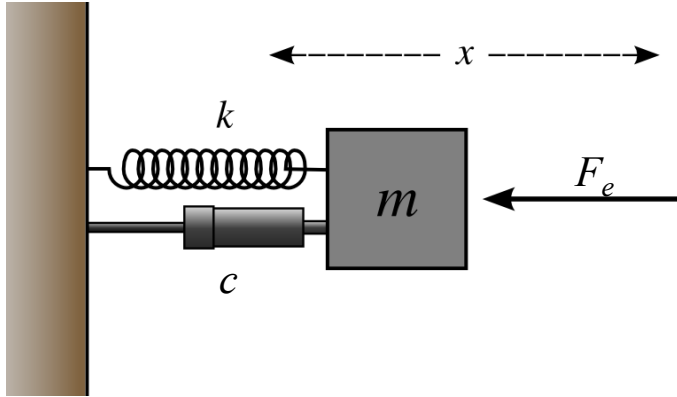


Mass-spring-damper system



Without external forces:

$$F_s = -kx$$

$$F_d = -cv = -c\dot{x}$$

$$\Sigma F = ma = m\ddot{x}$$

Equilibrium condition

$$m\ddot{x} + c\dot{x} + kx = 0$$

If an external acceleration acts on the mass, F_e adds up: $m\ddot{x} + c\dot{x} + kx - F_e = 0$

From a time-domain differential equation, to a linear equation in the domain of Laplace transforms:

$$mX(s)s^2 + cX(s)s + kX(s) = -ma \quad \rightarrow \quad \frac{X(s)}{a} = -\frac{1}{s^2 + \frac{c}{m}s + \frac{k}{m}} = -\frac{m}{k} \cdot \frac{1}{1 + 2 \cdot \frac{\xi}{\omega_0} s + \frac{s^2}{\omega_0^2}}$$

At low frequencies ($\omega \ll \omega_0$):

$$x(t) \rightarrow -\frac{m}{k} \cdot a$$

Transfer Function between force and displacement

- Dependence on two characteristic parameters of the system, which are the resonance (angular) **frequency** ω_0 and the **quality factor** Q .

$$mX(s)s^2 + cX(s)s + kX(s) = F_{ext}(s)$$

$$X(s) = \frac{1/m}{s^2 + c/m s + k/m} F_{ext}(s)$$

$$T_{XF}(s) = \frac{X(s)}{F_{ext}(s)} = \frac{1/m}{s^2 + c/m s + k/m}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

*[eigenvalue solution for $b=0$]
 $m s^2 + k = 0 \rightarrow -m\omega_0^2 + k = 0 \rightarrow \omega_0 = k/m$*

$$Q = \omega_0 \frac{m}{b}$$

[amplification factor at resonance in underdamped systems]

$$T_{XF}(s) = \frac{X(s)}{F_{ext}(s)} = \frac{1/m}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

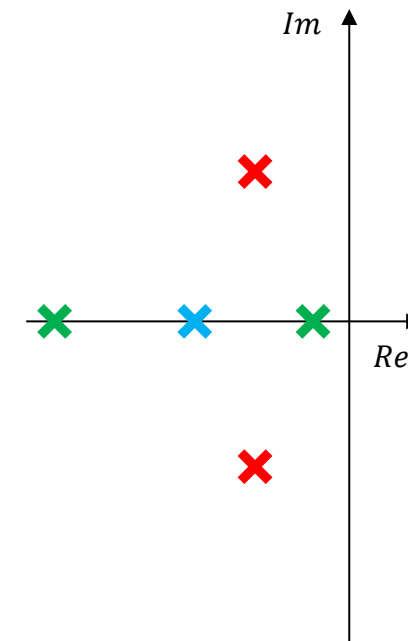
Poles of the Transfer Function

- The system has a **low-pass 2nd order transfer function** with **two singularities** which can be either complex conjugate, or coincident real, or split real poles.

$$T_{XF}(s) = \frac{1/m}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\Delta = \left(\frac{Q}{\omega_0}\right)^2 - 4\omega_0^2 = \left(\frac{1}{Q^2} - 4\right)\omega_0^2$$

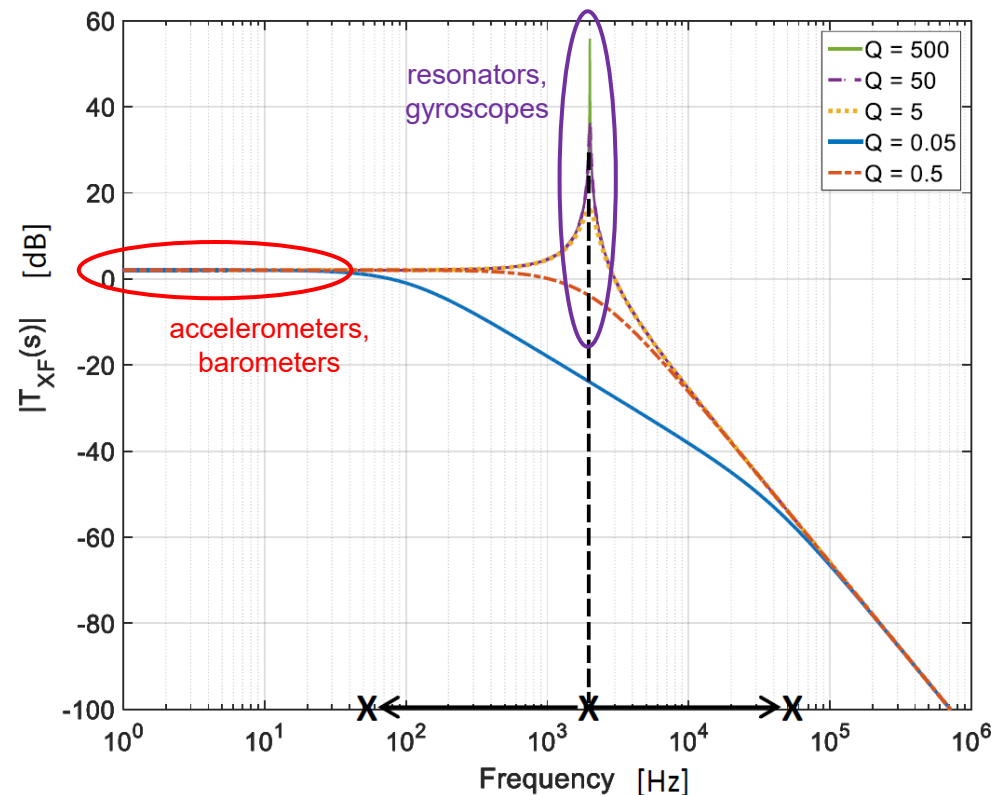
$$\frac{1}{Q^2} \geq 4$$



- split real poles:** $\Delta > 0 \rightarrow Q < 0.5$
- coincident real poles:** $\Delta = 0 \rightarrow Q = 0.5$
- complex conjugate poles:** $\Delta < 0 \rightarrow Q > 0.5$

Transfer Function modulus function of Q

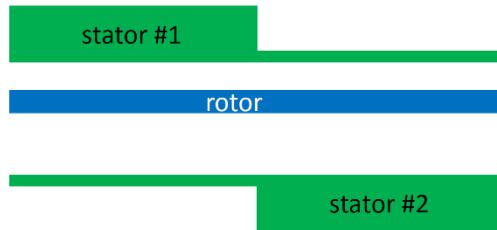
- MEMS devices operate in different regions of the transfer function:
 - accelerometers**, **microphones** and **pressure sensors** typically operate under forces occurring far **before resonance**; as we will see, they usually have relatively low Q factors (typically < 10 , or < 1);
 - resonators** (including the gyroscopes drive) operate **at resonance** (few tens kHz to few MHz) and require **high quality factors** (typically few thousand to ten thousand);
 - other **sensors** (gyroscopes, magnetometers...) operate **slightly before the resonance** frequency (off-resonance or mode-split operation), due to a modulation in frequency of the applied forces;
 - no devices operate beyond the resonance frequency.



Capacitive sensing configuration

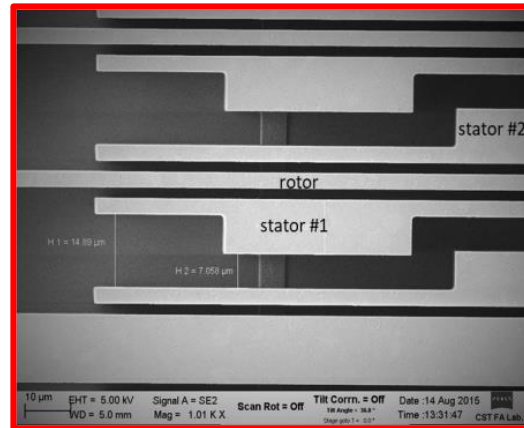
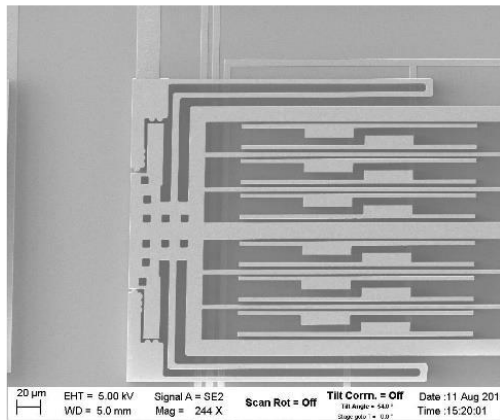
- Moving parts of microsystems are called “rotors” (even if they do not rotate).
- Fixed electrodes are, again from the analogy above, called **stators**.

Example 1: one parallel-plate (*gap varying*) differential transduction



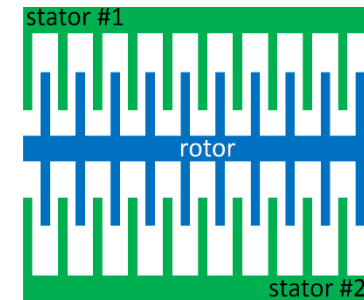
$$C_1 = \epsilon_0 \frac{AN}{g - x}$$

$$C_2 = \epsilon_0 \frac{AN}{g + x}$$



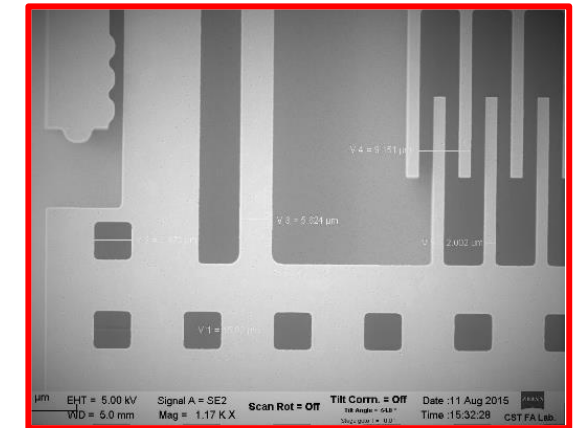
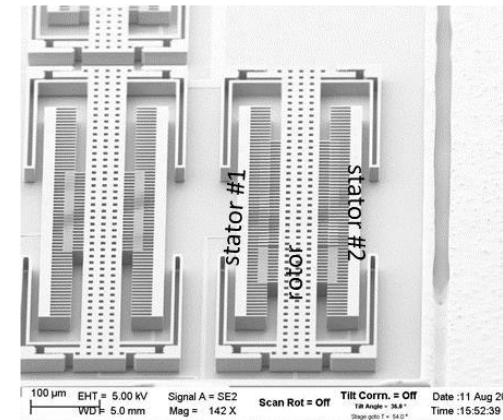
Motion direction ↑

Example 2: N comb-finger (*area varying*) differential transductions



$$C_1 = \frac{2\epsilon_0 h(x_0 + x)N}{g}$$

$$C_2 = \frac{2\epsilon_0 h(x_0 - x)N}{g}$$

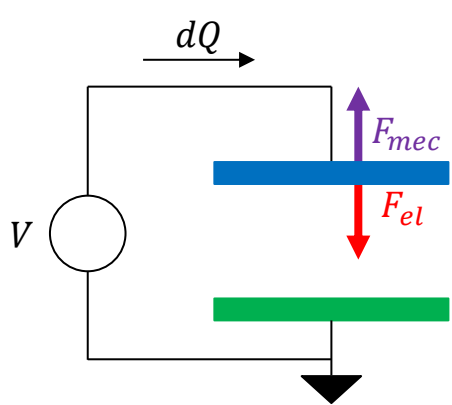


Electrostatic force between two parallel plates

- In equilibrium conditions, electrostatic forces balance the mechanical force
- Any variation in the energy E_c stored in the capacitor is given by the work W done by the mechanical or electrical forces $\rightarrow dE_c = dW_{mech} + dW_{elec}$
- **Single-ended configuration**

If a voltage is applied, a charge Q arises on the capacitor plates, as $Q = CV$.

Charges of opposite sign on the plates generate an electric field and thus an electrostatic force which is **always attractive**.



$$\begin{cases} dE_c = d\left(\frac{1}{2}CV^2\right) = \frac{V^2}{2}dC \\ dW_{mech} = -F_{mech} \\ dW_{elec} = VdQ = Vd(CV) = V(CdV + VdC) = V^2dC \end{cases}$$

$$F_{mech}dx = \frac{V^2}{2}dC$$

(electrostatic energy stored in a capacitor)

(work done for a displacement dx)

(electrostatic work done to change the voltage over a capacitor)

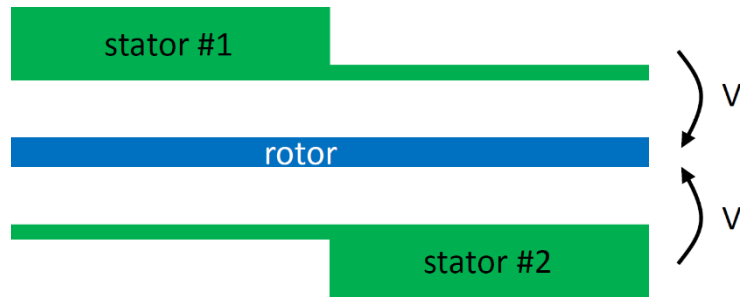
$$C = \epsilon_0 \frac{AN}{g - x} \xrightarrow{\text{derivative}} dC = -\epsilon_0 \frac{AN}{(g - x)^2} dx$$

$$F_{elec} = -F_{mech} = \frac{V^2}{2} \frac{\epsilon_0 AN}{(g - x)^2} dx$$

$$m\ddot{x} + c\dot{x} + kx = F_{ext} + F_{elec}$$

Differential configuration

- The situation can be easily extended by considering the differential configuration with opposite forces:



$$F_{elec} = \frac{V^2}{2} \frac{\epsilon_0 AN}{(g - x)^2} dx - \frac{V^2}{2} \frac{\epsilon_0 AN}{(g + x)^2} dx$$

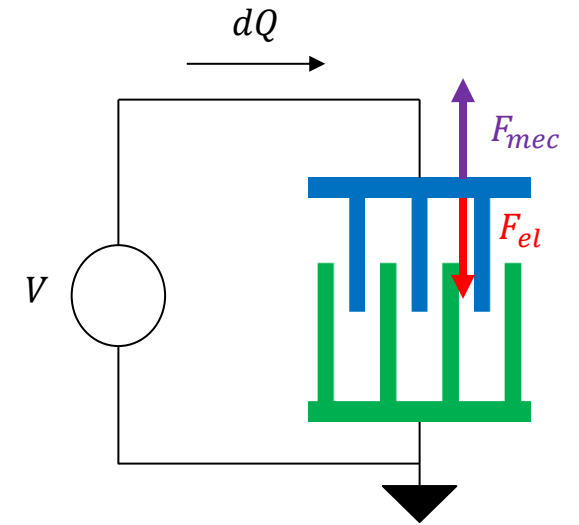
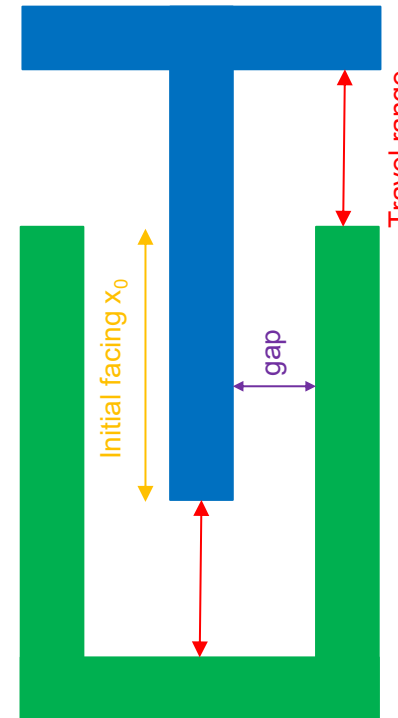
- Note that the applied force is a **function of the displacement** itself, both in single-ended and in differential configurations

Electrostatic force between comb fingers

- In equilibrium conditions, electrostatic forces balance the mechanical force
- Comb-finger capacitors are of the area-varying type:
 - the facing area is $A = h(x_0+x)$, h being the process height and x_0 the facing length at rest
 - for N fingers on the rotor, a factor 2 should be added as the facing is on two sides

$$C = \frac{2\varepsilon_0 AN}{g} = \frac{2\varepsilon_0 h(x + x_0)N}{g} \xrightarrow{\text{derivative}} dC = \frac{2\varepsilon_0 hN}{g} dx$$

$$F_{elec} = \frac{V^2}{2} \frac{2\varepsilon_0 hN}{x} dx = \frac{V^2 \varepsilon_0 hN}{x} dx$$



$$dE_C = dW_{mec_h} + dW_{elec}$$

$$F_{mech} = -\frac{V^2}{2} dC$$



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DEGLI STUDI
DI BERGAMO

MEMS devices – Accelerometer and Gyroscope

A.Y. 2025/2026

RELATORI

Elisa Riceputi

SEDE

University of Bergamo, Dalmine

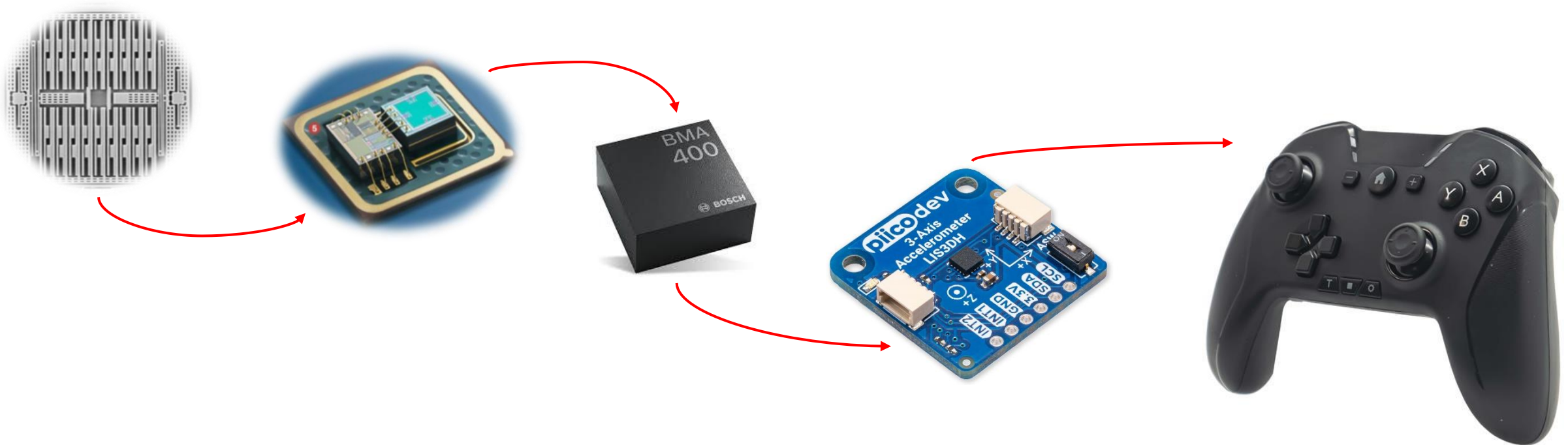
DATA

09/03/2026

Motivations and goals

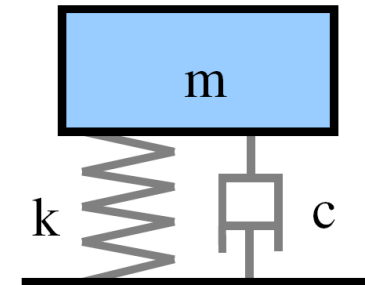
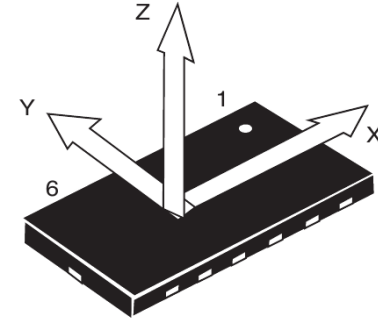
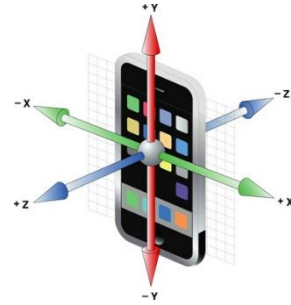
- Accelerometers are the “most intuitive” MEMS devices and were the first MEMS sensors to be commercialized in automotive (airbag activation, 1993), electrical appliances (washing machines, 2001), and consumer markets (gaming joysticks, 2006):

acceleration → inertial force → displacement → capacitive (or other) variation

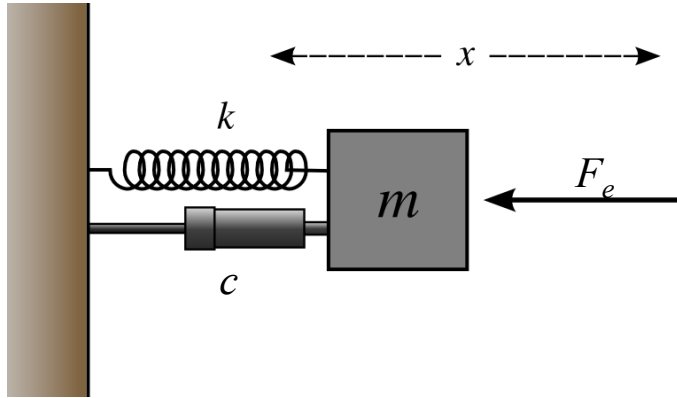


Accelerometer

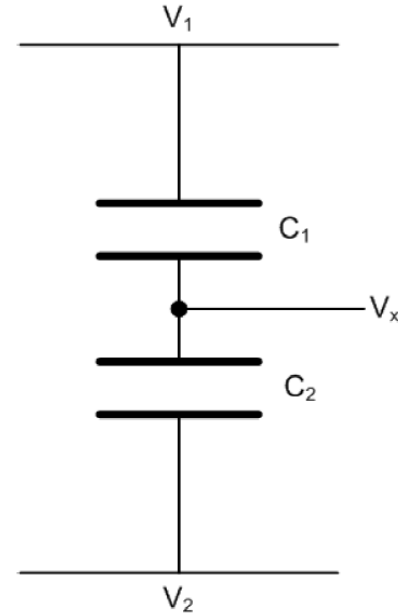
- Measures **proper** linear accelerations
 - Gravity g
 - External accelerations
- Application fields
 - Inertial navigation systems
 - Vibration monitoring
 - Smartphone
- All accelerometers (MEMS and not) are based on the **mass-spring-damper** system



Mass-spring-damper + capacitive displacement measurement



$$x(t) \rightarrow -\frac{m}{k} \cdot a$$

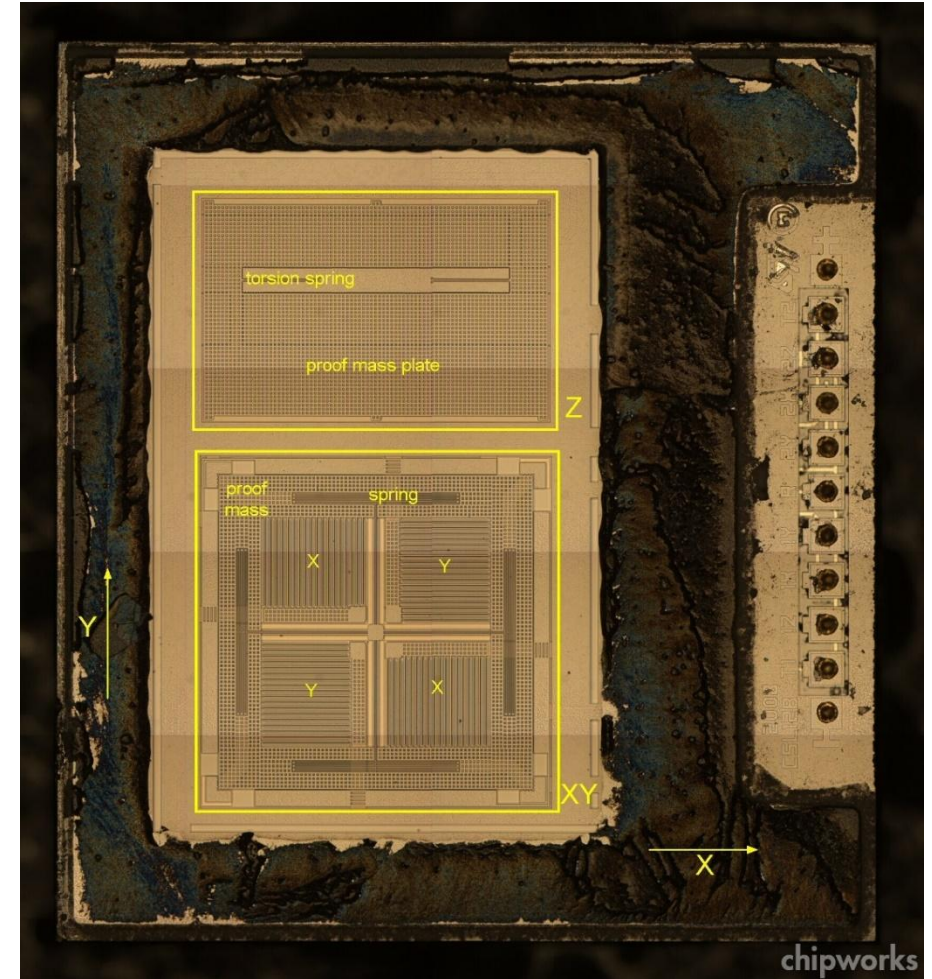


$$V_x \propto -\frac{V_0}{g} x$$

$$V_x \propto \frac{V_0}{g} \cdot \frac{m}{k} \cdot a$$

3D MEMS accelerometer

- A first mobile mass is anchored to the substrate by means of springs (X and Y in figure) → movements are constrained to lie on the XY plane
- The **mass displacement along X and Y** directions are measured by means of capacitive combs
- A second mass is anchored to the substrate by means of a spring → upward and downward constrained movement (Z axis)
- Mass and substrate (both made by silicon) form a capacitor → **mass vertical displacement** measured capacitively

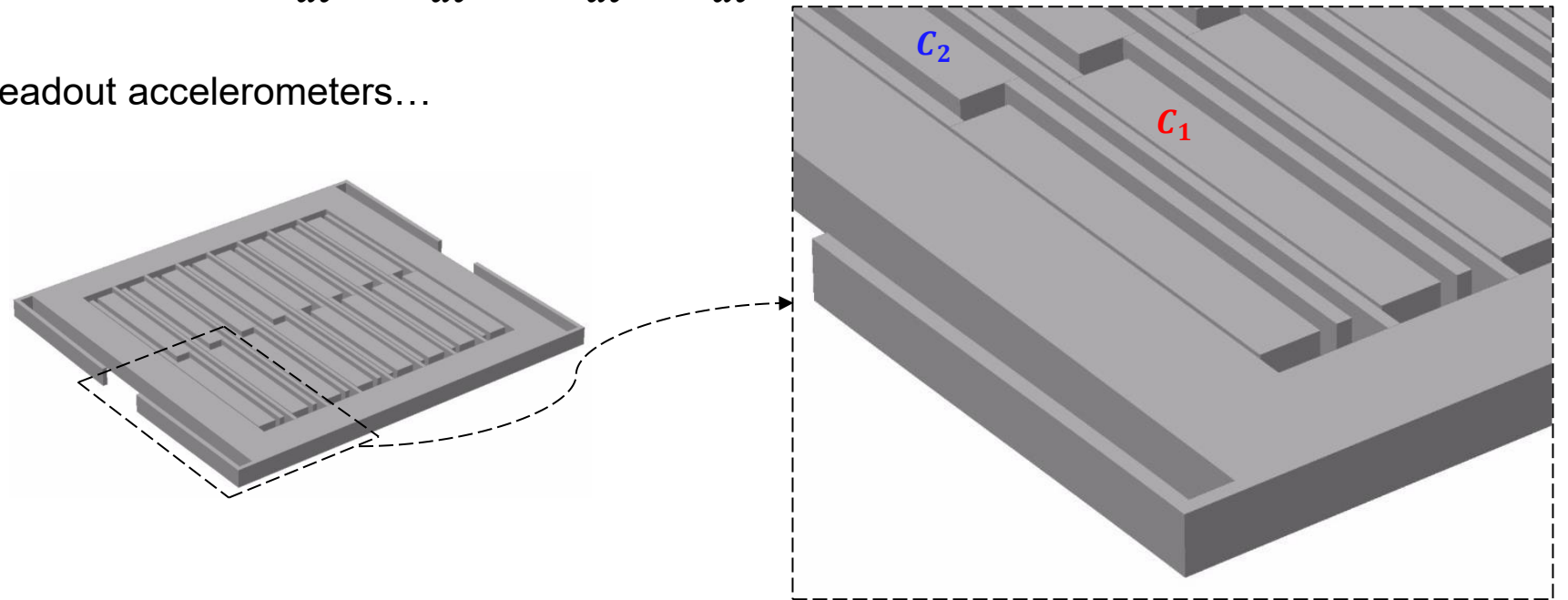


Readout approaches for MEMS accelerometers

- In order to readout a signal from an accelerometer, we need to **capacitively measure its displacement** to recover the information on accelerations.
- Note: the capacitance is **variable!** The general expression of the **current** through such a capacitor, biased by a generic voltage V , is:

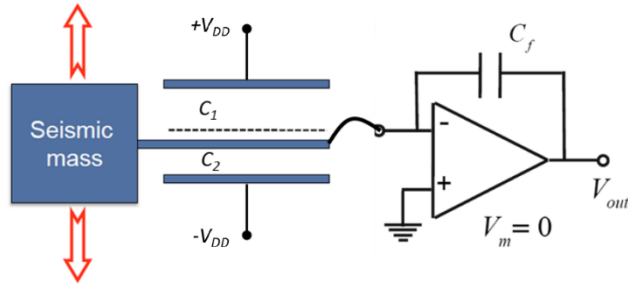
$$i_c = \frac{dQ}{dt} = \frac{c(CV)}{dt} = C \frac{dV}{dt} + V \frac{dC}{dt}$$

- There is not a single way to readout accelerometers...



Parallel-plate (PP) readout configurations

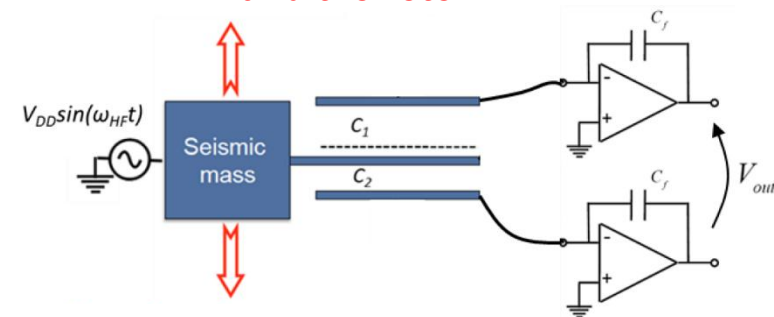
Application of a constant DC voltage $\pm V_{DD}$ between each stator and the rotor



$$i_c = C \frac{dV}{dt} + V \frac{dC}{dt} = V \frac{dC}{dt}$$

This term is proportional to the capacitance derivative: not to lose the stationary (DC) value of the acceleration, **we need to integrate the signal**. The feedback impedance is capacitive.

Application of a modulated AC voltage between each stator and the rotor



$$i_c = C \frac{dV}{dt} + V \frac{dC}{dt} = C \frac{dV}{dt}$$

This term dominates for very-high frequency AC signals (the derivative is indeed, proportional to the AC frequency). Any feedback impedance is okay.

- For didactic goals the use of a DC voltage is initially appropriate;
 - so we will start by analyzing an axel in the condition given in the left figure (anyway note that the final expression of the sensitivity is the same for both configurations!);
 - initial assumption: no bias current of the OpAmp (otherwise this is continuously integrated by Cf, leading to output voltage saturation!)

Example of a differential capacitive sensing configuration

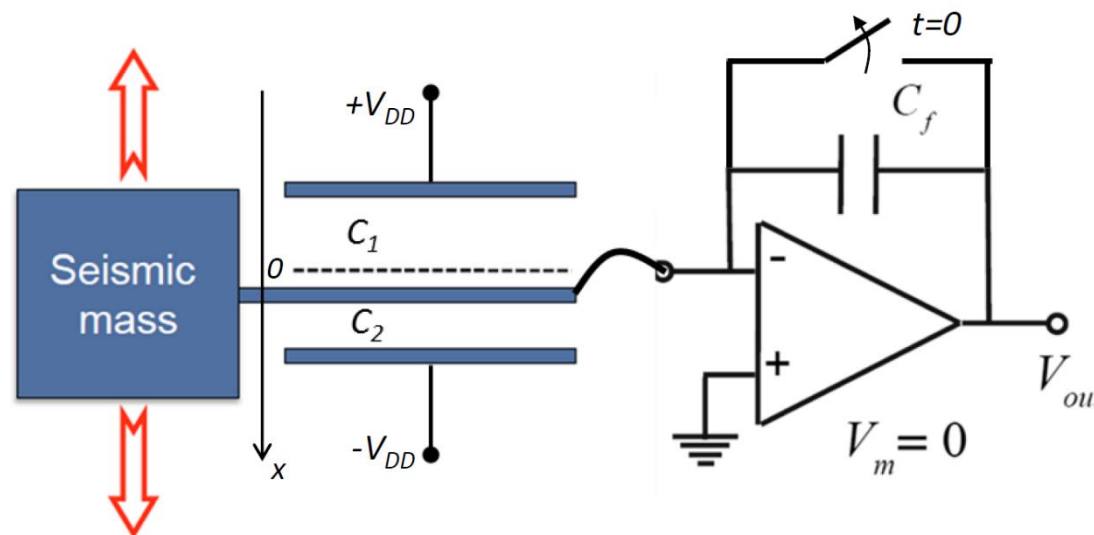
- Assume that the accelerometer is initially in the rest position and that C_f is initially discharged:

$$V_m = 0$$

$$Q_f = 0$$

$$C_1 = C_2 = \frac{\epsilon_0 AN}{g} = C_0$$

- If the position changes by a quantity x , we have opposite changes in the value of the differential capacitances:
 - for $x > 0$ (as in the figure) $\rightarrow C_1$ decreases and C_2 increases



$$\begin{cases} C_1 = \frac{\epsilon_0 AN}{g + x} \\ C_2 = \frac{\epsilon_0 AN}{g - x} \end{cases}$$

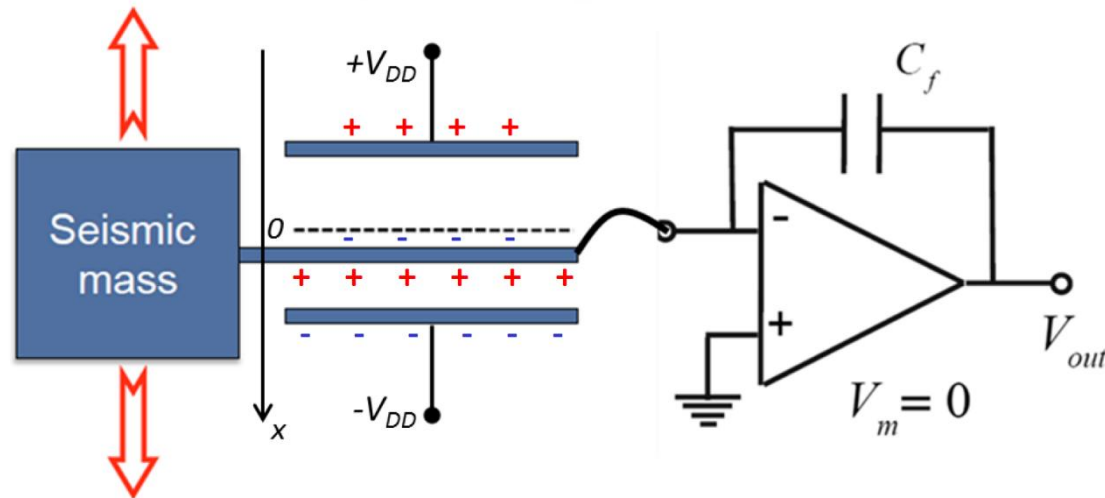
Charge transfer – imbalance

- The **charge on the rotor** has the sign in the figure and is given by:

$$Q_1 = -C_1 V_{DD}$$

$$Q_2 = C_2 V_{DD}$$

- While the values for $x = 0$ are equal and opposite (charge neutrality), for positive displacements a net charge on the rotor appears:

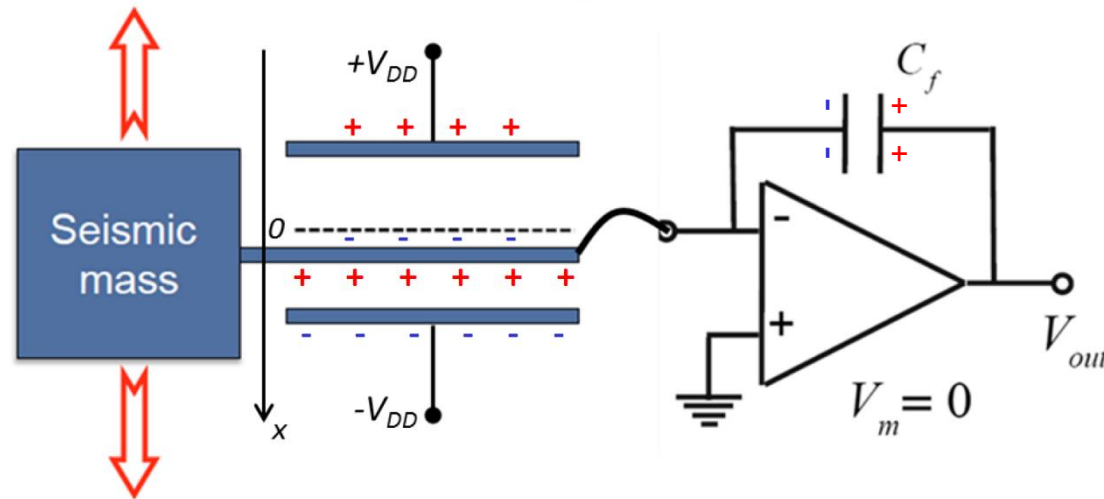


$$\Delta Q_m = -C_1 V_{DD} + C_2 V_{DD} > 0$$

Charge transfer – amplifier

- The OpAmp input is an ideal high impedance → the charge (i.e. the current flow) is delivered through the **feedback capacitance** ($\Delta Q_m = Q_f$)
- The **output voltage** of the operational amplifier changes by a quantity:

$$\Delta V_{out} = \frac{\Delta Q_m}{C_f} = \frac{-C_1 V_{DD} + C_2 V_{DD}}{C_f} = \frac{V_{DD}}{C_f} (C_2 - C_1)$$



Output voltage vs displacement

- Detailed calculation of ΔQ_m as a function of displacement x :

$$\Delta Q_m = V_{DD}(C_2 - C_1) = V_{DD} \left(\frac{\varepsilon_0 AN}{g-x} - \frac{\varepsilon_0 AN}{g+x} \right) = V_{DD} \frac{\varepsilon_0 AN}{g} \left(\frac{1}{1-x/g} - \frac{1}{1+x/g} \right) = V_{DD} C_0 \left(\frac{2x/g}{1-(x/g)^2} \right)$$

- For **small displacement** x of the suspended mass with respect to the **air gap** g between parallel plates:

- for $x \ll g \Rightarrow \frac{x}{g} \approx 0$

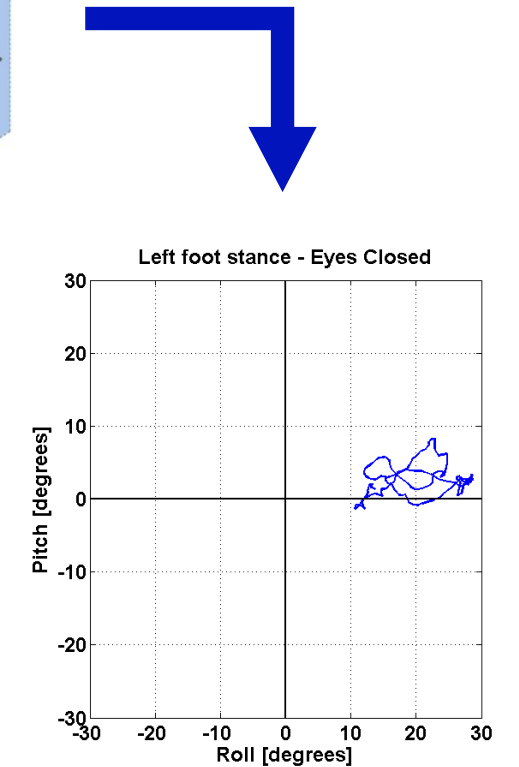
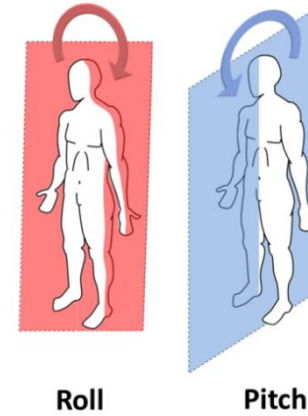
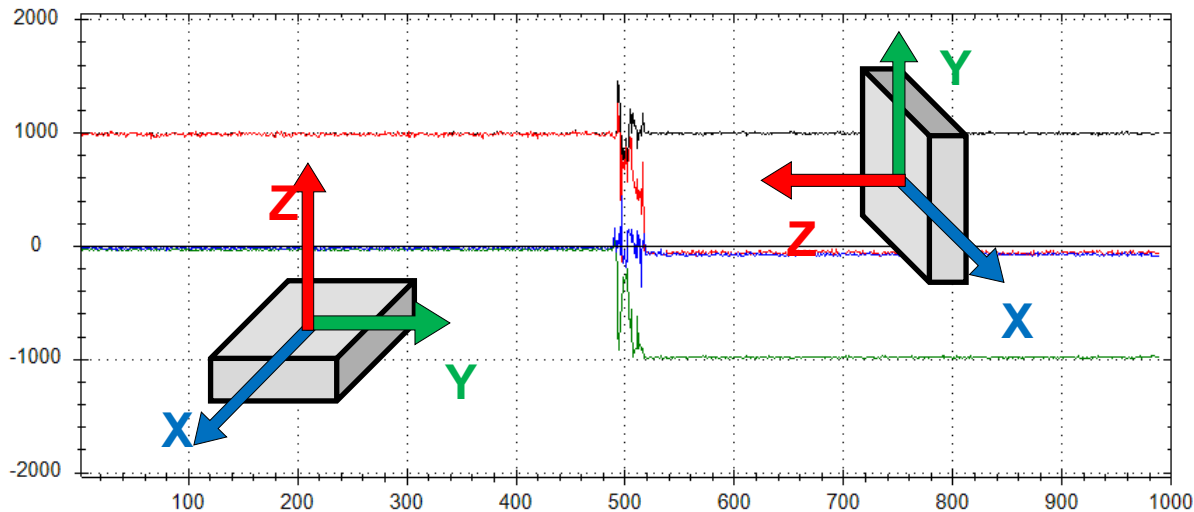
$$\lim_{\frac{x}{g} \rightarrow 0} \Delta Q_m = 2V_{DD} C_0 \frac{x}{g}$$

- The **output voltage** can be calculated as a **function of the displacement** x generated by the external acceleration:

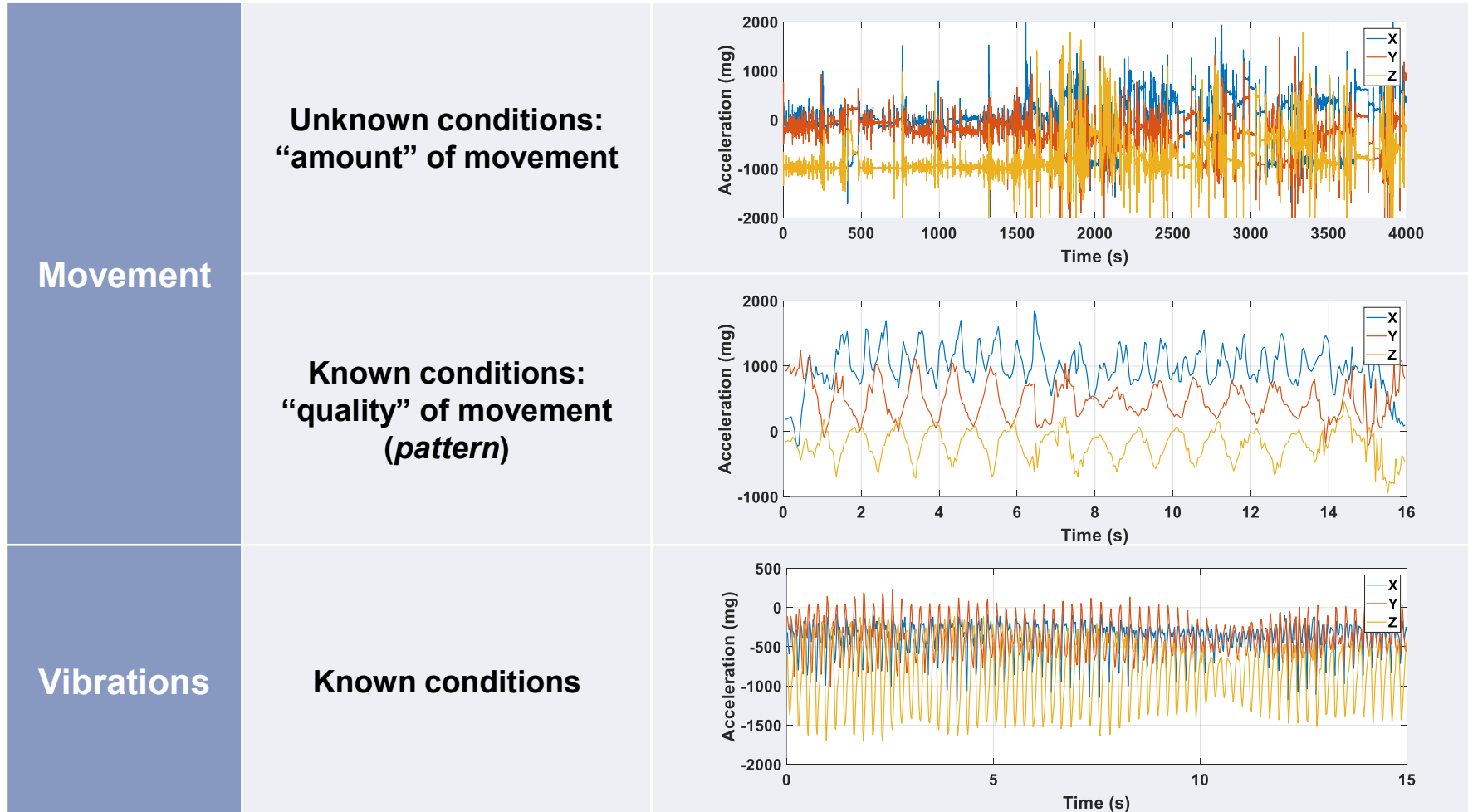
$$\Delta V_{out} = \frac{\Delta Q_m}{C_f} = 2 \frac{V_{DD} C_0}{C_f g} x$$

Application example: static conditions

- Orientation and inclination
 - Chest inclination
 - Device position (vertical, horizontal)



Application example: dynamic conditions

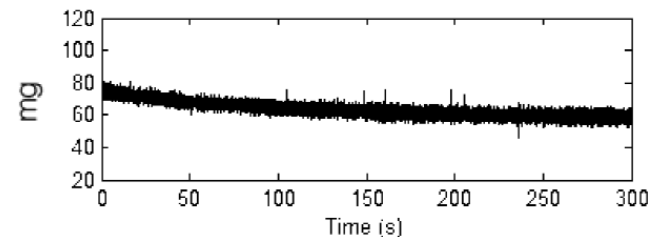
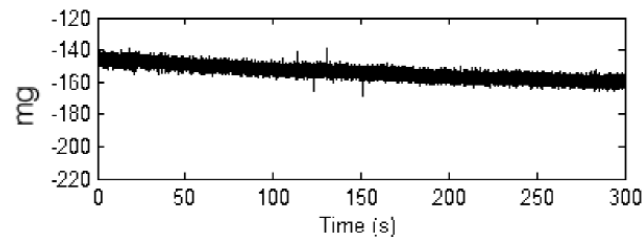


Accelerometer non-idealities

- Measured acceleration: gravity + external accelerations

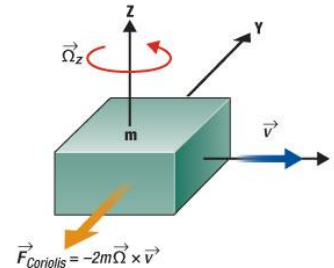
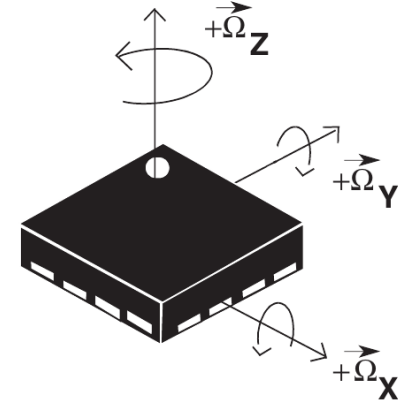
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} + \begin{bmatrix} A_{ext,x} \\ A_{ext,y} \\ A_{ext,z} \end{bmatrix}$$

- Thermal drift
 - Variation of $\pm 1,5\%$ of the sensitivity between $-40\text{ }^\circ\text{C}$ and $+85\text{ }^\circ\text{C}$
 - Temperature-dependent variation: $\pm 0,5\text{ mg}/^\circ\text{C}$



Gyroscope

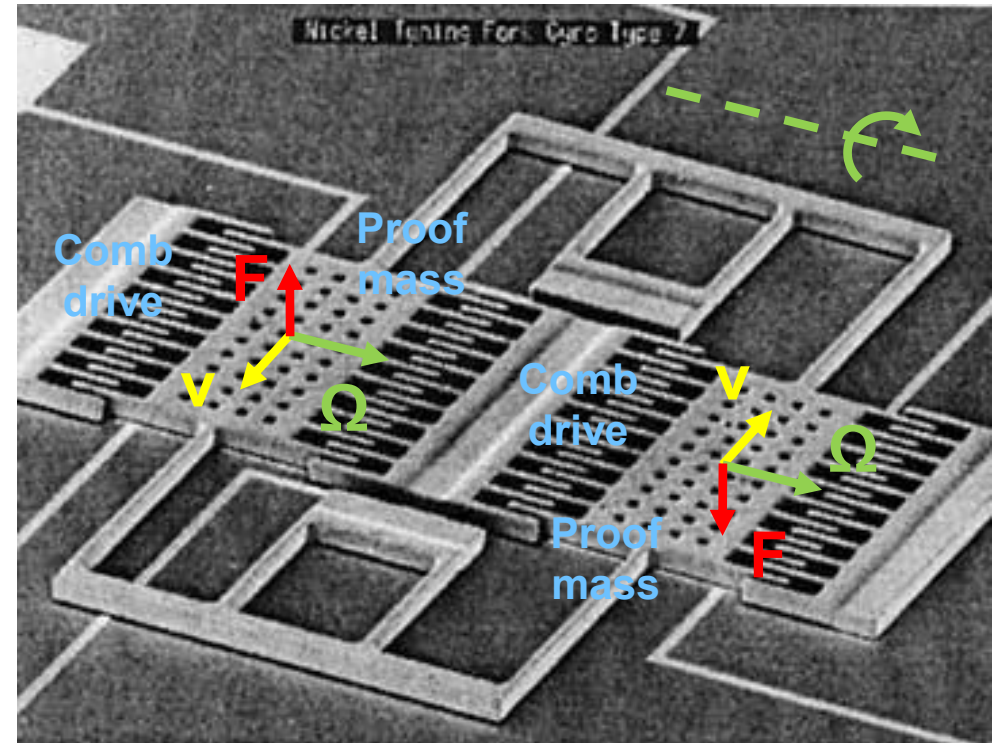
- Measures the angular velocity
- Application fields
 - Vibration monitoring
 - Automotive (ESP)
 - Image stabilizer
 - Videogames
- MEMS gyroscope: based on the mechanical movement of the MEMS structure due to the **Coriolis force**



$$\mathbf{F}_C = m \mathbf{a}_C = -2m \boldsymbol{\Omega} \times \mathbf{v}$$

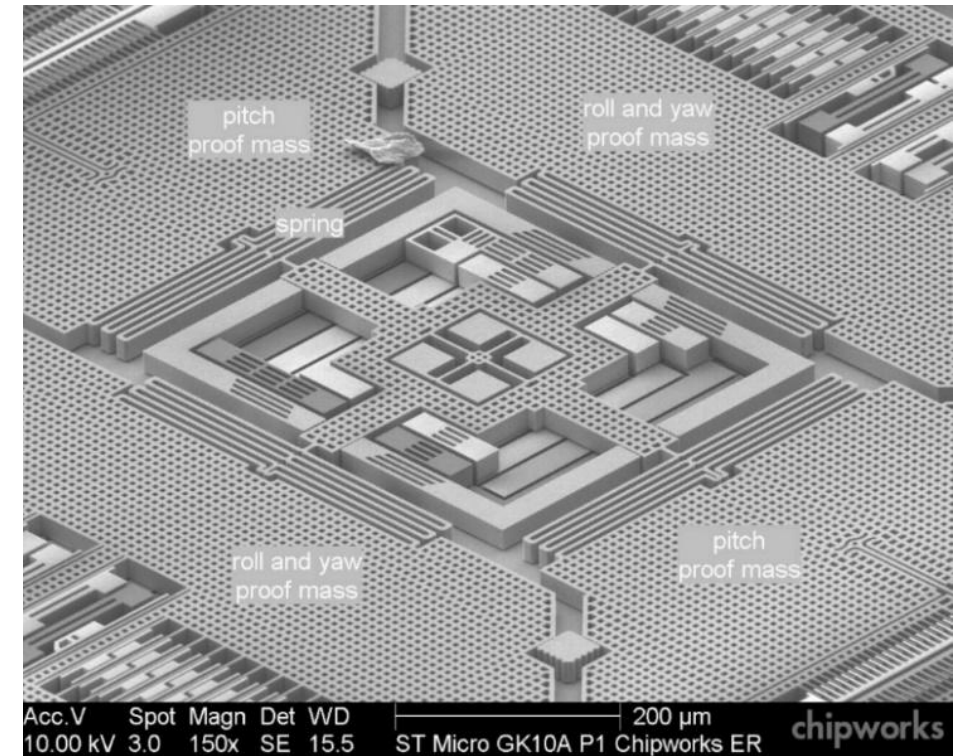
1D MEMS gyroscope

- Two proof masses positioned above two fixed conductive plates → two capacitors
- Comb drives make the masses vibrate in opposite directions
- The support integrating the sensor (and thus the masses) undergoes a **rotation** Ω
- Two forces F_C generate on the masses, making them moving upwards/downwards of a quantity $x \propto F_C$
- Displacement is measured by means of the two capacitors → $V = f(\omega)$

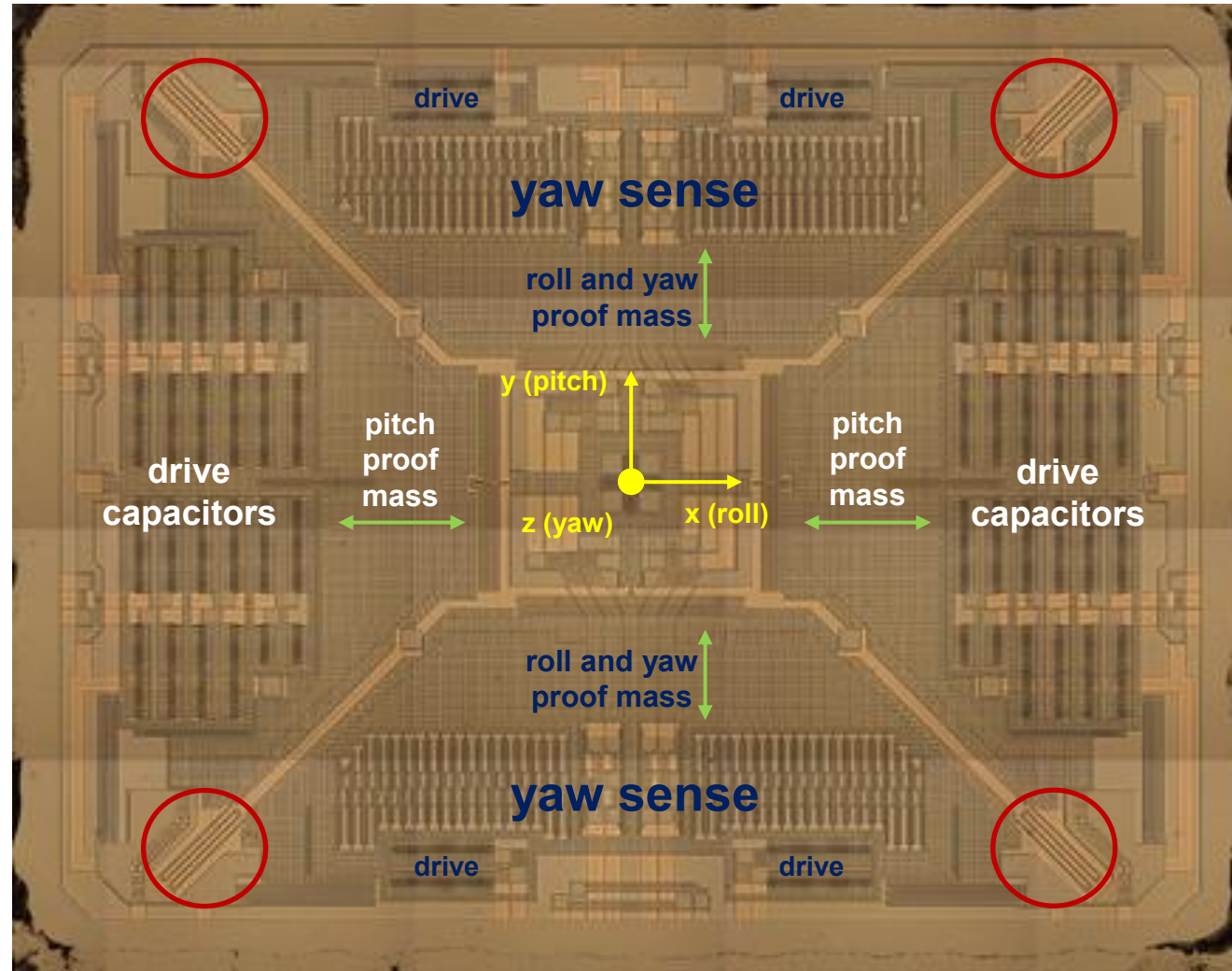


3D MEMS gyroscope

- 3 gyroscopes sensitive to rotations (*roll*, *pitch*, *yaw*) along 3 axes (x , y , z)
- Comb drives make the *pitch* proof mass vibrate
- Springs located in the corners transfer the vibrations to the *roll* and *yaw* proof masses
- A rotation along any of the three axes leads to a displacement of the related proof mass due to the Coriolis force: capacitive measurement either by means of plates placed under the masses (in case of x and y), or capacitive combs at the ends (in case of z)

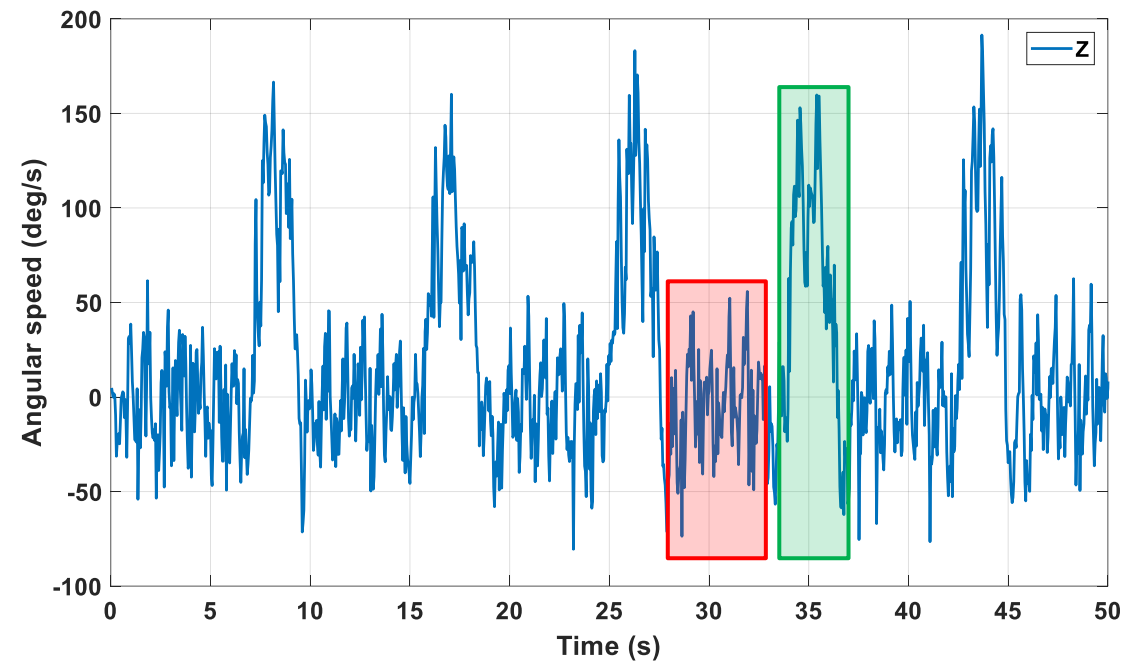


3D MEMS gyroscope



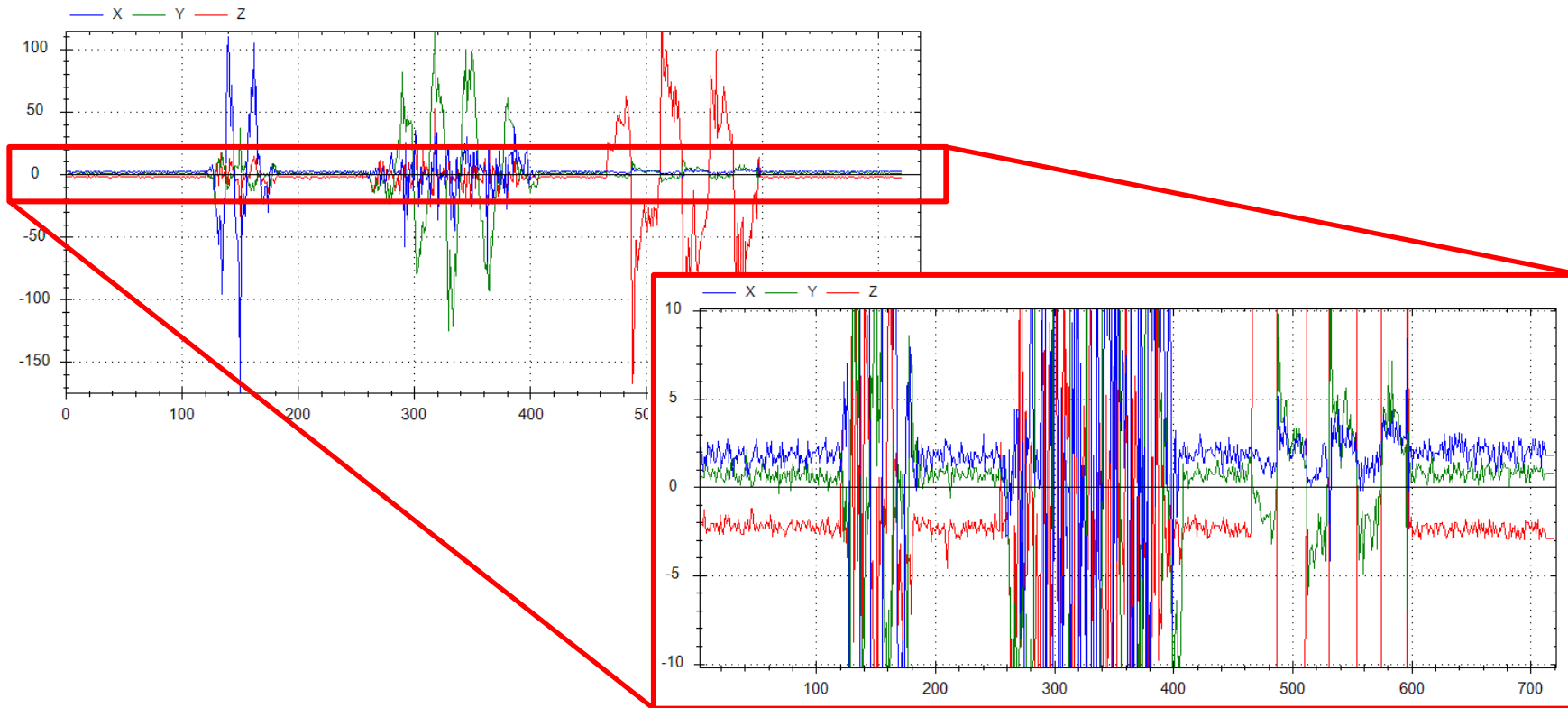
Application example: dynamic conditions

- Vertical rotation speed of the chest
 - Rotations **due to walking** (low intensity signal)
 - Rotations **due to the test execution** (high intensity signal)



Gyroscope non-idealities

- High power consumption due to the oscillating masses
- Characterized by an offset of few deg/s on each axis



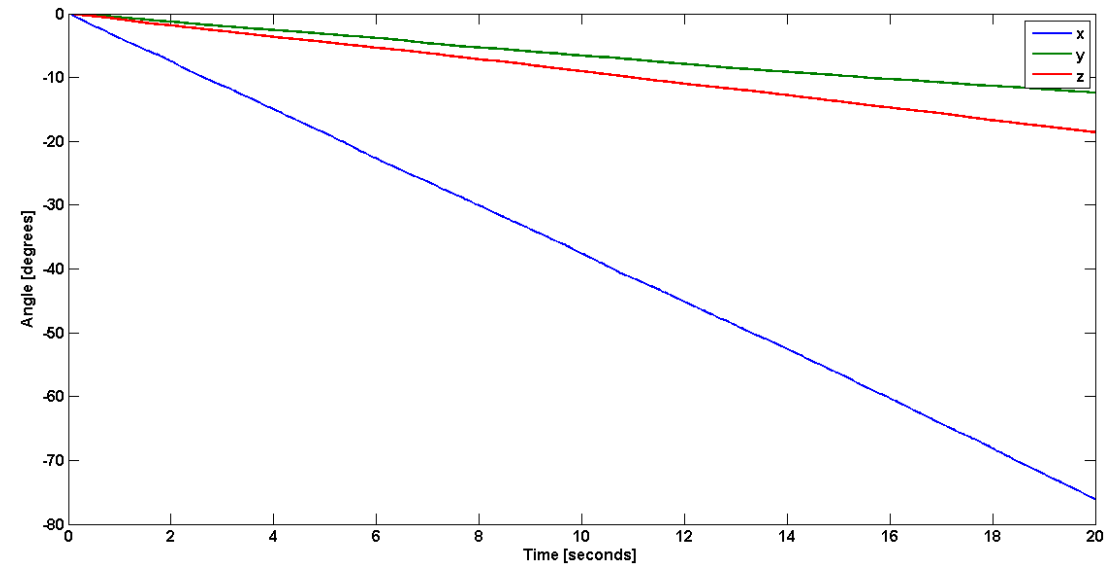
Effect of the non-ideality

- How is it possible to compute angular displacement when angular velocity is known?

$$\boldsymbol{\theta} = \int_0^T \boldsymbol{\Omega}(t) dt$$

$$\delta\boldsymbol{\theta} \approx \boldsymbol{\Omega}(t) \cdot t_s \quad \rightarrow \quad \boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \boldsymbol{\Omega}(t) \cdot t_s$$

- Drift effect due to the axes' offset!



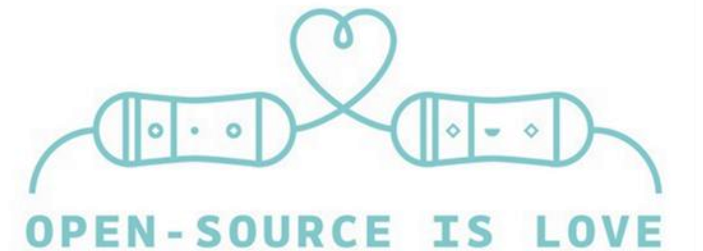
Laboratory activity

Arduino

- **What is Arduino?**

“Arduino designs, manufactures, and supports electronic devices and software, allowing people around the world to easily access advanced technologies that interact with the physical world. Our products are straightforward, simple, and powerful, ready to satisfy users’ needs from students to makers and all the way to professional developers.”

- Programmable platform (module/shield) with opensource schematic and layout and opensource Integrated Development Environment (IDE)
- Language based on C/C++



Arduino IDE – Next Lesson Requirements

- Find the downloadable software at the following link:

<https://www.arduino.cc/en/software>

- Last release → Arduino IDE 2.0.4

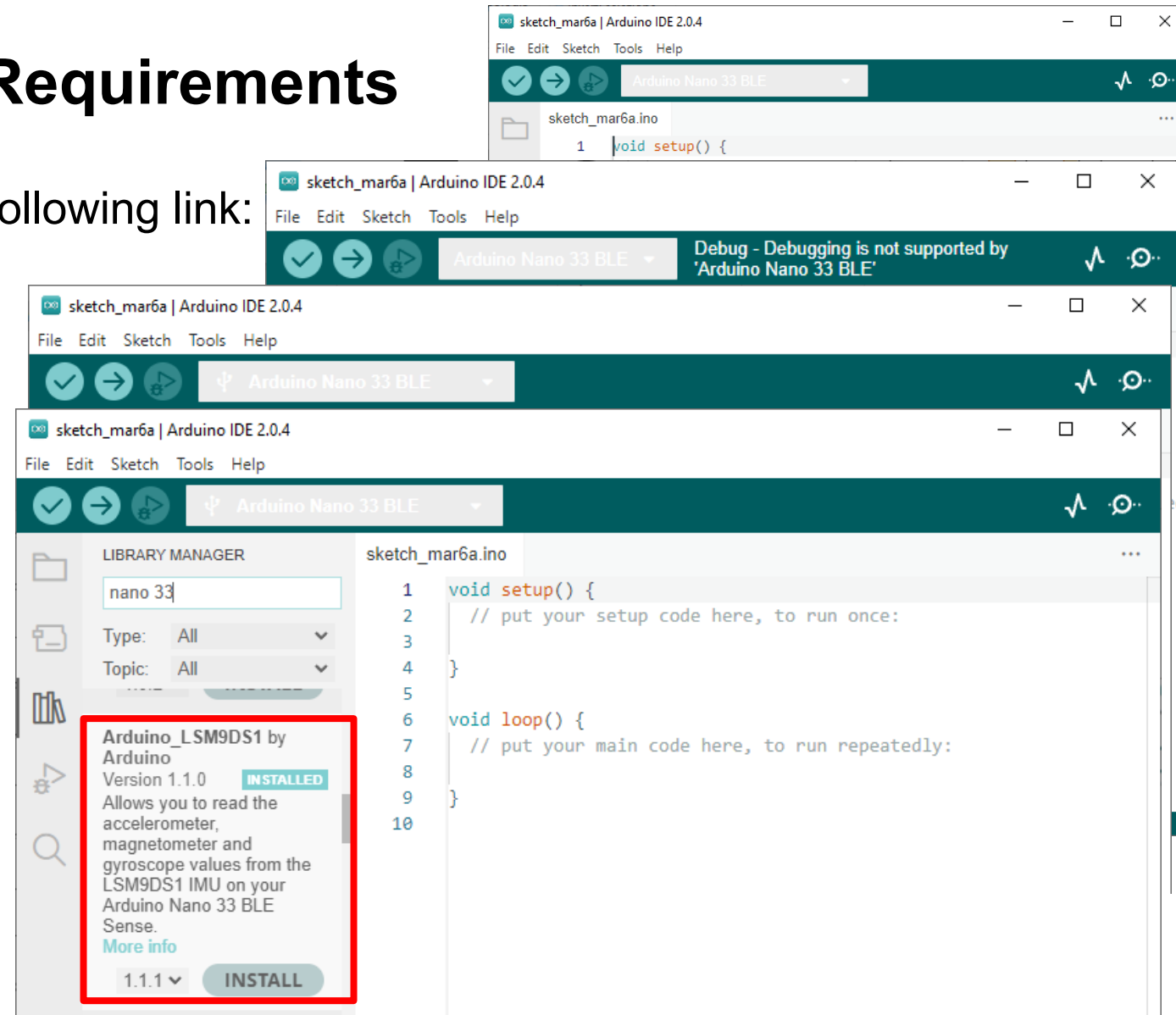
- Install ArduinoBLE drivers

- Board manager (Arduino Mbed OS Nano Boards)
- Library manager (Arduino BLE, Arduino_LSM9DS1)

- Online editor:

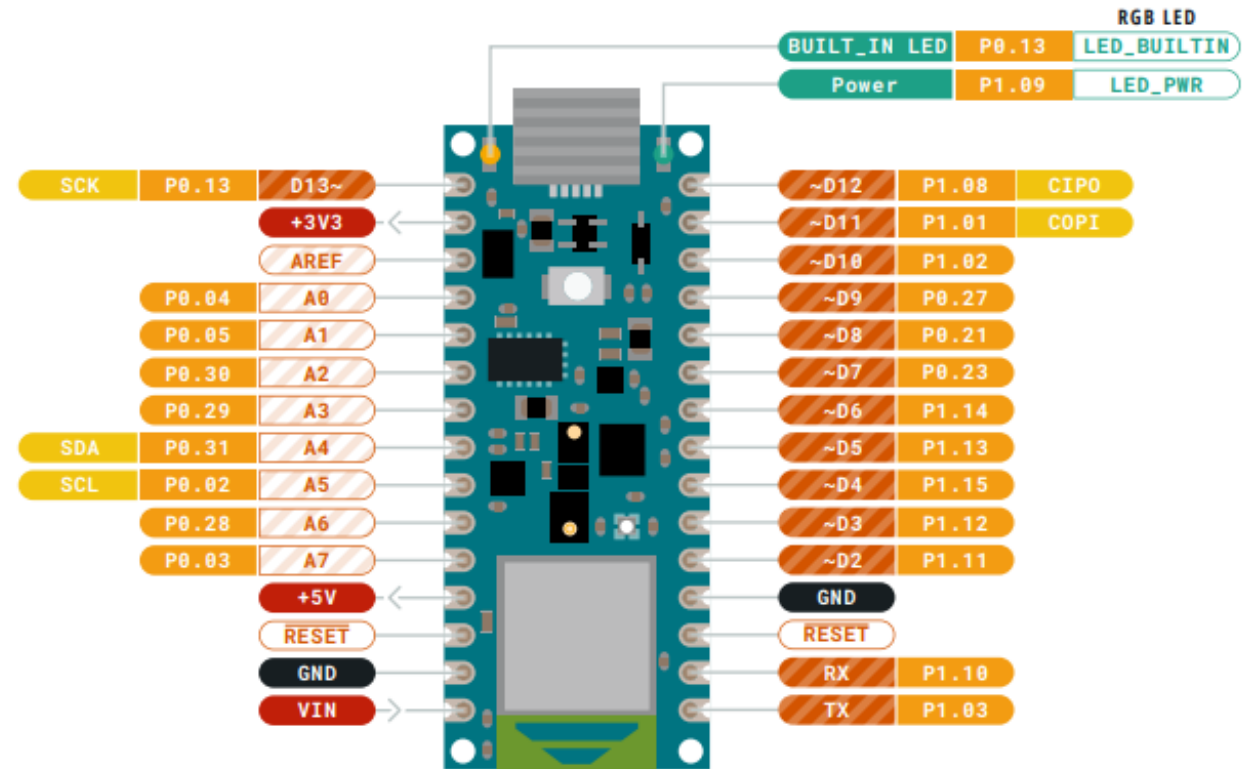
<https://create.arduino.cc/editor/>

- Please have a look at Ohm's Law!



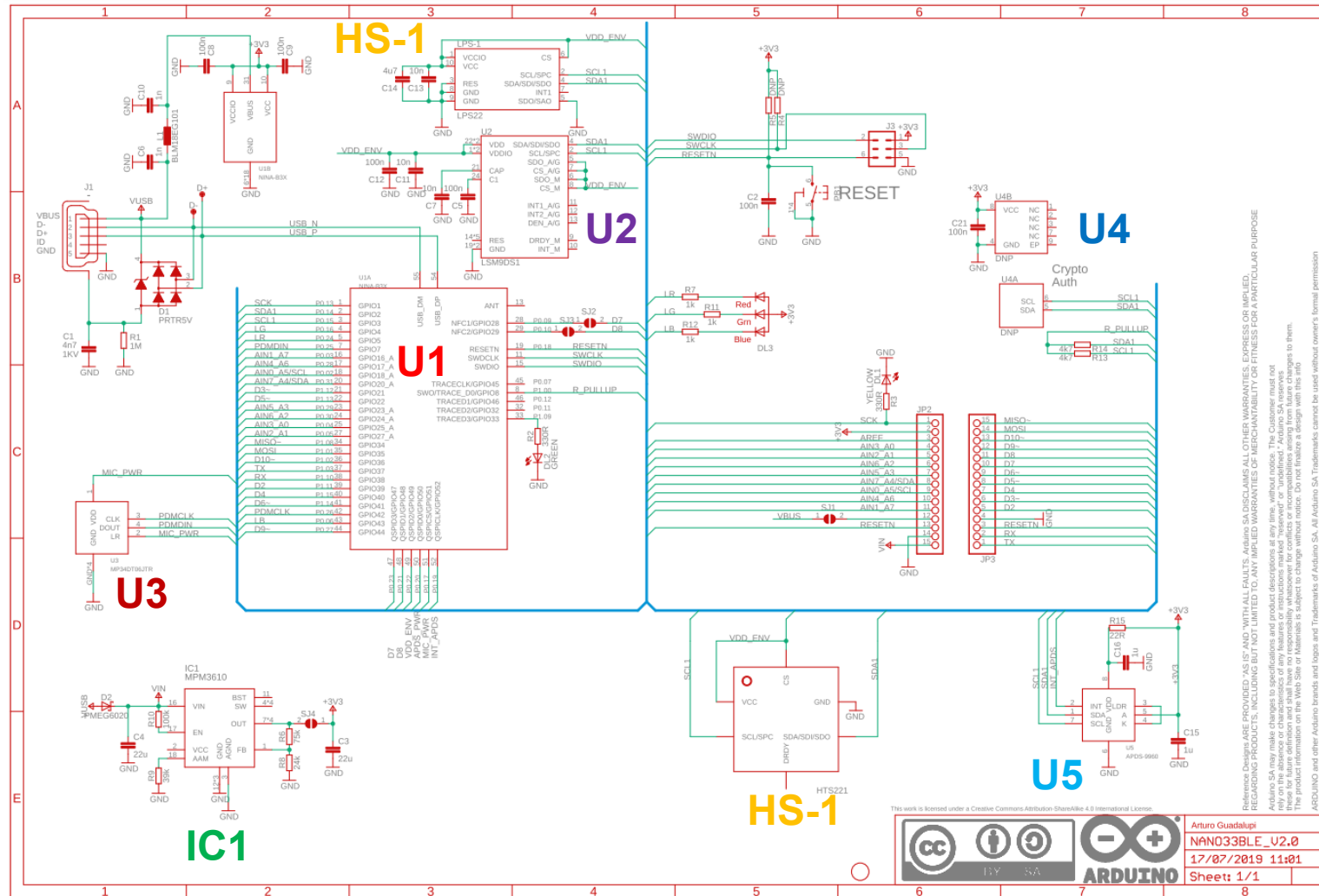
Arduino Nano 33 BLE Sense

- Power: 3.3 V, 15 mA DC for I/O pin
- NINA B306 module (**U1**):
 - Nordic nRF52840 (μ C, 32-bit, 64 MHz)
 - 256 KB SRAM, 1 MB flash
 - BLE 5.0 multi-protocol
- ATECC608A (Cryptographic chip, **U4**)
- MPM3610 (DC-DC – Power System, **IC1**)
- Sensors:
 - LSM9DS1 (IMU 9 axis, **U2**)
 - LPS22HB (Barometer and temperature, **HS-1**)
 - HTS221 (Relative humidity, **HS-1**)
 - APDS-9960 (Proximity, light, RGB, gestures, **U5**)
 - MP34DT05 (Digital microphone, **U3**)



[Pinout-NANOsense_latest.pdf \(arduino.cc\)](#)

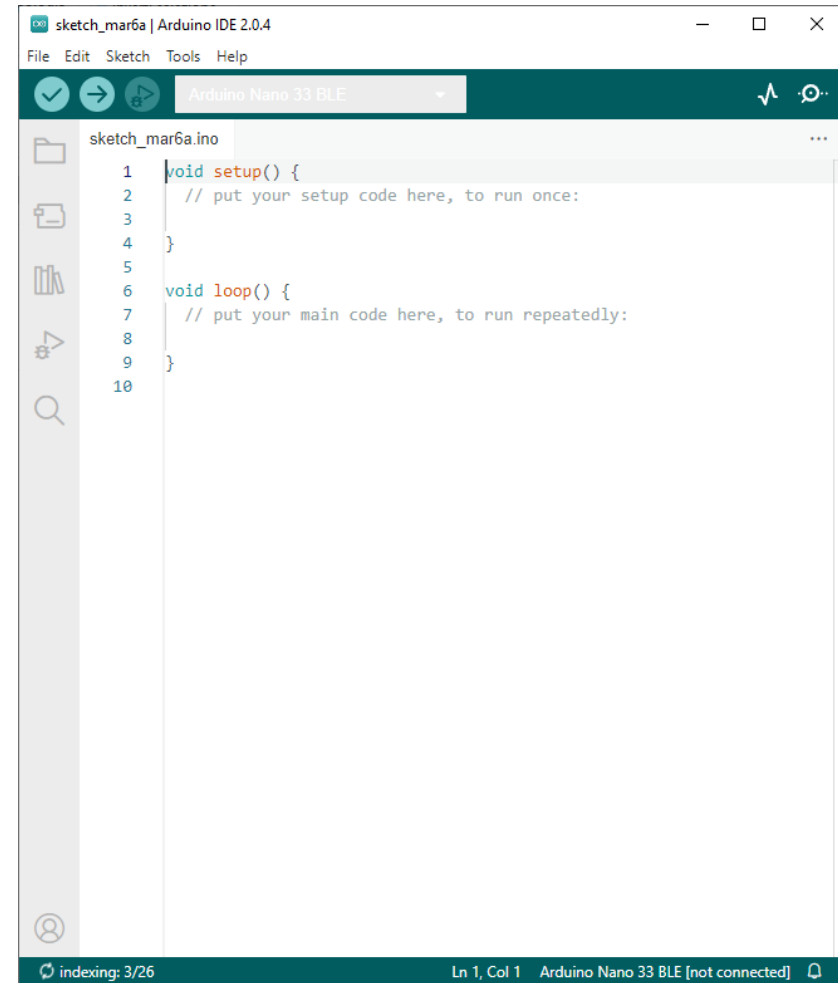
Arduino Nano 33 BLE Sense schematic



https://content.arduino.cc/assets/NANO33BLE_U2.0_sch.pdf

Arduino IDE – Next Lesson Requirements

- Find the downloadable software at the following link:
<https://www.arduino.cc/en/software>
- Last release → Arduino IDE 2.0.4



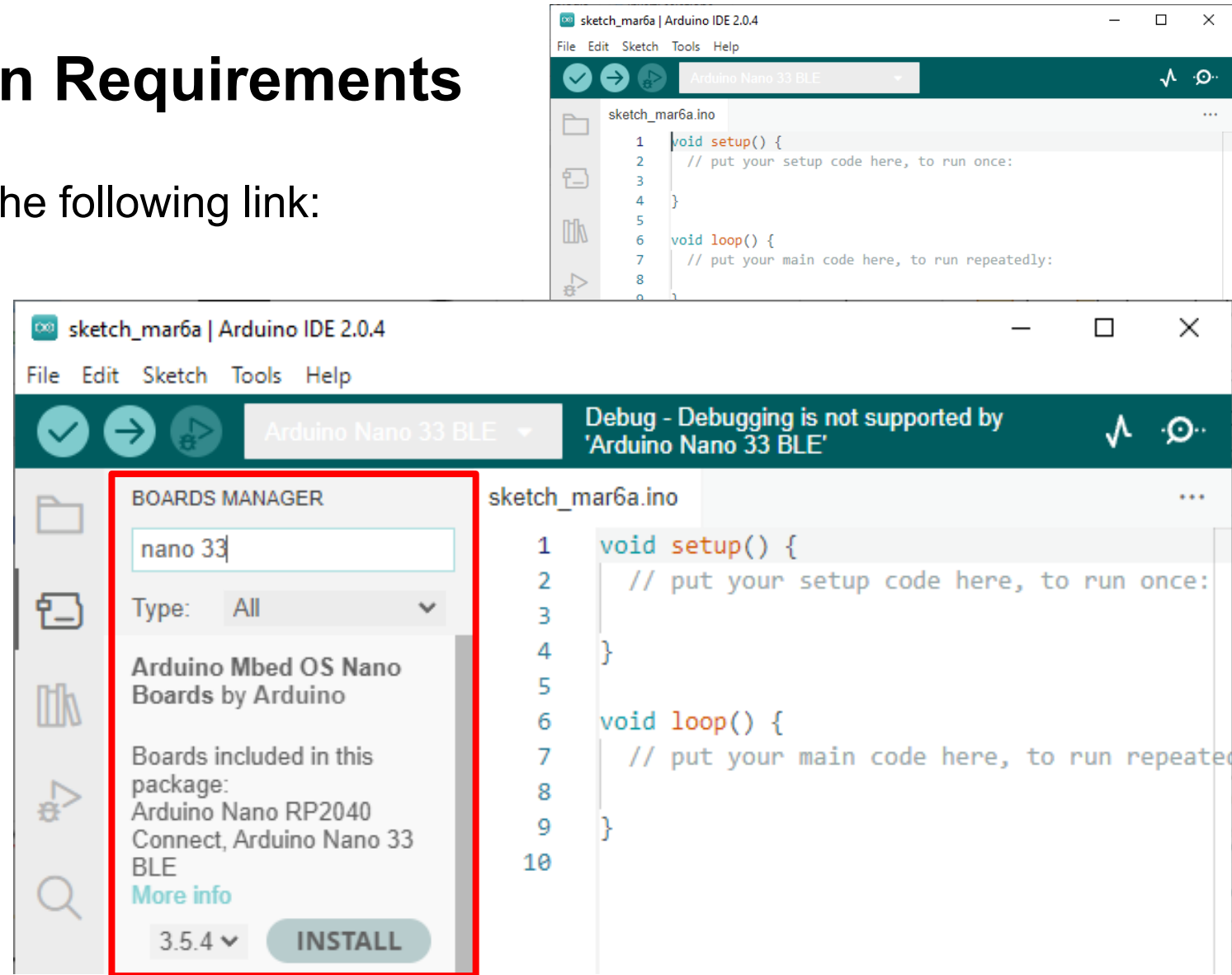
The screenshot displays the Arduino IDE 2.0.4 interface. The window title is "sketch_mar6a | Arduino IDE 2.0.4". The menu bar includes "File", "Edit", "Sketch", "Tools", and "Help". The toolbar shows icons for file operations and a dropdown menu currently set to "Arduino Nano 33 BLE". The main editor area shows the following code in "sketch_mar6a.ino":

```
1 void setup() {  
2   // put your setup code here, to run once:  
3  
4 }  
5  
6 void loop() {  
7   // put your main code here, to run repeatedly:  
8  
9 }  
10
```

The status bar at the bottom indicates "indexing: 3/26" on the left and "Ln 1, Col 1 Arduino Nano 33 BLE [not connected]" on the right.

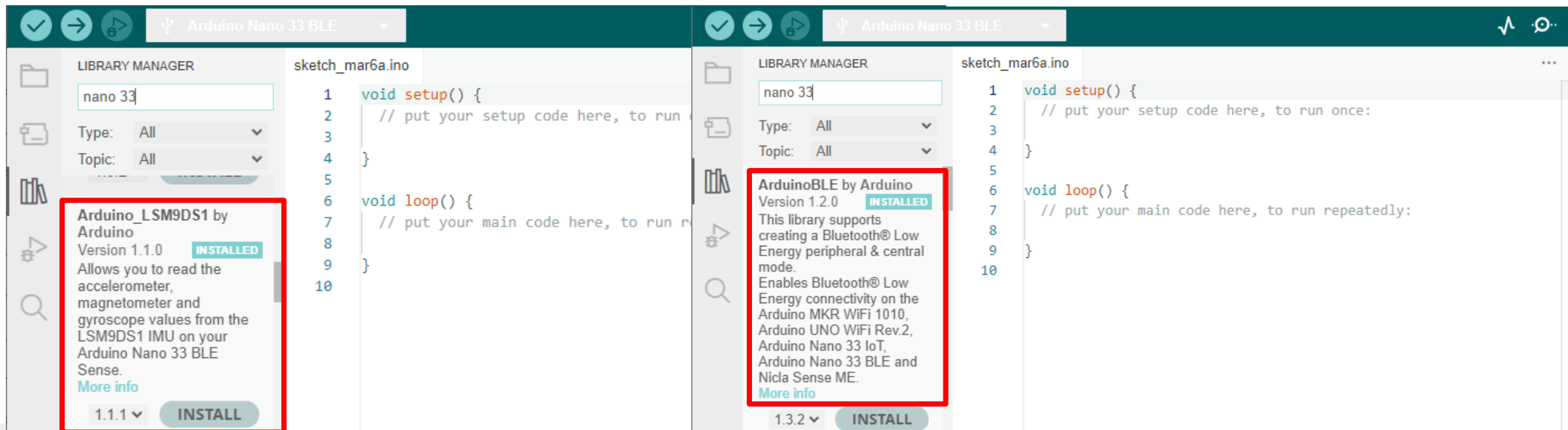
Arduino IDE – Next Lesson Requirements

- Find the downloadable software at the following link:
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- Last release → Arduino IDE 2.0.4
- Install ArduinoBLE drivers
 - Board manager (**Arduino Mbed OS Nano Boards**)



Arduino IDE – Next Lesson Requirements

- Find the downloadable software at the following link: <https://www.arduino.cc/en/software>
- Last release → Arduino IDE 2.0.4
- Install ArduinoBLE drivers
 - Board manager (Arduino Mbed OS Nano Boards)
 - Library manager (**Arduino BLE, Arduino_LSM9DS1**)



Arduino IDE – Next Lesson Requirements

- Find the downloadable software at the following link:

<https://www.arduino.cc/en/software>

- Last release → Arduino IDE 2.0.4

- Install ArduinoBLE drivers

- Board manager (Arduino Mbed OS Nano Boards)
- Library manager (Arduino BLE, Arduino_LSM9DS1)

- Online editor:

<https://create.arduino.cc/editor/>

- Please have a look at Ohm's Law!

