -2K + 4h - 3K + h =$$

$$= -5K + 5h = 0 \; ?$$

$$h = K$$

$$K \neq h \qquad h \neq K$$

3

$$S = \{ \underline{x} \in \mathbb{R}^5 \mid 3x_1 + x_3 - 2x_4 = 0 \wedge x_2 = 0 \}$$

$$\begin{cases} 3x_1 + x_3 - 2x_4 = 0 \\ x_2 = 0 \end{cases}$$

$$\begin{cases} x_3 = -3x_1 + 2x_4 \\ x_2 = 0 \end{cases}$$

$$\underline{x} = \begin{bmatrix} x_1 \\ 0 \\ -3x_1+2x_4 \\ x_4 \\ x_5 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ -3 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$\dim S = 3$

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