UNIVERSITÀ
DEGLI STUDI
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Dipartimento
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22059 – APPLIED TOPICS IN MANAGEMENT ENGINEERING

Excel, Access and Matlab

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MATLAB

What is MATLAB?

- MATLAB stands for MATrix LABoratory.
- It is a high-performance language for technical computing.
- It integrates computation, visualization, and programming environment.

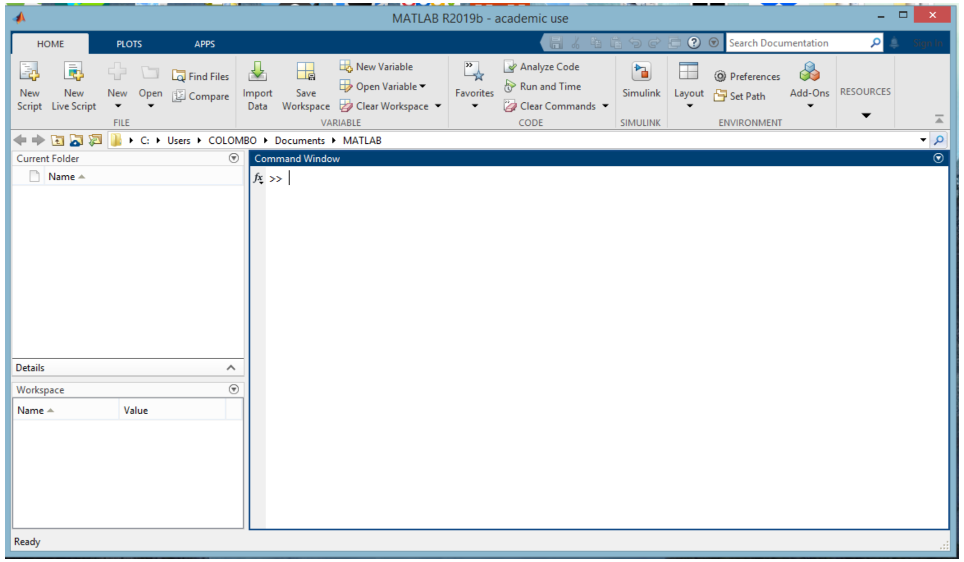

Fig.1: MATLAB Interface

AGENDA

Lecture XI

- MATRICES
  - Array (Vettore)
  - Identity matrix

- RECIPROCAL METHOD
  - Theoretical background
  - Example
  - Generalization
  - Complex example with MATLAB

- FURTHER MATERIAL

MATRICES

- Everything in MATLAB is represented by matrices.
  - Variables are also a special case of matrix, having dimension 1 × 1.
  - A matrix contains elements numbered by row (*i*) and column (*j*).
- For example:

$$\begin{array}{cc} & \text{Columns} \quad + \\ \text{Rows} & \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \\ + & \end{array}$$

The element in row 1 and column 2.

Array (Vettore)

- If the matrix has only one dimension it becomes an array:
  - A *row array* is a 1 x n matrix .
  - A *column array* is a n x 1 matrix.

- The array is the main data structure used by MATLAB.
- Each array is composed of elements (variables) characterized by a type.
- Each variable can store a value of a specific data type.

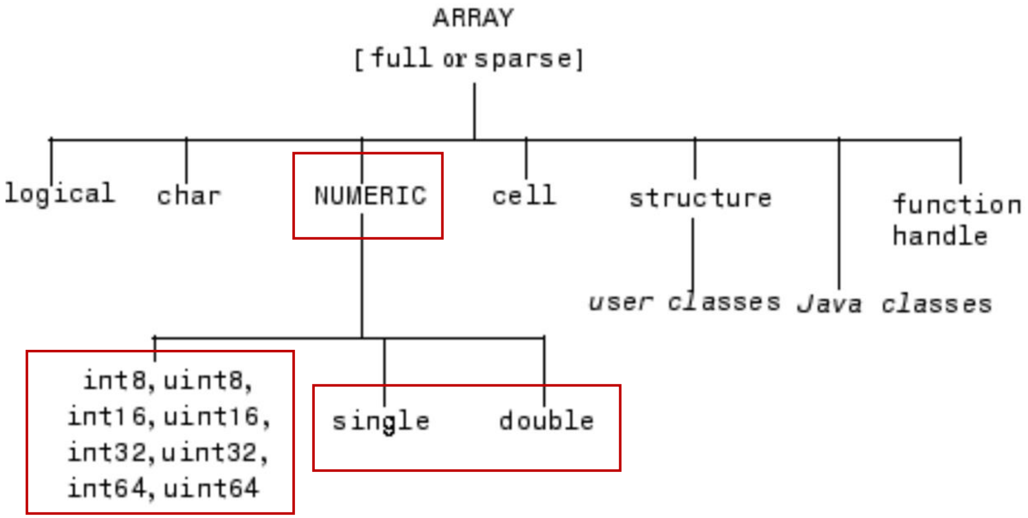

Fig.2: Data types

Array (Vettore)

- An array is an indexed collection of variables (elements) of the same type.
- For example:
  - Array composed of detected temperatures.
  - Array composed of school grades.

- An array in MATLAB is created by writing the elements that compose it within a pair of square brackets.
- Row array → Elements must be separated by a comma or a space.
  - `v= [17,23,3,42] or v= [17 23 3 42]`
- Column array → Elements must be separated by a semicolon or you can write a row array followed by a transposing operator (‘).
- `v= [17;23;3;42] or v= [17 23 3 42]’`

Array (Vettore)

- MATLAB displays row arrays horizontally and column arrays vertically.
- The `disp` function shows the content of a variable:

```
>> v=[17,23,3,42];
>> disp(v)
    17    23     3    42
```

```
>> v=[17;23;3;42];
>> disp(v)
    17
    23
     3
    42
```

Fig.3: Use of the MATLAB `disp` function

Array (Vettore)

- You can also find the size and the length of an array.
- `Size` → It returns the number of rows and colums composing the array.

```
>> size(v)

ans =

     1     4
```

Rows ← 1 4 → Columns

- `Length` → It returns the maximum dimension of an array (number of elements).

```
>> length(v)

ans =

     4
```

Fig.4: `size` and `length` functions

NB.
You can do the same also for matrices.

Identity matrix

- You can generate an Identity matrix.
- An Identity matrix is a matrix composed by ones on main diagonal and zeros elsewhere.
- You must use the `eye` function.
  - `eye (number of rows, number of columns)`
- For example:

```
>> eye(4,4)

ans =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

4 x 4 identity matrix.

Fig.5: `eye` function

RECIPROCAL METHOD

THEORETICAL BACKGROUND

MATLAB

- Several are the methods used to calculate the cost of a good.
- The most precise techniques allow to spread the costs of the service centres over the costs of the production centres.
- You can use one of these 4 methods:
  - One-step direct method
  - Two-step direct method
  - Step-down method
  - Reciprocal method → It is the best one.

Why?

It is the only mechanism which correctly carries out exchanges between service centres.

RECIPROCAL METHOD

Example

Allocate the costs of the two service centres to the two production centres:

	SCE1	SCE2	PCE1	PCE2	Total
Costs before allocation (million):	900	174	600	300	1974
Days dedicated to the centre:					
SCE1		24	36	60	120
SCE2	3		24	3	30

RECIPROCAL METHOD - Example

- First of all, you have to set up a system:

Y_1 → New value of the SCE1

Y_2 → New value of the SCE2

Internal cost of the SCE1

amount of cost from the SCE2

$$\begin{cases} Y_1 = 900 + \frac{3}{30} * Y_2 \\ Y_2 = 174 + \frac{24}{120} * Y_1 \end{cases}$$

Internal cost of the SCE2

amount of cost from the SCE1

RECIPROCAL METHOD - Example

- Now, you have to solve it.

MATLAB

$$\begin{cases} Y_1 = 900 + \dfrac{1}{10} * Y_2 \\ Y_2 = 174 + \dfrac{1}{5} * \left(900 + \dfrac{1}{10} * Y_2\right) \end{cases} \longrightarrow \begin{cases} Y_1 = 900 + \dfrac{1}{10} * Y_2 \\ Y_2 = 174 + 180 + \dfrac{1}{50} * Y_2 \end{cases}$$

$$\begin{cases} Y_1 = 900 + \dfrac{1}{10} * Y_2 \\ Y_2 = 354 * \dfrac{50}{49} \end{cases} \longrightarrow \begin{cases} Y_1 = 900 + \dfrac{1}{10} * 361{,}22 \\ Y_2 = 361{,}22 \end{cases} \longrightarrow \begin{cases} Y_1 = 936{,}12 \\ Y_2 = 361{,}22 \end{cases}$$

RECIPROCAL METHOD - Example

- Finally, you have to calculate the new cost of PCE1 and PCE2.

MATLAB

$$PCE_1 = \overbrace{\frac{36}{120} * Y_1 + \frac{24}{30} * Y_2}^{\text{amount of cost from the SCEs}} + 600 \longrightarrow \text{Initial cost of the PCE1}$$

$$PCE_2 = \underbrace{\frac{60}{120} * Y_1 + \frac{3}{30} * Y_2}_{\text{amount of cost from the SCEs}} + 300 \longrightarrow \text{Initial cost of the PCE2}$$

RECIPROCAL METHOD - Example

- You have to replace Y_1 and Y_2 with the values calculated in the previous step:

$$PCE_1 = \frac{36}{120} * 936{,}12 + \frac{24}{30} * 361{,}22 + 600 = 1169{,}81$$

$$PCE_2 = \frac{60}{120} * 936{,}12 + \frac{3}{30} * 361{,}22 + 300 = 804{,}18$$

Generalization

- In the example above, it was easy to calculate the new costs of the service and production centres as there were few variables involved.

How would we behave if there were *n* service centers and *m* production centers?

- The calculations would become longer and more demanding.
- Therefore, the Reciprocal method can be generalized.

MATLAB

Generalization

- Matrix A is the square matrix of order *n* whose generic element a_{ij} represents the percentage of *i* service centre resources consumed by service centre *j*.

- Matrix B is the rectangular matrix of size *n x m* whose generic element b_{ij} represents the percentage of the *i* service centre resources consumed by the production centre *j*.

- X represents the array column composed by the costs of the individual service centres to be allocated to the production centres.

- Y is the array column composed by the equivalent costs of the service centres.

Generalization

- So, the generic i-th element of the Y array can be described as follows:

$$y_i = x_i + \sum_{j=1}^{n} a_{ij} y_j$$

- In matrix terms:

$$Y = X + A'Y$$

- Resolving with respect to Y:

$$Y = (I - A')^{-1} X$$

Generalization

- Now, the direct method can be applied to move from the equivalent cost of the Y service centres to the cost allocated to the production centres.

- PC is the array column of order m that represents the cost allocated over the production centres. Each i-th element is:

$$PC_i = \sum_{j=1}^{n} b_{ij} y_j$$

- In matrix terms:

$$CP = B'Y = B'(I - A')^{-1}X$$

RECIPROCAL METHOD

- To easly solve these expressions you can use MATLAB!
- Create a M-File Function so that you can recall the “reciprocalmethod” function every time you want.
- Click on New, then, Function.
- The Editor will appear.

Formal parameters

reciprocalmethod.m

```matlab
function Z = reciprocalmethod(A,B,X,C)
%function that performs the reciprocal method
n=max(size(X)); %define the order of the array column X
I=eye(n); %generate the Identity matrix of order n
Y=(inv(I-A'))*X; %calculate the array column Y
Y=(B'*Y); %calculate the array composed
%by the cost allocated over the production centers
Z=Y+C; %total cost of the production centre
end
```

Fig.6: Function File

Complex example with MATLAB

	SCE1	SCE2	SCE3	SCE4	PCE1	PCE2	PCE3	PCE4	PCE5
Costs before allocation (million):	200	150	320	50	500	650	400	220	150
% dedicated to the centre:									
SCE1		0.16	0.09	0.32	0.05	0	0.13	0.09	0.16
SCE2	0.38	0.07	0	0.14	0.05	0.26	0.06	0.04	
SCE3	0.06	0.48	0.14			0.06	0.13	0.10	0.03
SCE4	0.02	0.20	0.17	0.05	0.04	0.36	0.08	0.02	0.06

Complex example with MATLAB

- First of all, you have to manually define:
  1. Service centre allocation matrix on service centres → SCEM
  2. Matrix of service centre allocations on production centres →SPCM
  3. Array column of internal costs of SCEs → SCIC
  4. Array colum with Internal costs of the PCEs → PCEIC

```
>> SCEM =[0,0.16,0.09,0.32;0.38,0.07,0,0.14;0.06,0.48,0.14,0;0.02,0.20,0.17,0.05];
>> SPCM=[0.05,0,0.13,0.09,0.16;0.05,0.26,0.06,0.04,0;0,0.06,0.13,0.10,0.03;0.04,0.36,0.08,0.02,0.06];
>> SCIC=[200;150;320;50];
>> PCEIC=[500;650;400;220;150];
```

Fig.7: Definition of matrices and arrays

Complex example with MATLAB

- Then, you have to call the `reciprocalmethod` function entering the correct input:

```
>> PCEs=reciprocalmethod(SCEM,SPCM,SCIC,PCEIC)
```

Fig.8: How to recall the function

Complex example with MATLAB

MATLAB

- Select Invio.

- MATLAB will show the result:

Current parameters

```
>> PCEs=reciprocalmethod(SCEM,SPCM,SCIC,PCEIC)

PCEs =

  560.3095
  920.3097
  573.7140
  334.2355
  251.4313
```

You can save the result in a variable.

Fig.9: Costs of the PCEs

NB.

The formal parameters and the current parameters may have different names.
The function associates the current parameters to the formal parameters considering their position.

FURTHER MATERIAL

To review and deepen the topics of this lecture

1. MATLAB online help.
2. https://www.youtube.com/watch?v=ITdMT5tfQsQ
3. https://www.youtube.com/watch?v=guk9vTlyN5k
4. https://www.youtube.com/watch?v=jf1yr4AbOFY