L'irragionevole efficacia della matematica nelle scienze (Naturali e umane)

Summer School

La matematica incontra l'Arte

e

la Tecnologia

San Pellegrino Terme 7-8-9 Settembre 2015

Mirko Degli Esposti | mirko.degliesposti@unibo.it

L'IRRAGIONEVOLE EFFICACIA DELLA MATEMATICA NELLE SCIENZE (NATURALI E UMANE)

Summer School

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DA DOVE COMINCIAMO.....DAGLI INIZI (ALMENO PER UNA VOLTA)

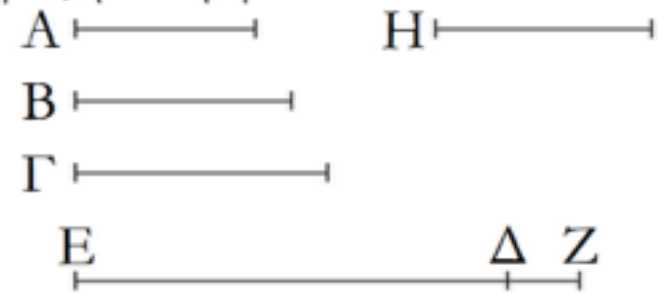
Da dove cominciamo.....dagli inizi (almeno per una volta)

Euclide d'Alessandria (325- 265 a.c.) -- "Elementi" Libro 9, Proposizione 20

Da dove cominciamo.....dagli inizi (almeno per una volta)

κ΄.

Οἱ πρῶτοι ἀριθμοὶ πλείους εἰσὶ παντὸς τοῦ προτεθέντος πλήθους πρώτων ἀριθμῶν.



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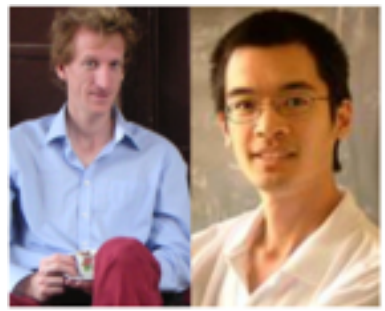
Euclide d'Alessandria (325- 265 a.c.) -- "Elementi" Libro 9, Proposizione 20

Da dove cominciamo.....dagli inizi (almeno per una volta)

2008

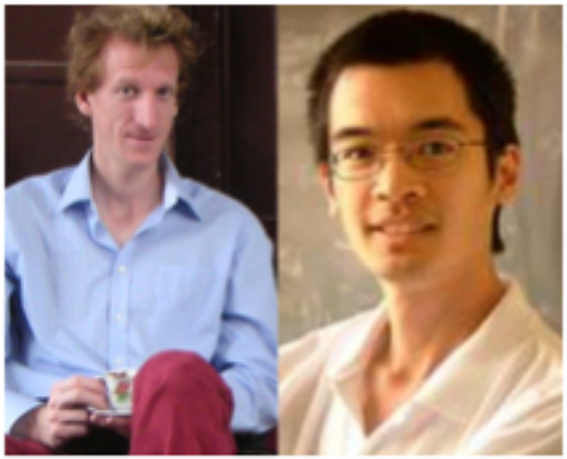


Ben Green and Terry Tao

Euclide d'Alessandria (325- 265 a.c.) -- "Elementi" Libro 9, Proposizione 20

I numeri Interi e i Numeri Primi

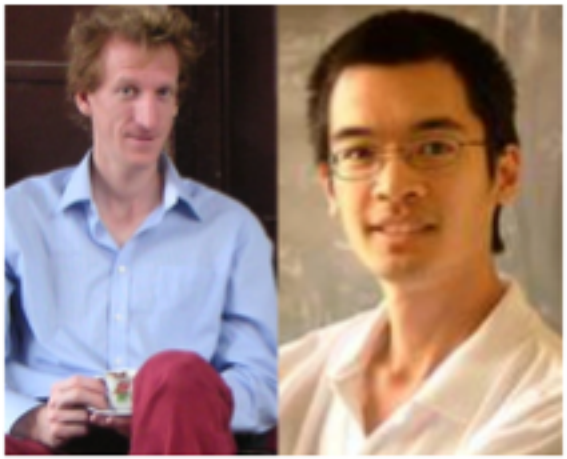
I numeri Interi e i Numeri Primi



Ben Green and Terry Tao

BEN GREEN:(Bristol, 1977), Università di Cambridge
TERRY TAO:(Adelaide, 1975), Università della California, medaglia fields 2006

I numeri Interi e i Numeri Primi



Ben Green and Terry Tao

Annals of Mathematics, **167** (2008), 481–547

The primes contain arbitrarily long arithmetic progressions

By Ben Green and Terence Tao*

Abstract

We prove that there are arbitrarily long arithmetic progressions of primes. There are three major ingredients. The first is Szemerédi's theorem, which asserts that any subset of the integers of positive density contains progressions of arbitrary length. The second, which is the main new ingredient of this paper, is a certain transference principle. This allows us to deduce from Szemerédi's theorem that any subset of a sufficiently pseudorandom set (or measure) of positive *relative* density contains progressions of arbitrary length. The third ingredient is a recent result of Goldston and Yıldırım, which we reproduce here. Using this, one may place (a large fraction of) the primes inside a pseudorandom set of "almost primes" (or more precisely, a pseudorandom measure concentrated on almost primes) with positive relative density.

BEN GREEN:(Bristol, 1977), Università di Cambridge
TERRY TAO:(Adelaide, 1975), Università della California, medaglia fields 2006

I numeri Interi e i Numeri Primi: partiamo dall'inizio...

Euclide d'Alessandria (325- 265 a.c.) -- "Elementi" Libro 9, Proposizione 20

I numeri Interi e i Numeri Primi: partiamo dall'inizio...

I numeri Primi sono i numeri interi **NON divisibili**: 2, 3,5,11,13,.....

I NUMERI INTERI E I NUMERI PRIMI: PARTIAMO DALL'INIZIO...

I NUMERI PRIMI SONO I NUMERI INTERI NON DIVISIBILI: 2, 3,5,11,13,.....

.....2741, 2749.....,1 299 709,.....1 79 424 673 .....

EUCLIDE D'ALESSANDRIA (325- 265 A.C.) -- "ELEMENTI" LIBRO 9, PROPOSIZIONE 20

I NUMERI INTERI E I NUMERI PRIMI: PARTIAMO DALL'INIZIO...

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.....2741, 2749.....,1 299 709,.....1 79 424 673 .....

EUCLIDE (ELEMENTI, LIBRO IX, PROPOSIZIONE 20):

TEOREMA:

ESISTONO INFINITI NUMERI PRIMI

χ΄.

Οἱ πρῶτοι ἀριθμοὶ πλείους εἰσὶ παντὸς τοῦ προτεθέντος πλήθους πρώτων ἀριθμῶν.

A H

B

Γ

E Δ Z

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EUCLIDE (ELEMENTI, LIBRO IX, PROPOSIZIONE 20):

TEOREMA:

ESISTONO INFINITI NUMERI PRIMI

DIMOSTRAZIONE: **REDUCTIO AD ABSURDUM**

ϰ΄.

Οἱ πρῶτοι ἀριθμοὶ πλείους εἰσὶ παντὸς τοῦ προτεθέντος πλήθους πρώτων ἀριθμῶν.

Α Η

Β

Γ

Ε Δ Ζ

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I Numeri Primi, Come Trovarli ?

Eratostene di Cirene (275- 195 a.C.)
Terzo bibliotecario della Biblioteca di Alessandria e precettore di Tolomeo IV

I Numeri Primi, Come Trovarli ?

Il Crivello di Eratostene

Eratostene di Cirene (275- 195 a.c.)
Terzo bibliotecario della Biblioteca di Alessandria e precettore di Tolomeo IV

I Numeri Primi, Come Trovarli ?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

OSTENE

Eratostene di Cirene (275- 195 a.c.)
Terzo bibliotecario della Biblioteca di Alessandria e precettore di Tolomeo IV

I Numeri Primi, Come Trovarli ?

Il Crivello di Eratostene

Eratostene di Cirene (275- 195 a.c.)
Terzo bibliotecario della Biblioteca di Alessandria e precettore di Tolomeo IV

IMPREVEDIBILI NUMERI PRIMI

GOLDBACH & EULERO (1742), HARDY- LITTLEWOOD (1923), VINOGRADOV (1937), ...

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- CONGETTURA DI GOLDBACH:

OGNI NUMERO PARI SI SCRIVE COME SOMMA DI DUE NUMERI PRIMI

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OGNI NUMERO PARI SI SCRIVE COME SOMMA DI DUE NUMERI PRIMI

- CONGETTURA DEI NUMERI GEMELLI:

ESISTONO INFINITI "TWIN PRIME": (5,7), (11,13),.....(137,139), ...

GOLDBACH & EULERO (1742), HARDY- LITTLEWOOD (1923), VINOGRADOV (1937), ...

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ESISTONO INFINITI "TWIN PRIME": (5,7), (11,13),.....(137,139), ...

,....., 3756801695685 · 2^666669 ± 1 (PRIMEGRID 2011: 200700 DIGITS....)

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,....., 3756801695685 · 2^666669 ± 1 (PRIMEGRID 2011: 200700 DIGITS....)

I NUMERI PRIMI SONO VERAMENTE IMPREVEDIBILI: NON SAPPIAMO QUANDO INCONTREREMO IL PROSSIMO.....(FATTORIZZAZIONE....)

GOLDBACH & EULERO (1742), HARDY- LITTLEWOOD (1923), VINOGRADOV (1937), ...

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MA COME SONO DISTRIBUITI TRA I NUMERI INTERI ?

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I NUMERI PRIMI SONO UNA POLVERE **CASUALE**/ALEATORIA.... FINE FINE.....

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CARLOS PARIS (HTTP://WWW.SIEVESOFCHAOS.COM/)

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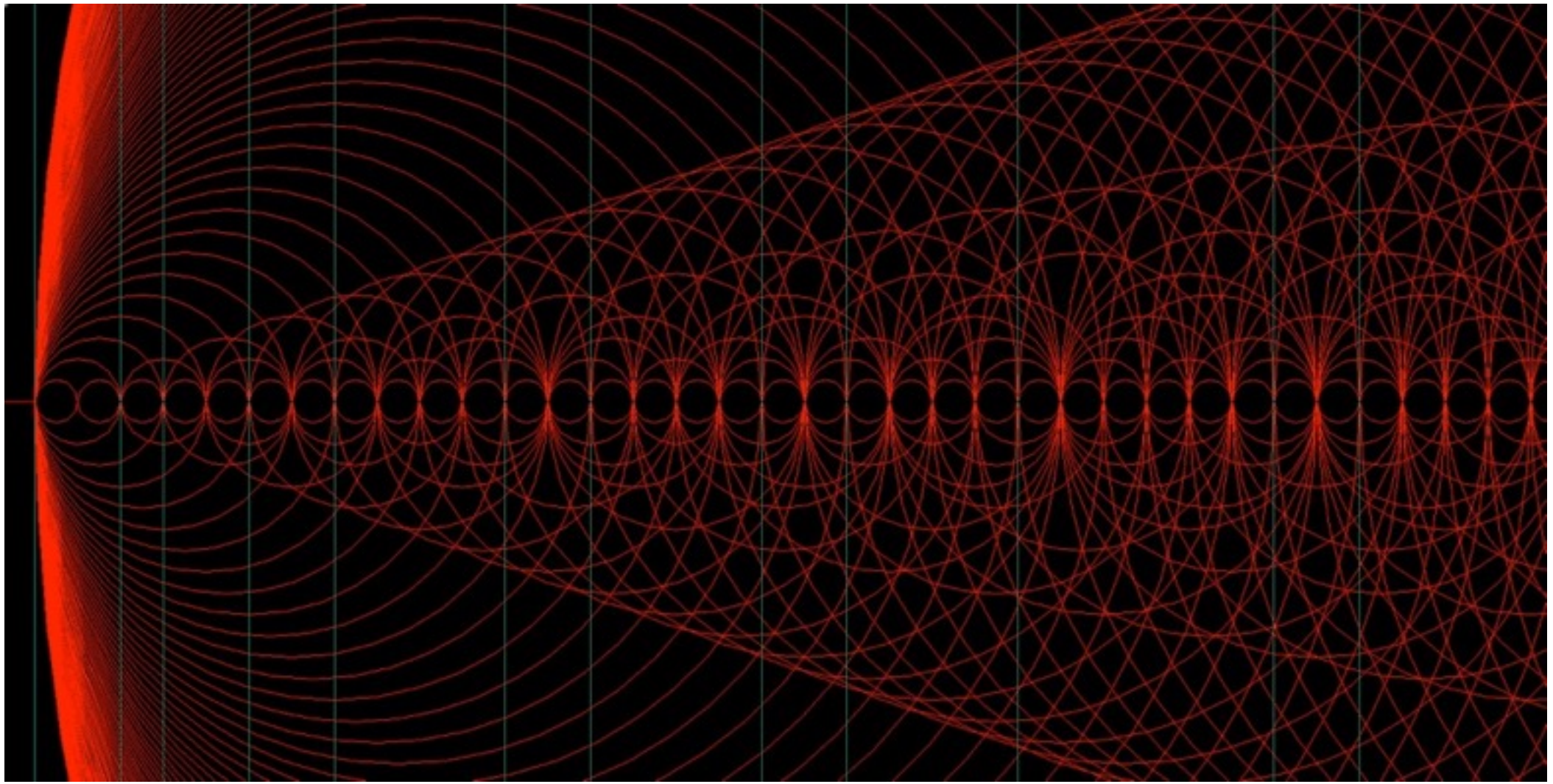

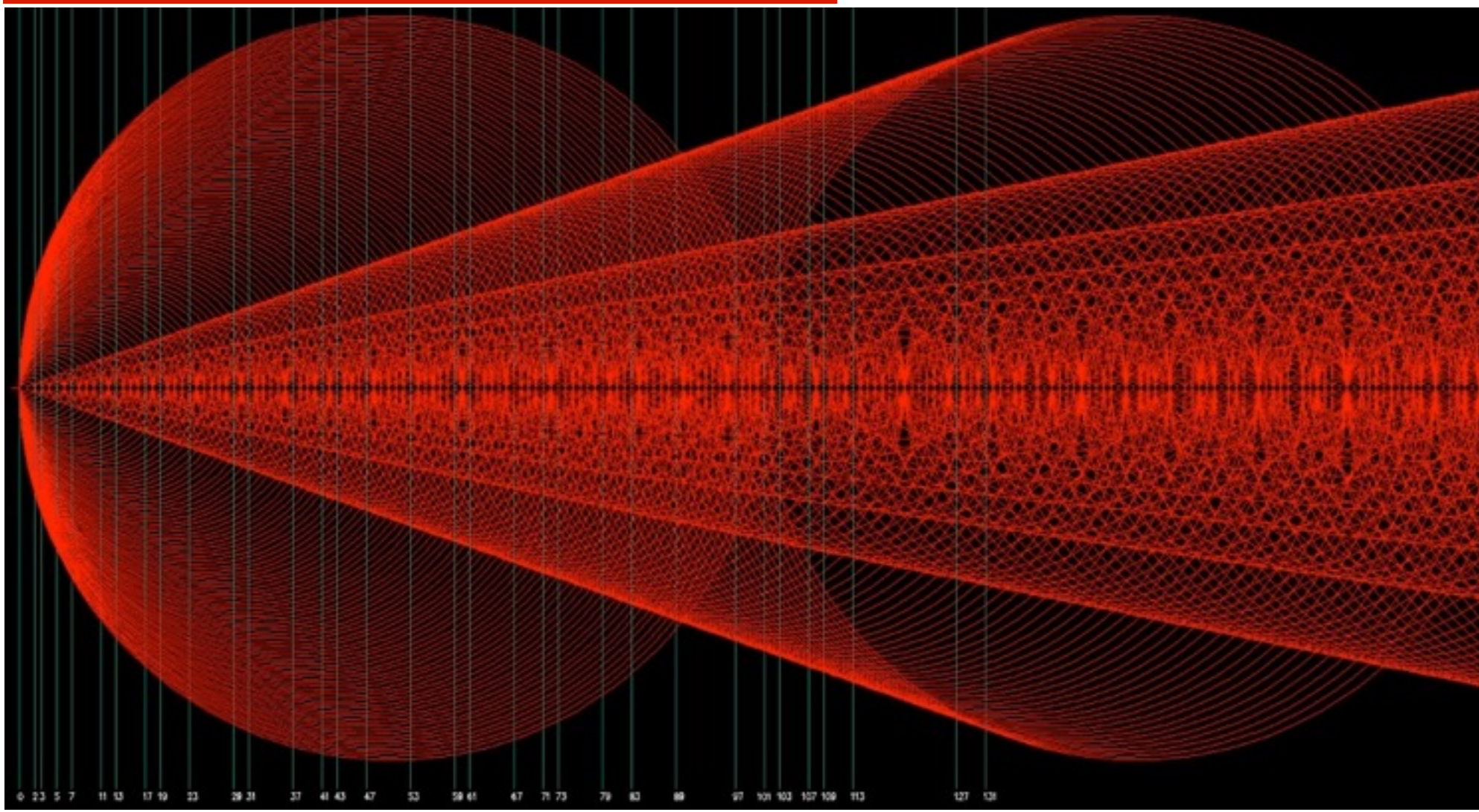
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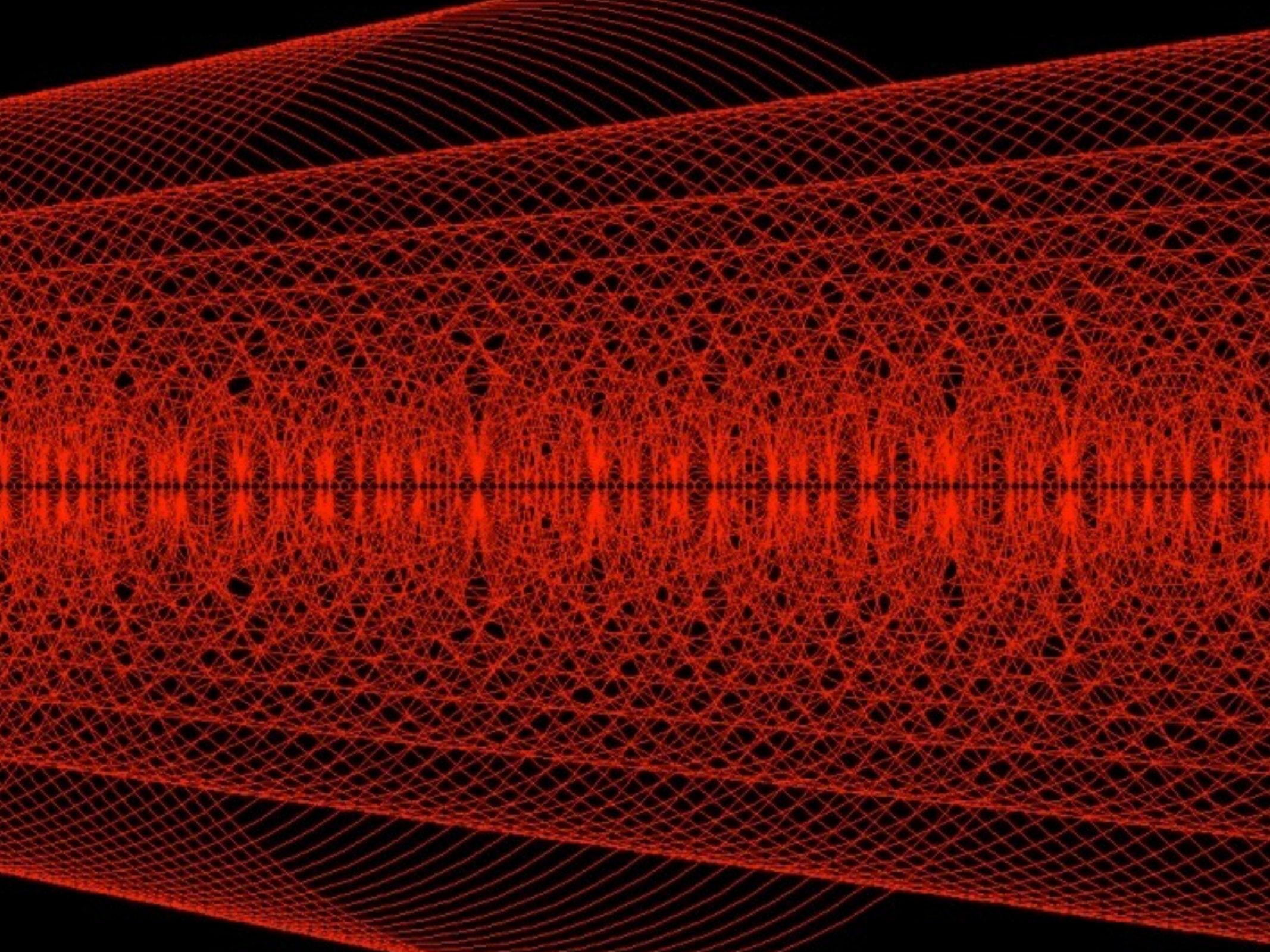

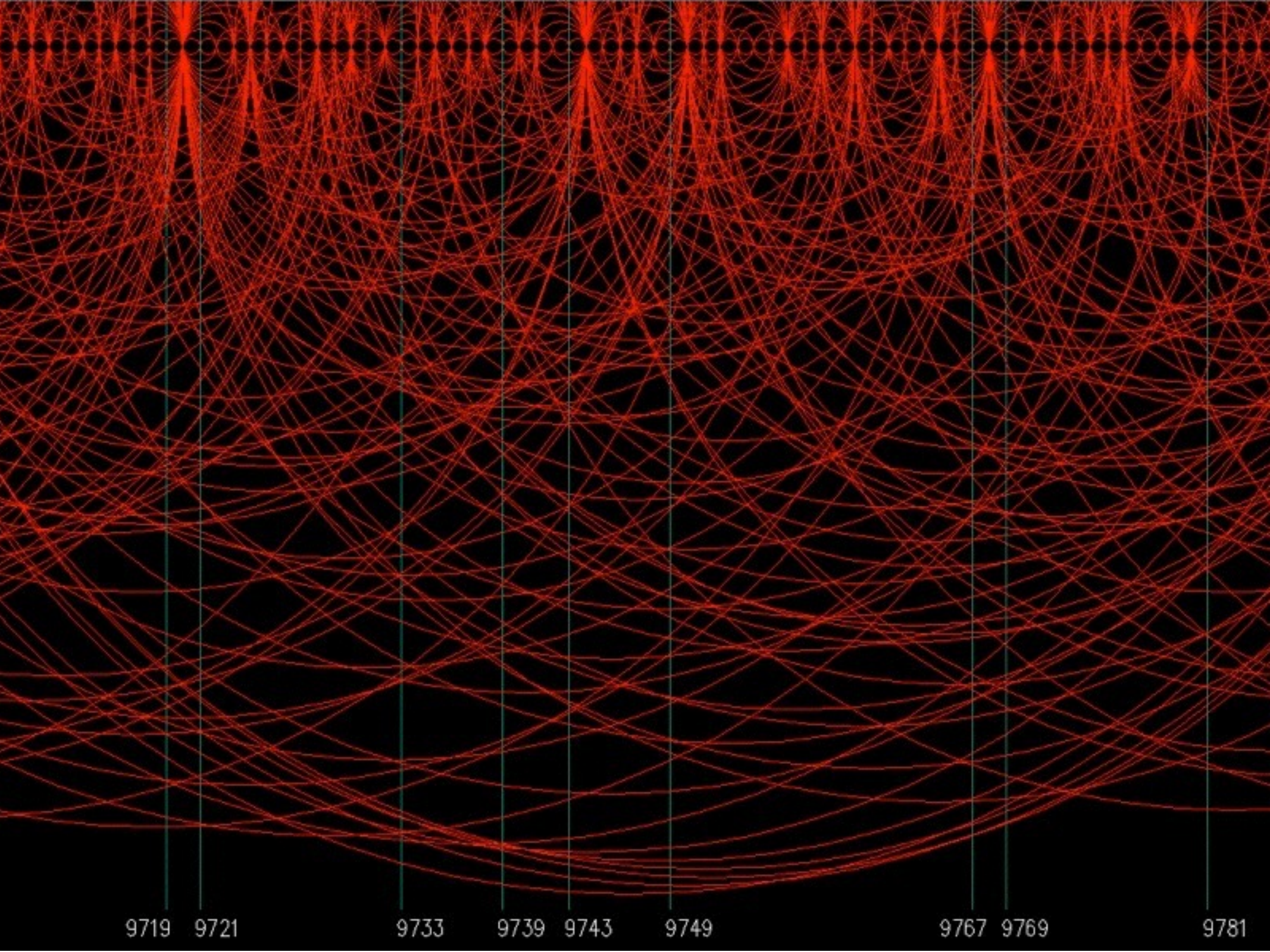
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IMPREVEDIBILI NUMERI PRIMI: MA CAMBIAMO PROSPETTIVA

TEOREMA DI DIRICHLET (1837)

IMPREVEDIBILI NUMERI PRIMI: MA CAMBIAMO PROSPETTIVA

PENSIAMO INVECE ORA ALLE "STRUTTURE" PIÙ ORDINATE.....

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PENSIAMO INVECE ORA ALLE "STRUTTURE" PIÙ ORDINATE.....

LE PROGRESSIONI ARITMETICHE:

TEOREMA DI DIRICHLET (1837)

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PENSIAMO INVECE ORA ALLE "STRUTTURE" PIÙ ORDINATE.....

LE PROGRESSIONI ARITMETICHE:

2,4,6,8,10,....., 1020,1022,.....(PASSI DI 2)

TEOREMA DI DIRICHLET (1837)

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PENSIAMO INVECE ORA ALLE "STRUTTURE" PIÙ ORDINATE.....

LE PROGRESSIONI ARITMETICHE:

2,4,6,8,10,....., 1020,1022,.....(PASSI DI 2)

13,26,39,....., 13x20,13x21,.....(PASSI DI 13)

TEOREMA DI DIRICHLET (1837)

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PENSIAMO INVECE ORA ALLE "STRUTTURE" PIÙ ORDINATE.....

LE PROGRESSIONI ARITMETICHE:

2,4,6,8,10,....., 1020,1022,.....(PASSI DI 2)

13,26,39,....., 13x20,13x21,.....(PASSI DI 13)

IN GENERALE: $a \times n$, $n=1,2,3,.....$

a, 2a,3a,4a,5a,.....,....

TEOREMA DI DIRICHLET (1837)

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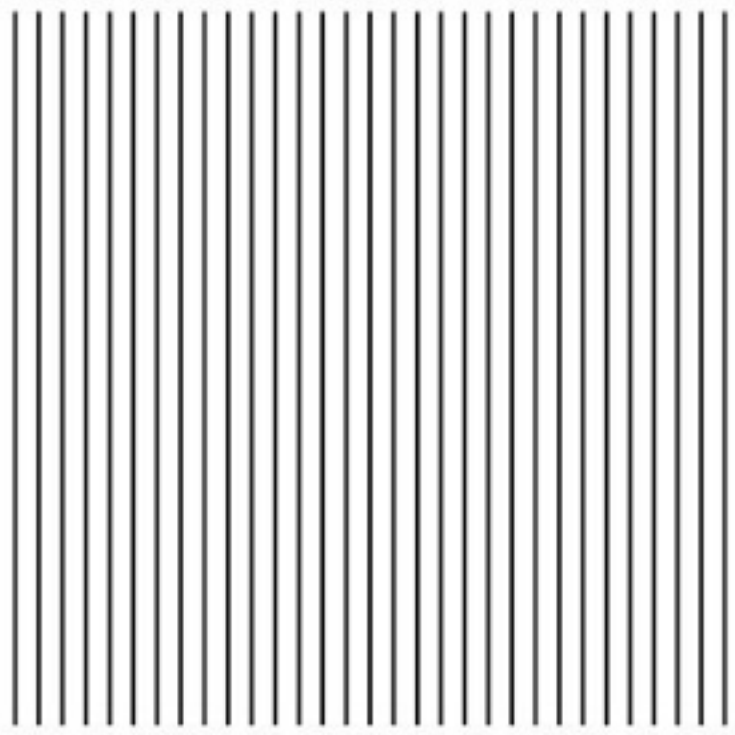

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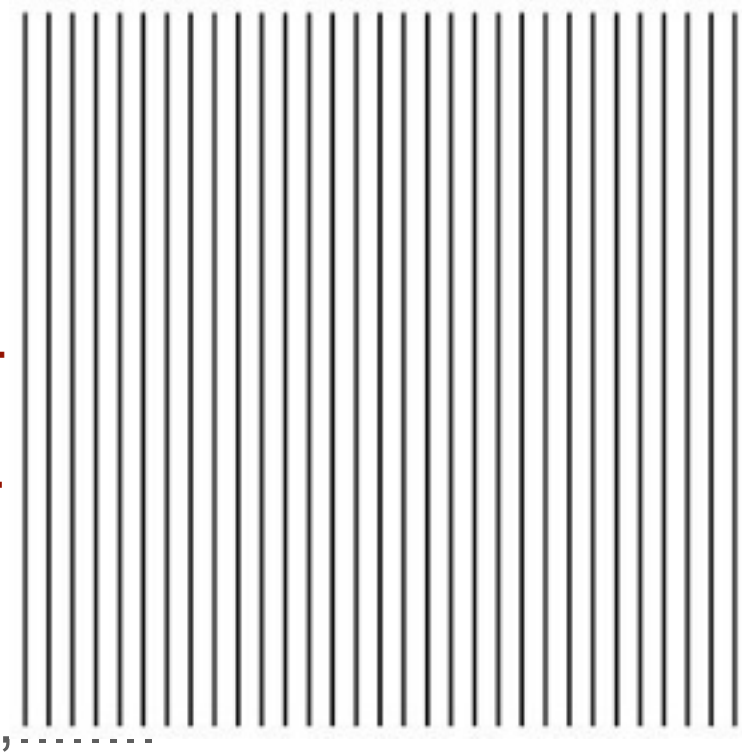

IN GENERALE: a x n, n=1,2,3,.....

a x n + b

a, 2a,3a,4a,5a,.....,....

TEOREMA DI DIRICHLET (1837)

a,b in Z

IMPREVEDIBILI NUMERI PRIMI: MA CAMBIAMO PROSPETTIVA

PENSIAMO INVECE ORA ALLE "STRUTTURE" PIÙ ORDINATE.....

LE PROGRESSIONI ARITMETICHE:

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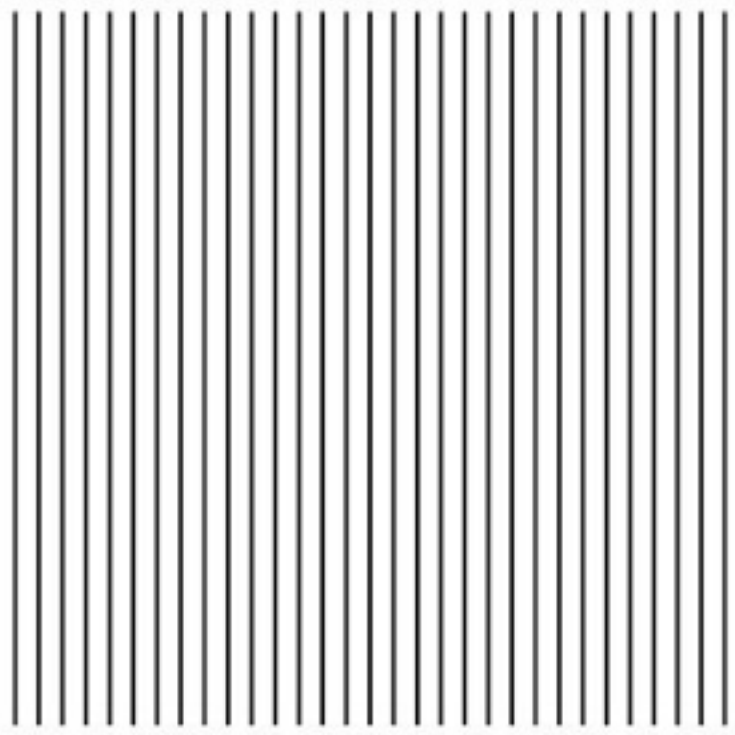
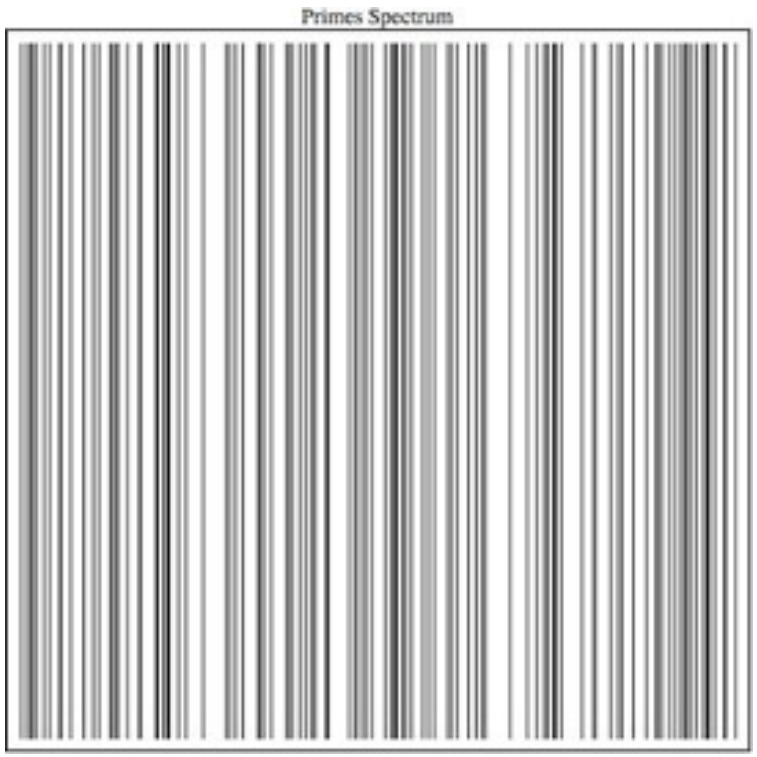


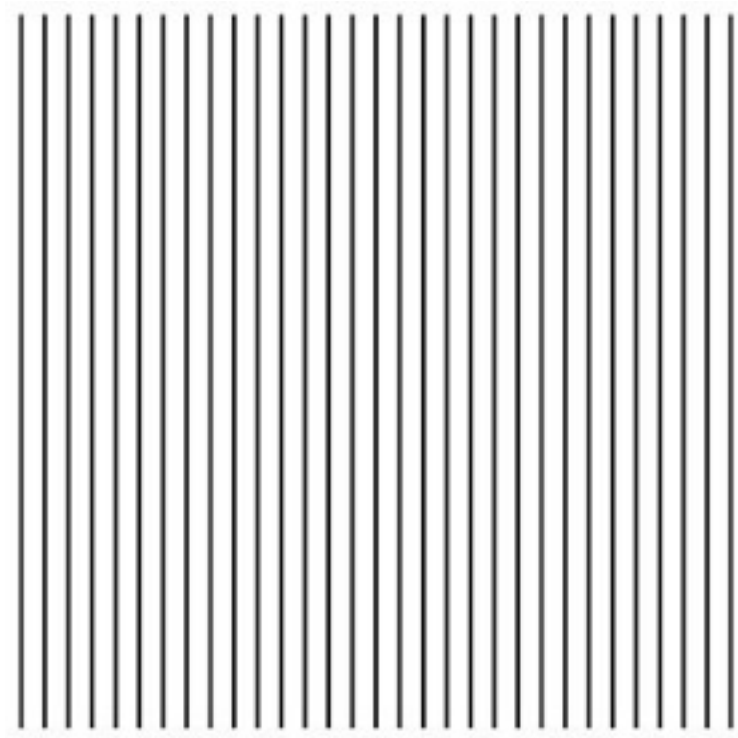
IN GENERALE: $a \times n$, n=1,2,3,.....

a, 2a,3a,4a,5a,.....,....

TEOREMA DI DIRICHLET (1837)

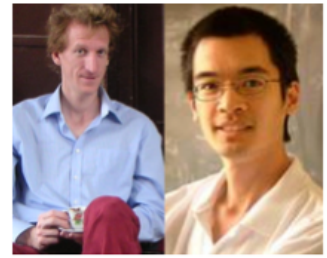
IMPREVEDIBILI NUMERI PRIMI: MA CAMBIAMO PROSPETTIVA





TEOREMA DI DIRICHLET (1837)

Il Teorema di Green & Tau (2008)



Ben Green and Terry Tao

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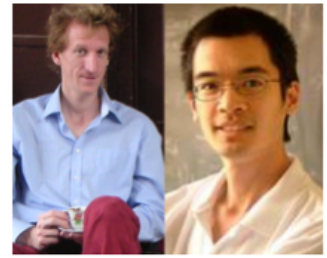
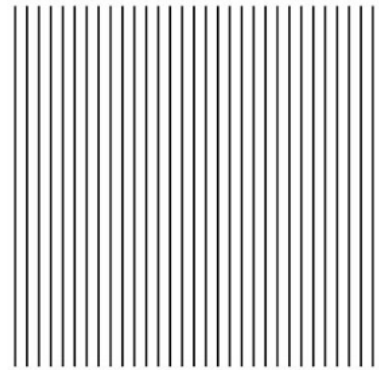
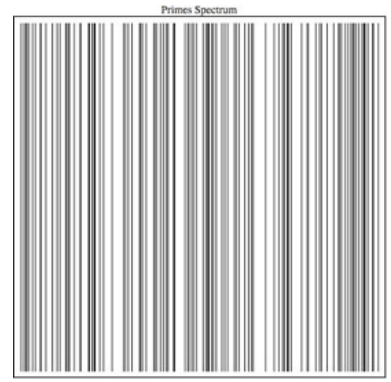


Ben Green and Terry Tao





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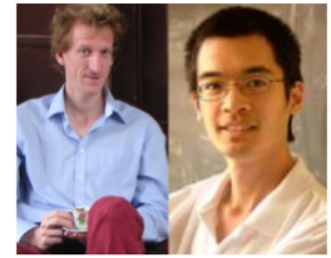
TERRY TAO:(Adelaide, 1975), Università della California, medaglia fields 2006

Il Teorema di Green & Tau (2008)

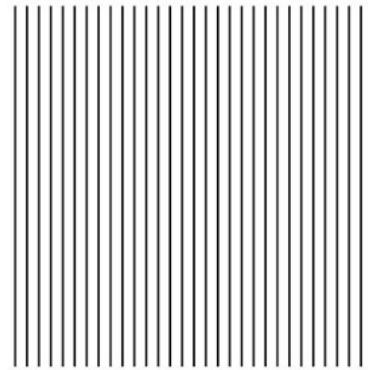


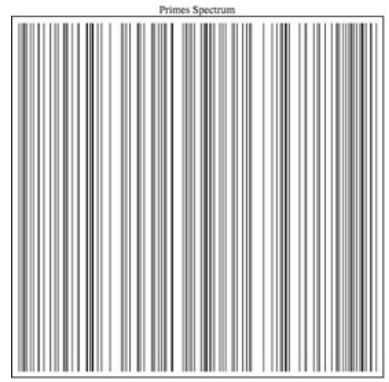
Ben Green and Terry Tao

Ordine & Disordine

Convivono perfettamente.....

(necessariamente)





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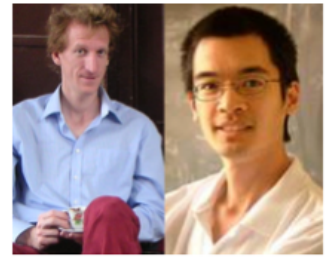


Ben Green and Terry Tao

Ordine & Disordine

Convivono perfettamente.....

(necessariamente)

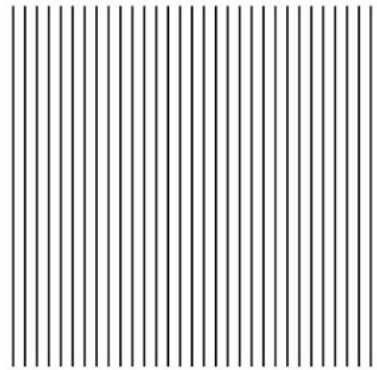
Annals of Mathematics, **167** (2008), 481–547

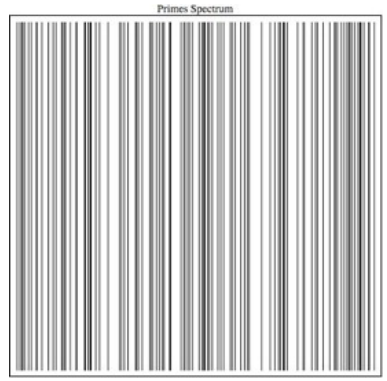
The primes contain arbitrarily long arithmetic progressions

By Ben Green and Terence Tao*

Abstract

We prove that there are arbitrarily long arithmetic progressions of primes. There are three major ingredients. The first is Szemerédi's theorem, which asserts that any subset of the integers of positive density contains progressions of arbitrary length. The second, which is the main new ingredient of this paper, is a certain transference principle. This allows us to deduce from Szemerédi's theorem that any subset of a sufficiently pseudorandom set (or measure) of positive *relative* density contains progressions of arbitrary length. The third ingredient is a recent result of Goldston and Yıldırım, which we reproduce here. Using this, one may place (a large fraction of) the primes inside a pseudorandom set of "almost primes" (or more precisely, a pseudorandom measure concentrated on almost primes) with positive relative density.





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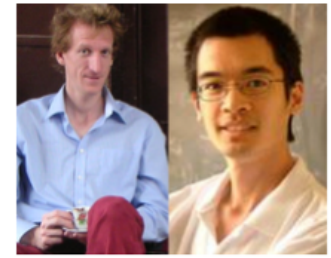

Ben Green and Terry Tao

We now apply Lemma 10.4 with $\sigma := 1/6m$ and $G := G_0G_1G_2$. Again by the Leibniz rule we have the bound (10.10), and furthermore

$$\|G\|_{C^r(\mathcal{D}_\sigma^m)} \leqslant O_m(1)O_{m,w(N)}(1)\left(\frac{\log R}{\log\log R}\right)^r \prod_{p|\Delta}\left(1+O_m(p^{-1/2})\right)$$

for all $0 \leqslant r \leqslant m$. From Lemma 10.6 and Lemma 10.4 we can then estimate (10.5) as

$$\leqslant (1+o_m(1))\left(\frac{W}{\phi(W)}\right)^m \log^m R \prod_{p|\Delta}\left(1+O_m(p^{-1/2})\right)$$
$$+ O_{m,w(N)}\left(\frac{\log^m R}{\log\log R}\right)\prod_{p|\Delta}\left(1+O_m(p^{-1/2})\right) + O_m(e^{-\delta\sqrt{\log R}}).$$

The claim (9.1) then follows by choosing $w(N)$ (and hence W) sufficiently slowly growing in N (and hence in R). Proposition 9.6 follows. □

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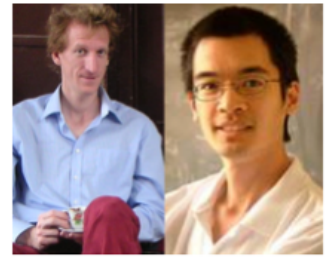

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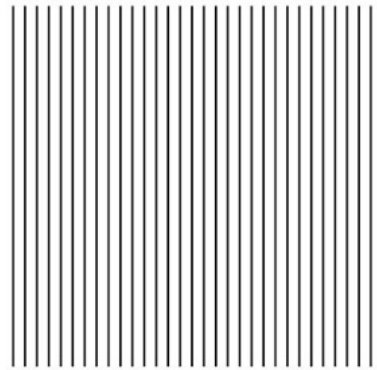

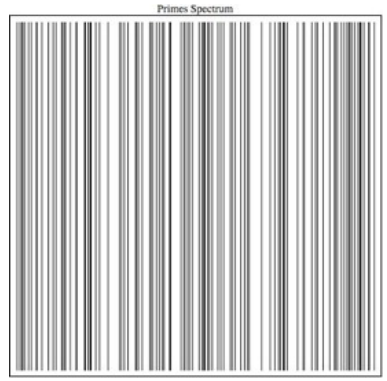

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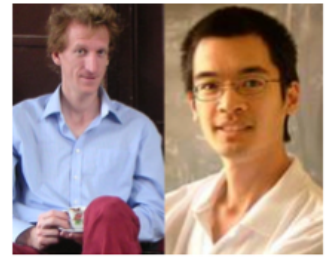

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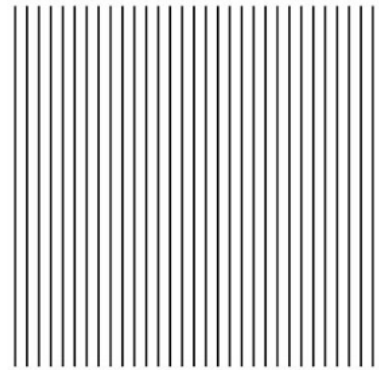

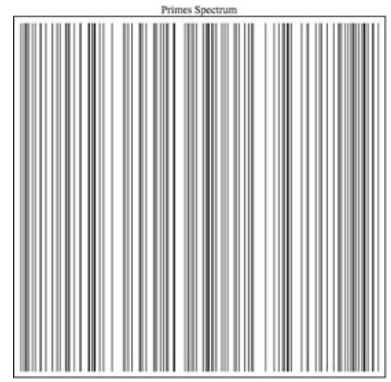

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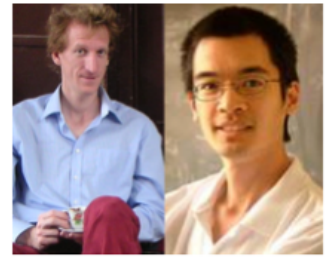

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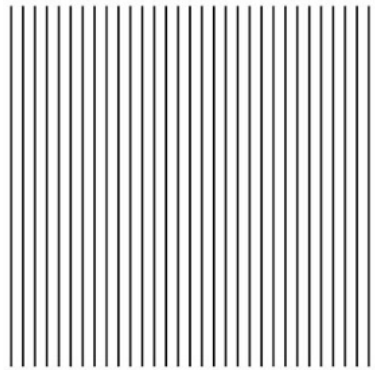
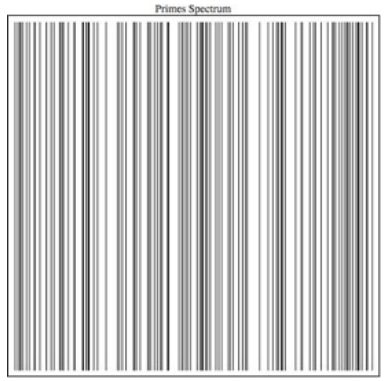
10 primi ordinati (a salti di 210)

199, 409, 619, 829, 1039, 1249, 1459, 1669, 1879, 2089





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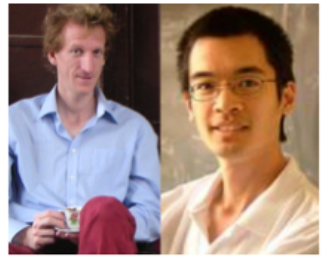

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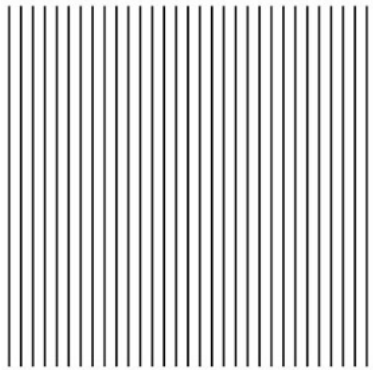
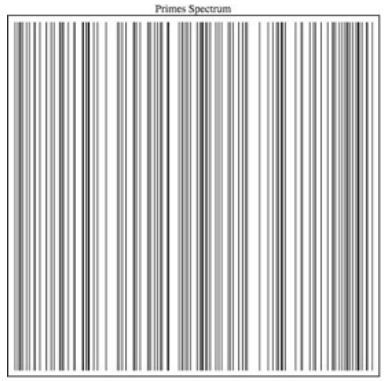
L'esempio più lungo noto....



43 142 746 595 714 191 + 23,681,770 · 223,092,870 · n



n=0,1,2,3,....,25

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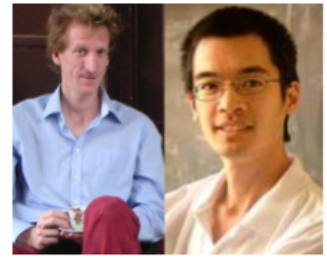

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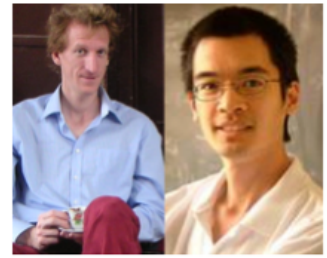

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ma cosa abbiamo Imparato ??

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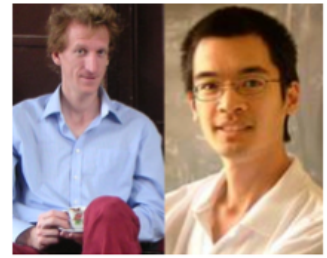

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MA COSA ABBIAMO IMPARATO ??

- SI PUÒ PASSARE LA VITA A STUDIARE QUESTE COSE

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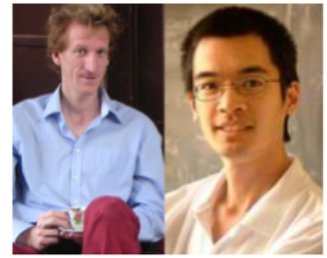

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- LE "STRUTTURE" INTERESSANTI (COMPLESSE) SI TROVANO LÀ DOVE L'ORDINE E STRUTTURE RIGIDE CONVIVONO CON IL NECESSARIO CASO.....

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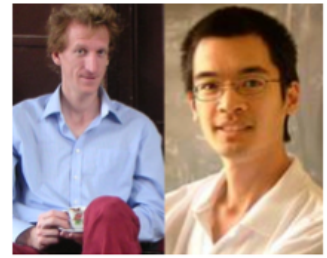

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...E LA MATEMATICA LO HA CAPITO TANTO TEMPO FA.....

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TRA ORDINE E CHAOS, UN PROFICUO PARADIGMA

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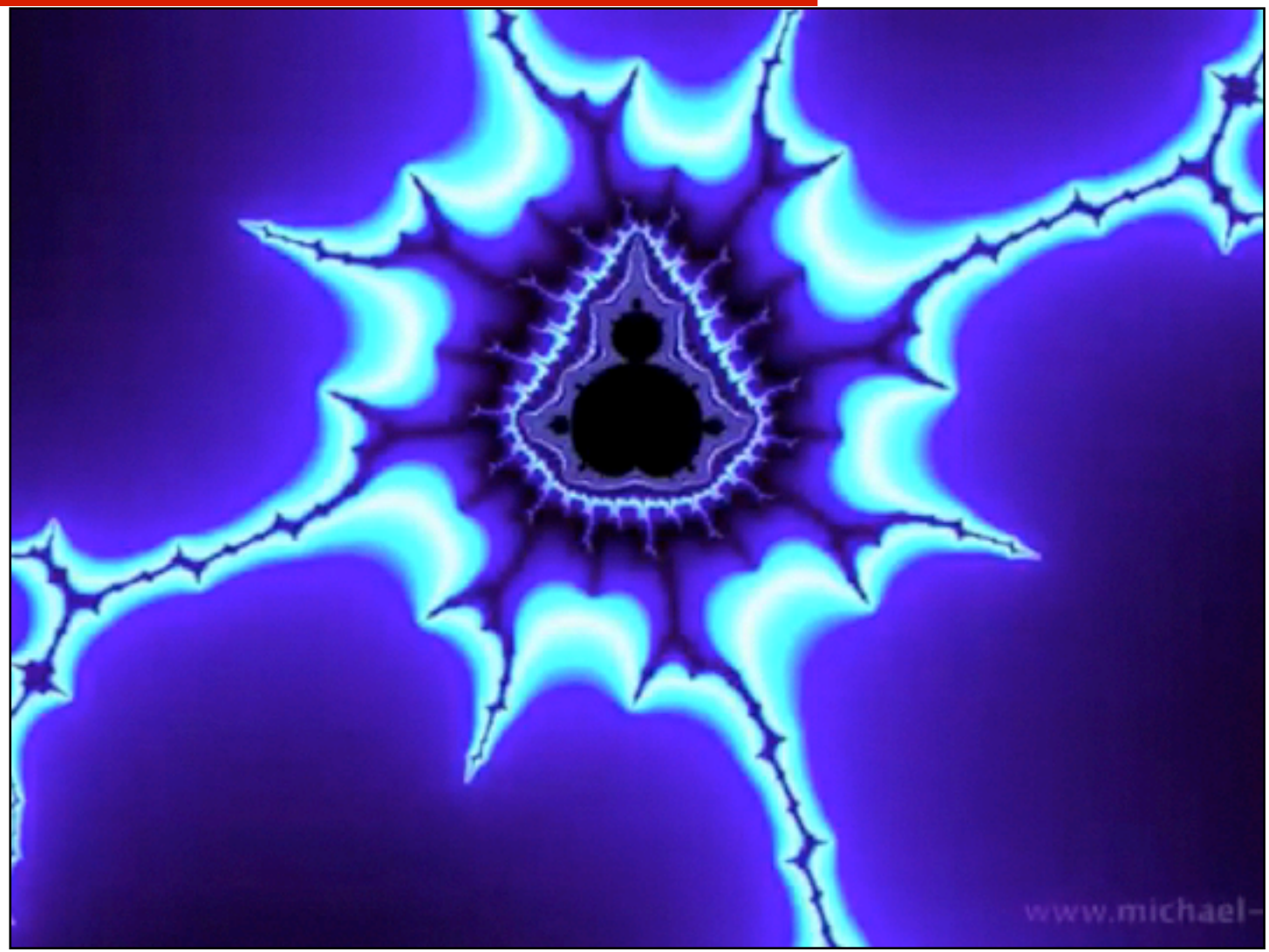
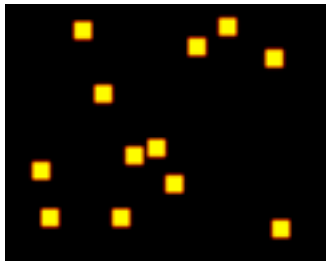
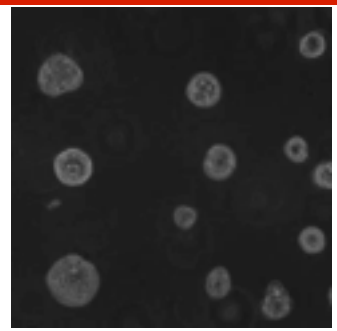
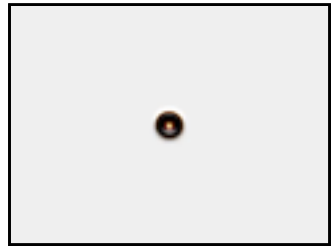


DA JULES HENRI POINCARÉ (1854 – 1912) ALLA TEORIA DEL CAOS

Non solo numeri.....

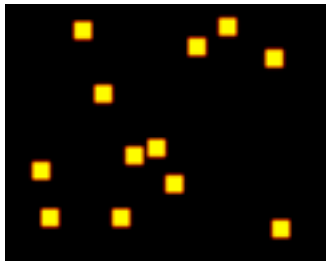


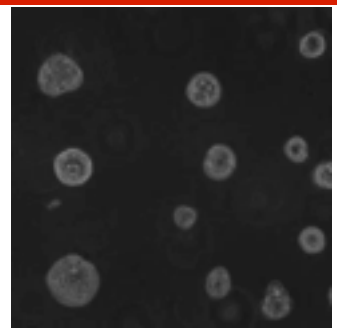


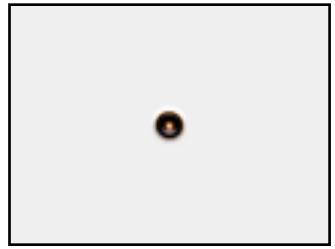


"The chemical basis of Morphogeneis" Alan Turing (1952): modello reazione-diffusione

Non solo numeri.....







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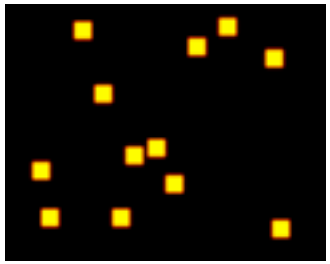
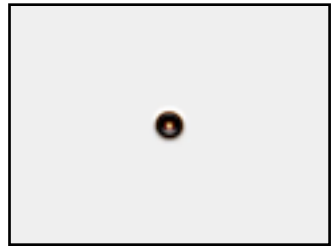
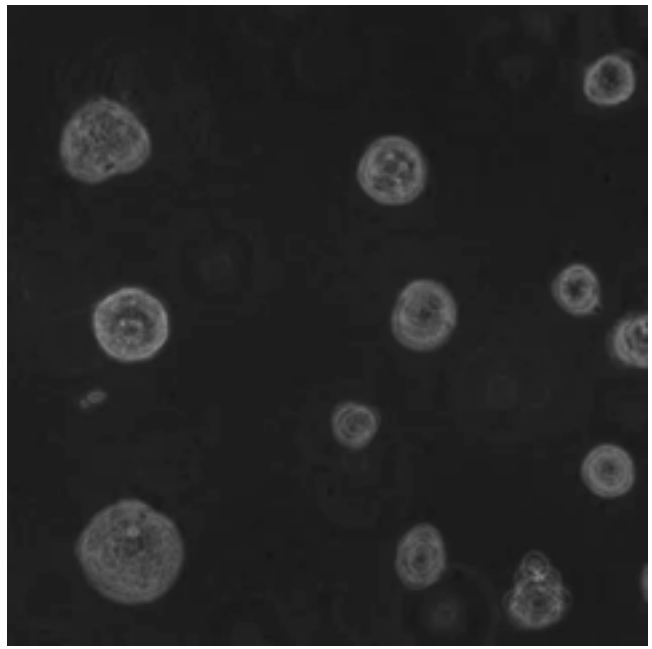


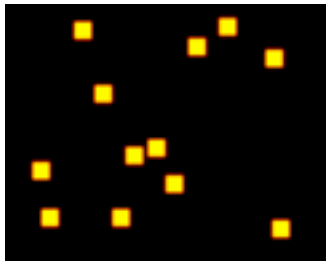


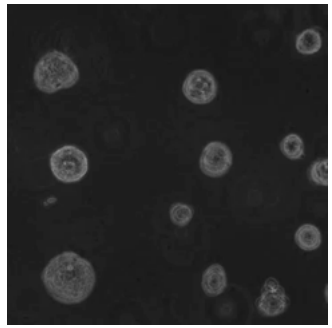


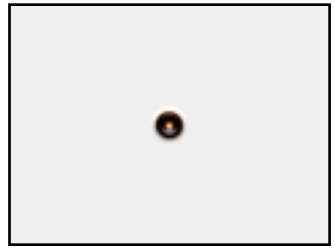
MCF10:
CELLULE EPITELIALI DEL SENO

"The chemical basis of Morphogeneis" Alan Turing (1952): modello reazione-diffusione

Non solo numeri.....







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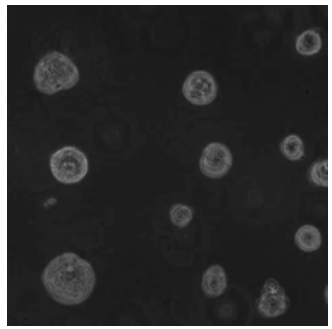

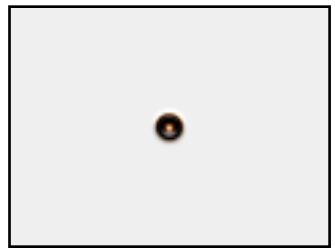

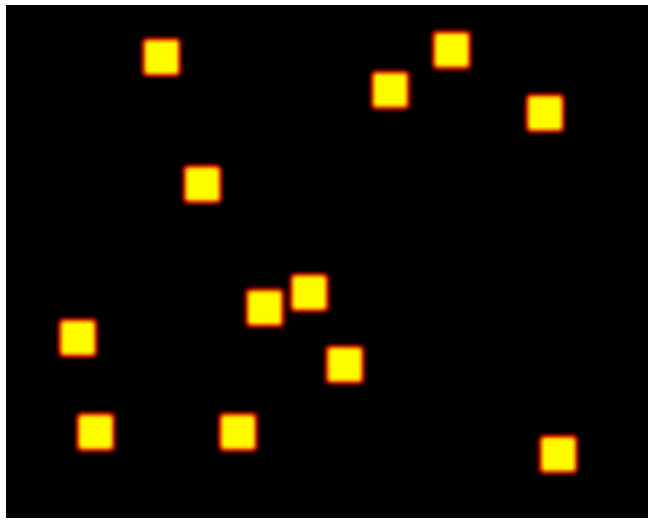

$$\partial_t u = D\partial_x^2 u + R(u),$$

KPP (Kolmogorov-Petrovsky-Piskounov)

"The chemical basis of Morphogeneis" Alan Turing (1952): modello reazione-diffusione

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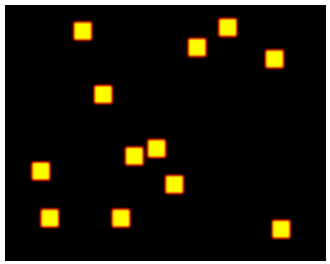

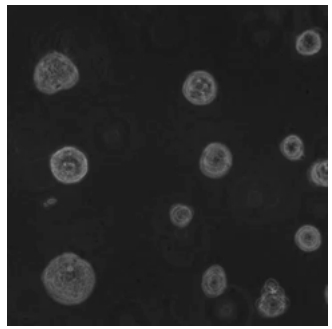

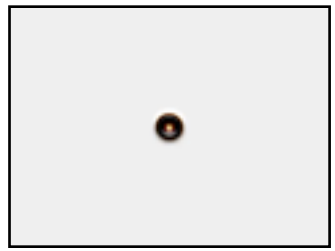

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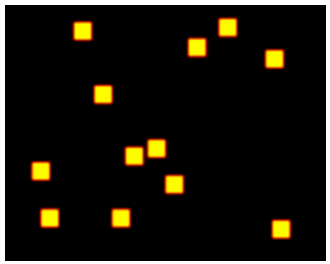
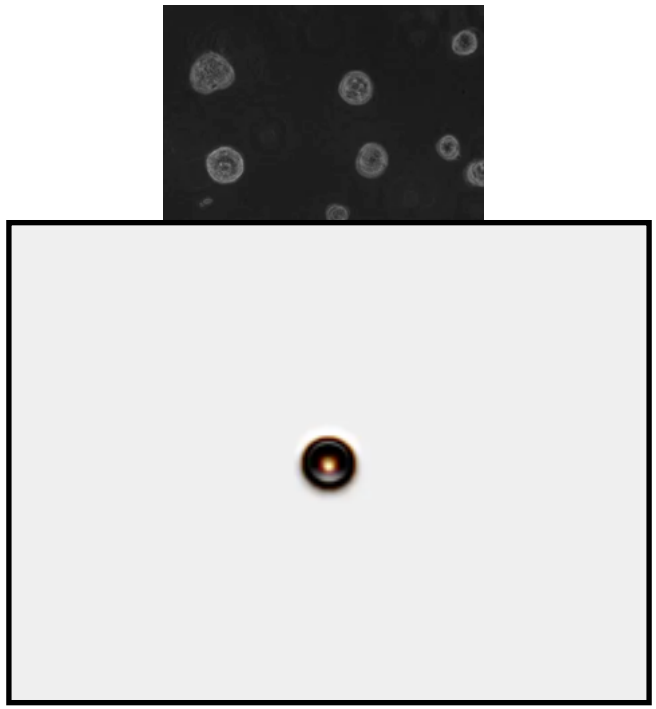
KPP (Kolmogorov-Petrovsky-Piskounov)

"The chemical basis of Morphogeneis" Alan Turing (1952): modello reazione-diffusione

Non solo numeri.....





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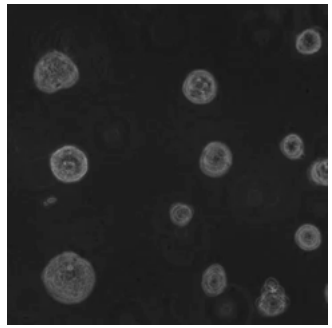

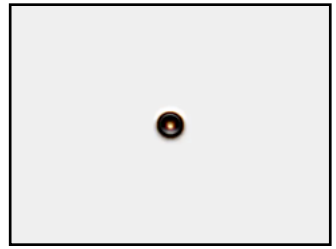

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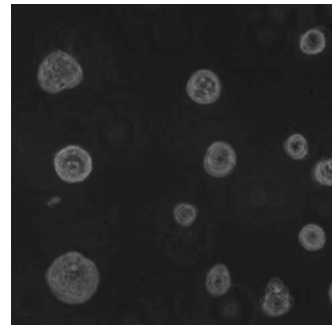

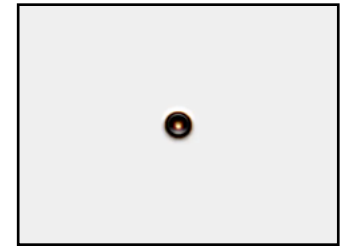

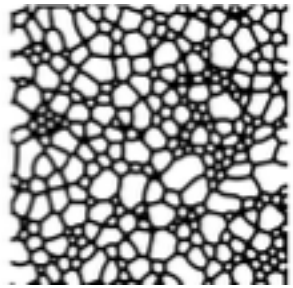

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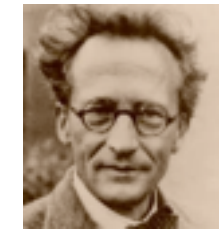

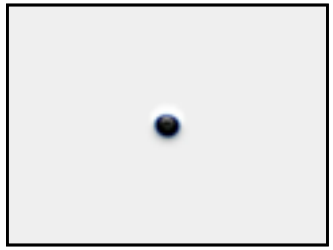

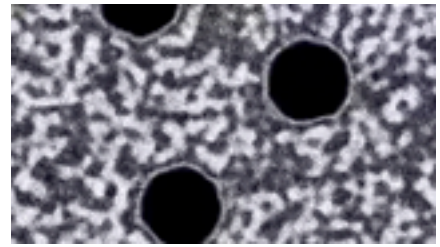

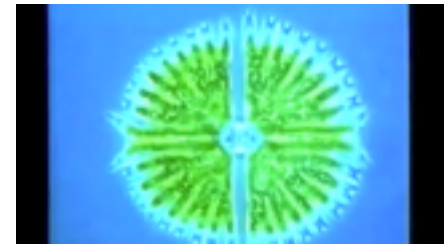

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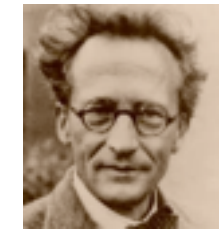

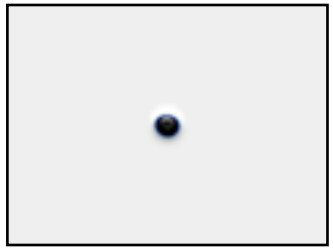

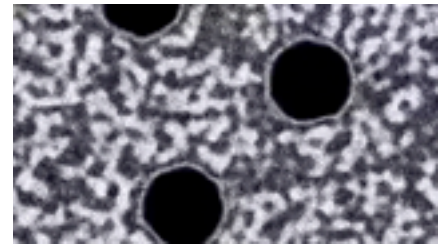

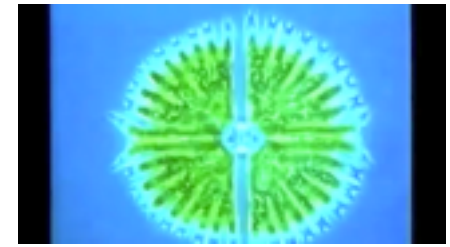

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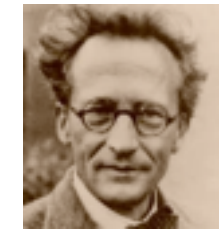

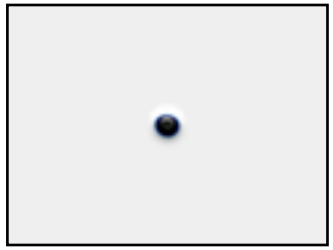

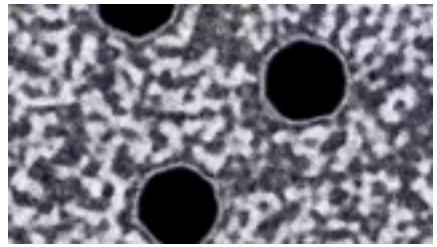

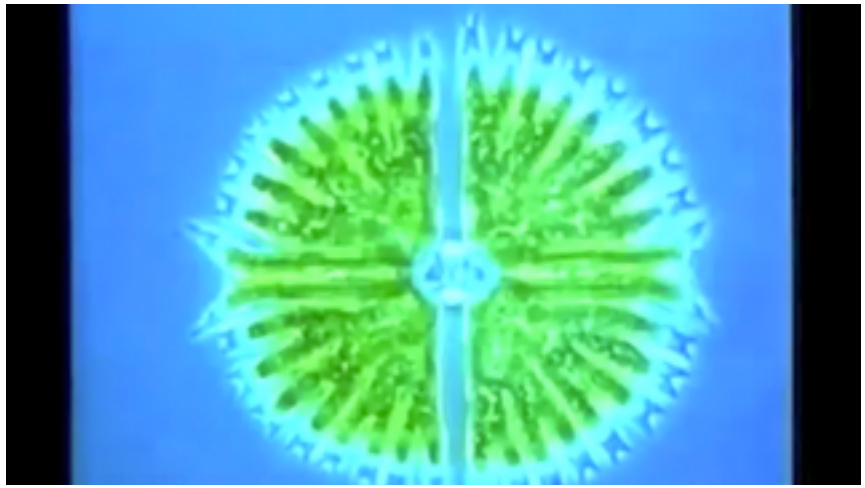

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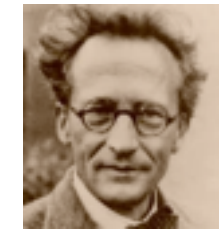

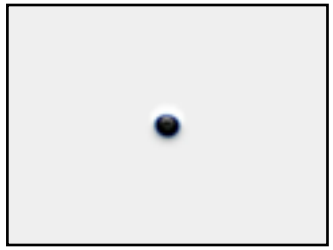

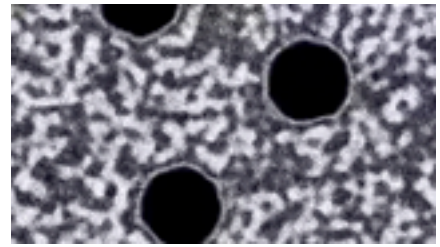

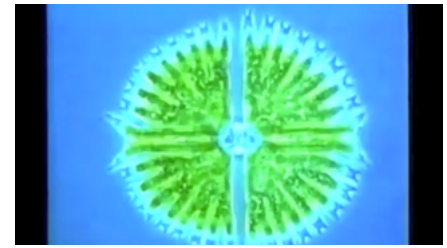

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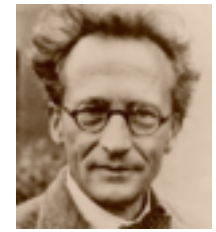

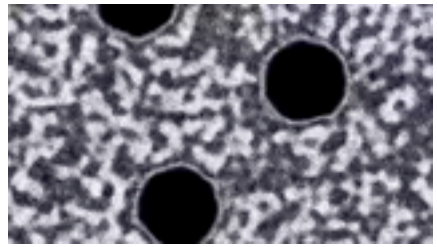

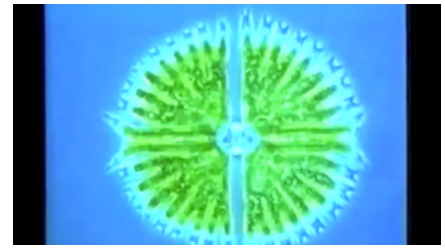

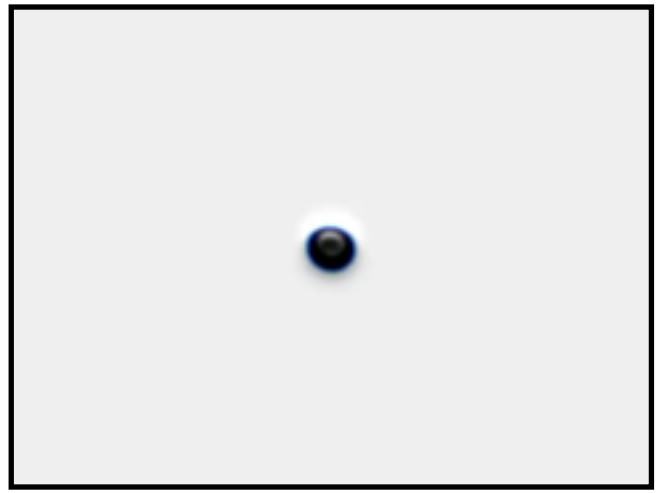

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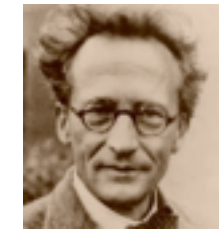

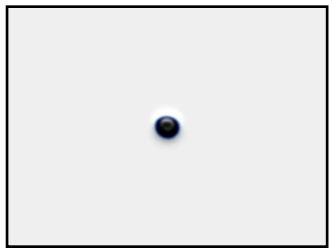

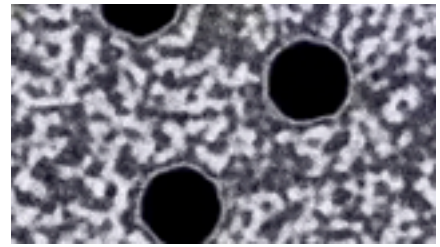

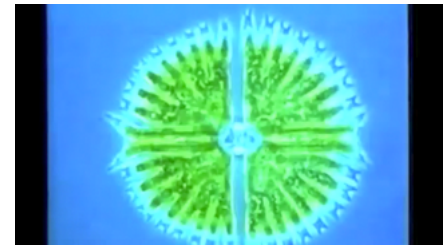

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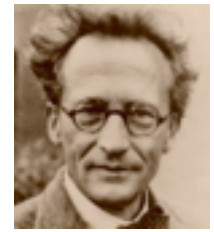

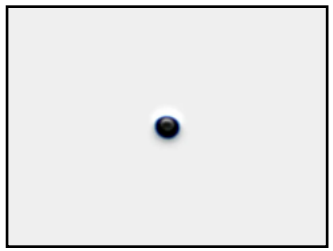

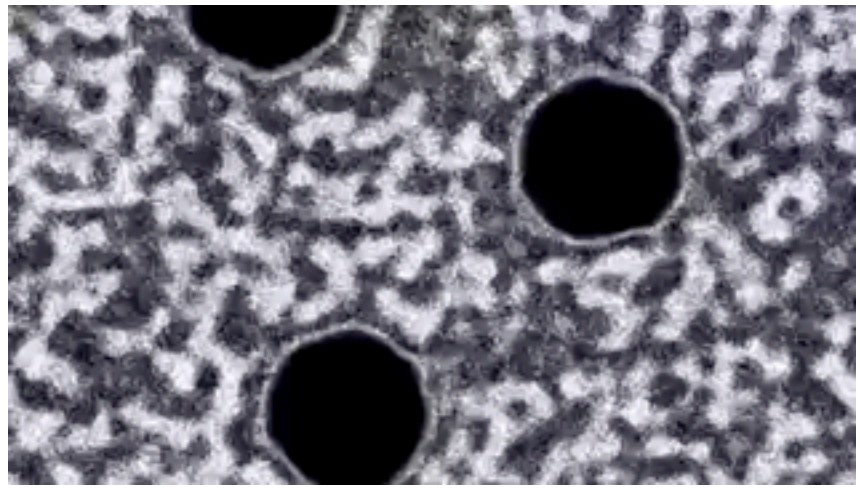

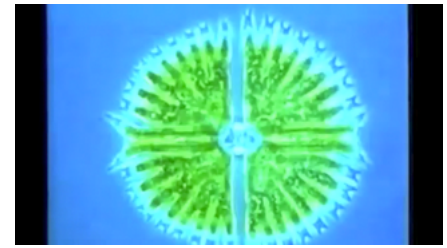

Gioco della Vita di Conway
(1970)

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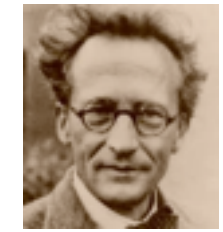

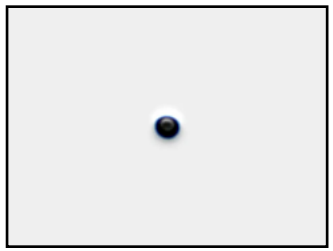

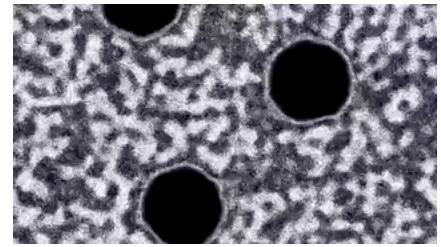

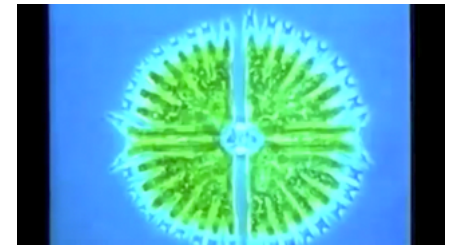

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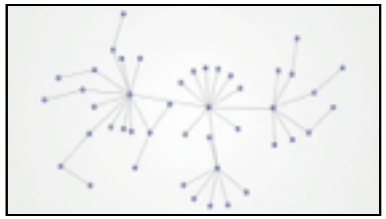

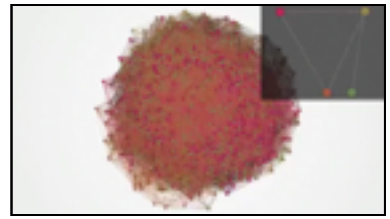

"A Mathematical Theory of Communication" (1949) Claude Shannon

& Network Theory

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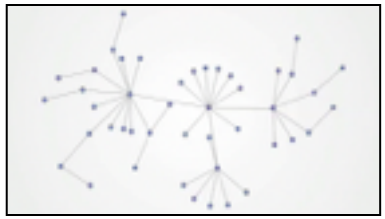

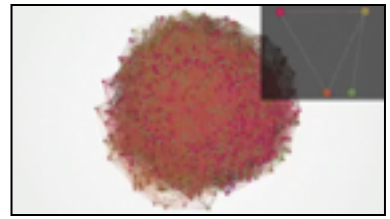

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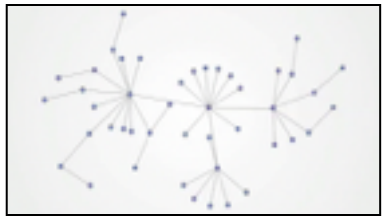

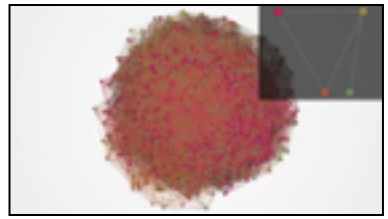

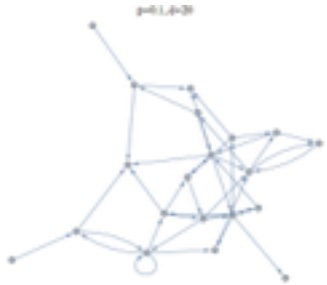

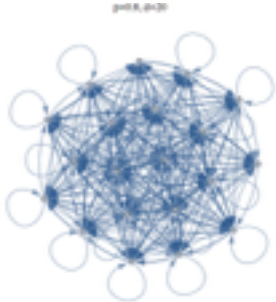

"On Random Graphs"
(1959)

Paul Erdős and Alfréd Rényi

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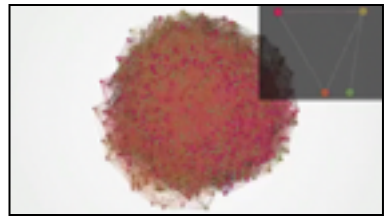

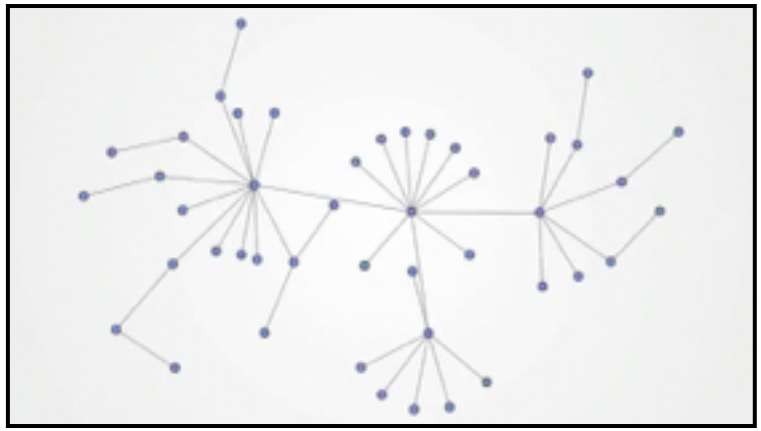

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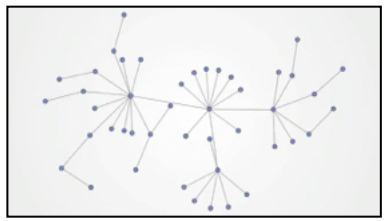

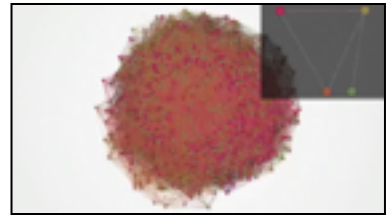

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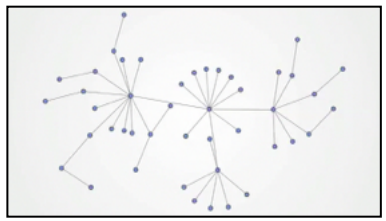

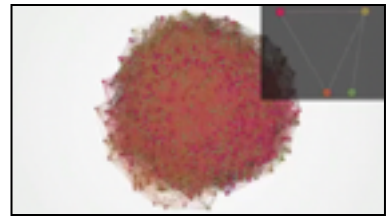

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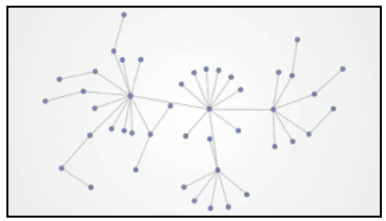

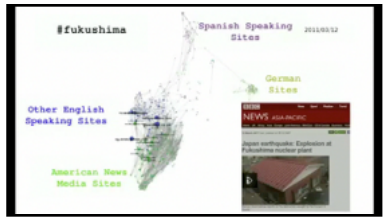

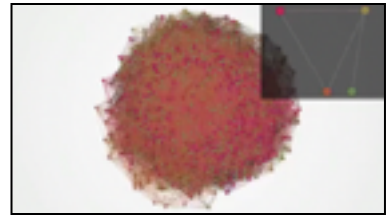

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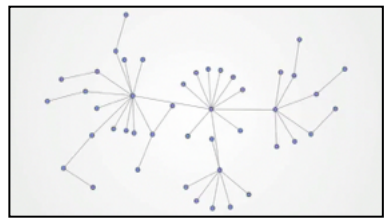

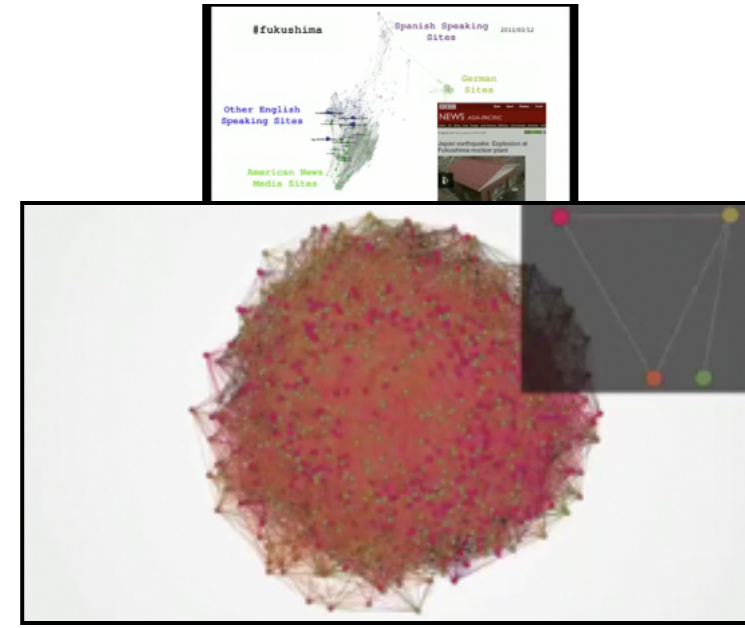

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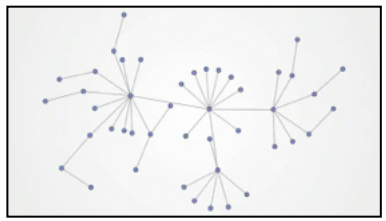
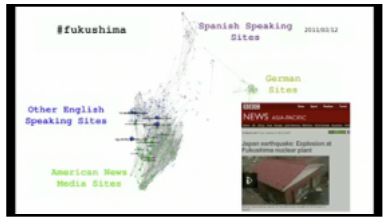
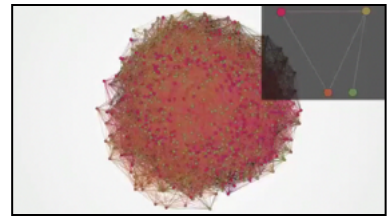
& Network Theory

Non solo numeri.....l'informazione







"A Mathematical Theory of Communication" (1949) Claude Shannon

& Network Theory

Non solo numeri……il linguaggio

"Language is a remarkable faculty by which humans convey thoughts to one another by means of a highly structured signal"

Steven Pinker

Non solo numeri.....il linguaggio

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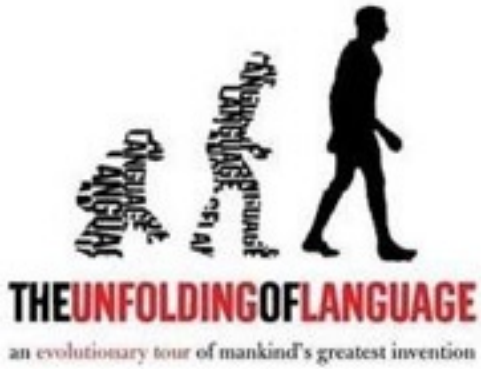
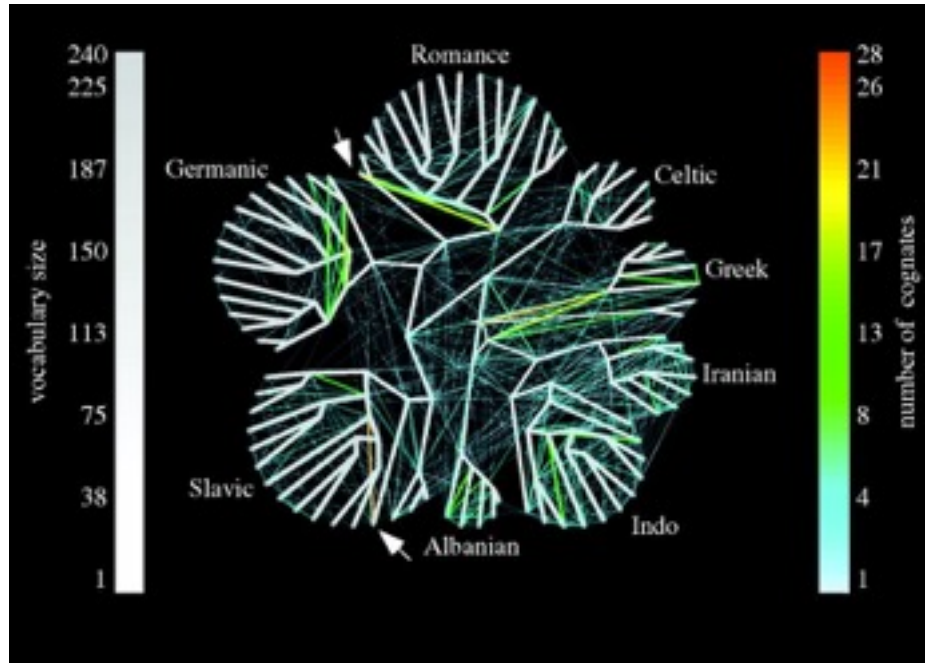
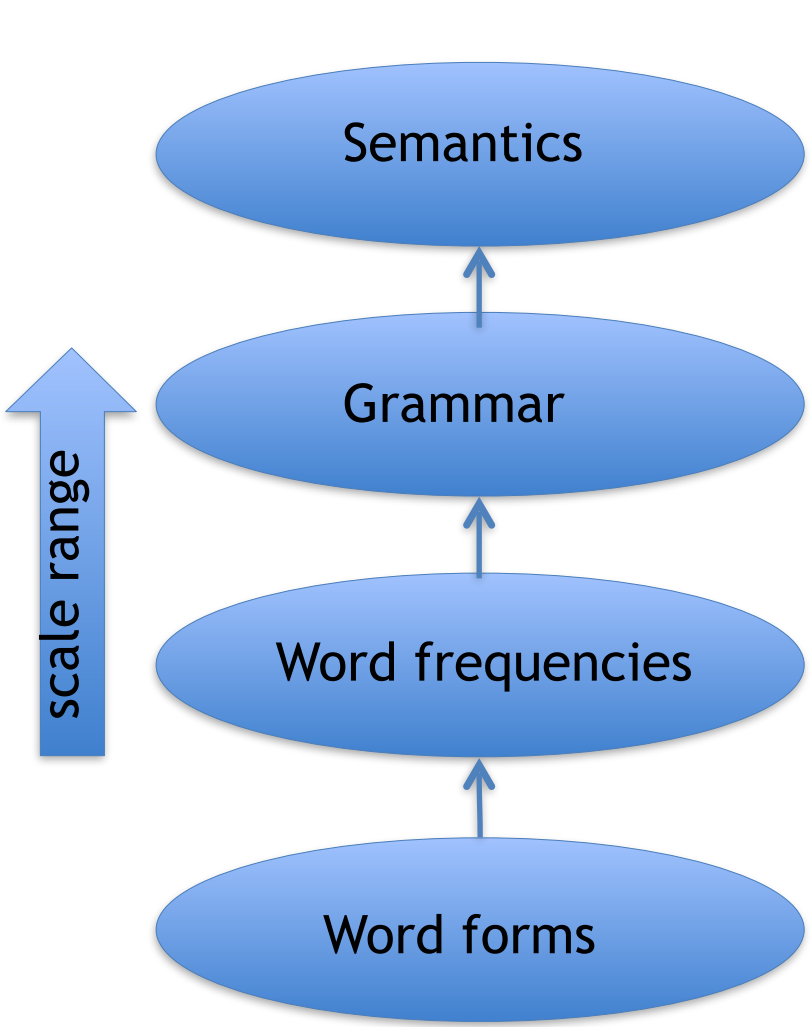




Towards a measure of semantic information



Nested hierarchies in language

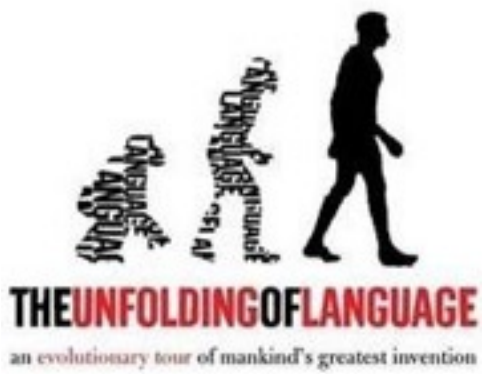
From sentence length to hundreds or thousands of words

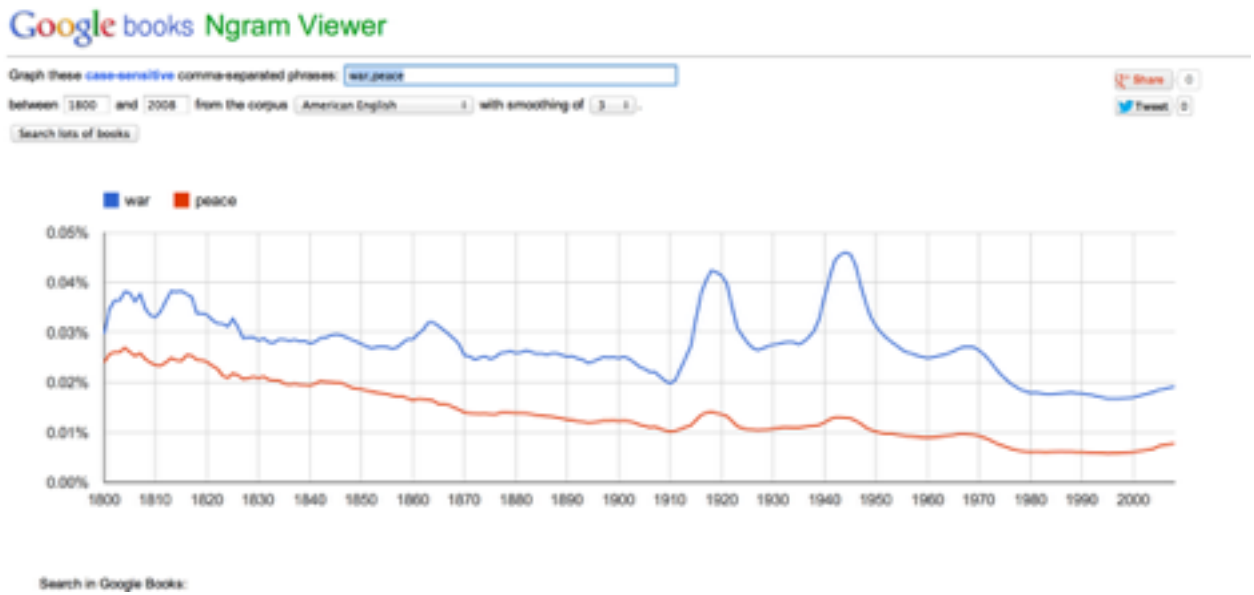
Sentence length range

Word length range

Montemurro & Zanette. Adv Comp Syst (2010)
Montemurro, Cortex (2014)

Non solo numeri.....il linguaggio





Non solo numeri.....il linguaggio

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r	*Æneid*		*Don Quijote*		*D. Copperfield*	
1	*et*	2146	*que*	20398	*the*	13763
2	*in*	761	*y*	17946	*I*	13472
3	*nec*	353	*de*	17935	*and*	12332
4	*per*	332	*la*	10225	*to*	10503
5	*ad*	321	*a*	9692	*of*	8748
6	*atque*	295	*en*	8077	*a*	7991
7	*non*	280	*el*	8066	*in*	6253
8	*cum*	240	*no*	6166	*that*	5388
9	*tum*	239	*los*	4697	*was*	5317
10	*quæ*	239	*se*	4644	*my*	5207
11	*est*	218	*con*	4154	*it*	5042
12	*nunc*	214	*por*	3838	*her*	3875
13	*aut*	214	*las*	3426	*you*	3730
14	*hæc*	212	*lo*	3414	*me*	3619
15	*iam*	203	*le*	3380	*he*	3610

NON SOLO NUMERI.....IL LINGUAGGIO

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r È IL RANK DI UNA PAROLA
n(r) IL NUMERO DELLE SUE OCCORRENZE
f(r) LA FREQUENZA (EMPIRICA)

r	*Æneid*		*Don Quijote*		*D. Copperfield*	
1	*et*	2146	*que*	20398	*the*	13763
2	*in*	761	*y*	17946	*I*	13472
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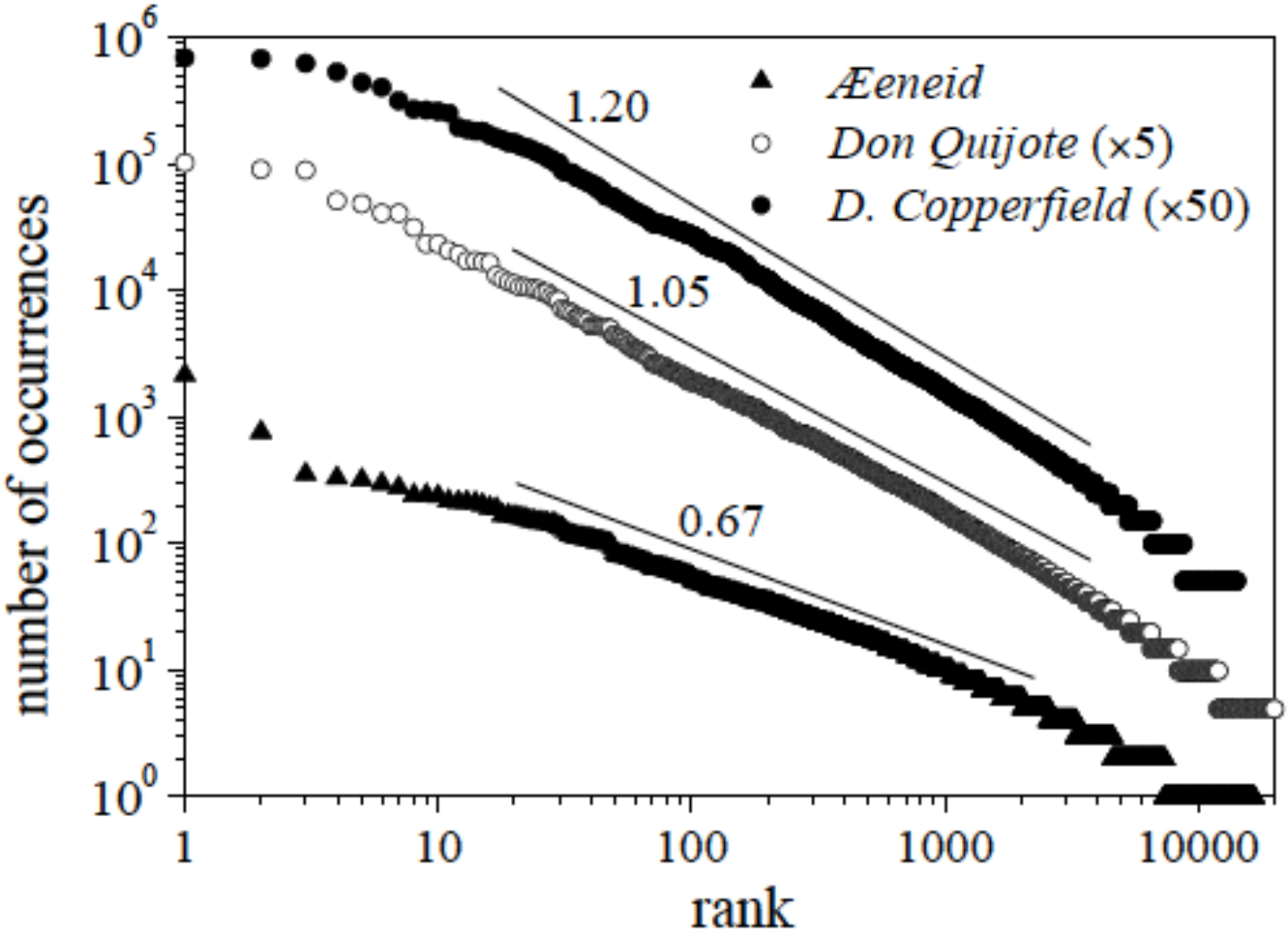
number of occurrences
rank
Æneid
Don Quijote (×5)
D. Copperfield (×50)
1.20
1.05
0.67
10^0
10^1
10^2
10^3
10^4
10^5
10^6
1
10
100
1000
10000

$$f(r) \sim r^{-z}$$

La legge di Zipf: EVERYWHERE !!!

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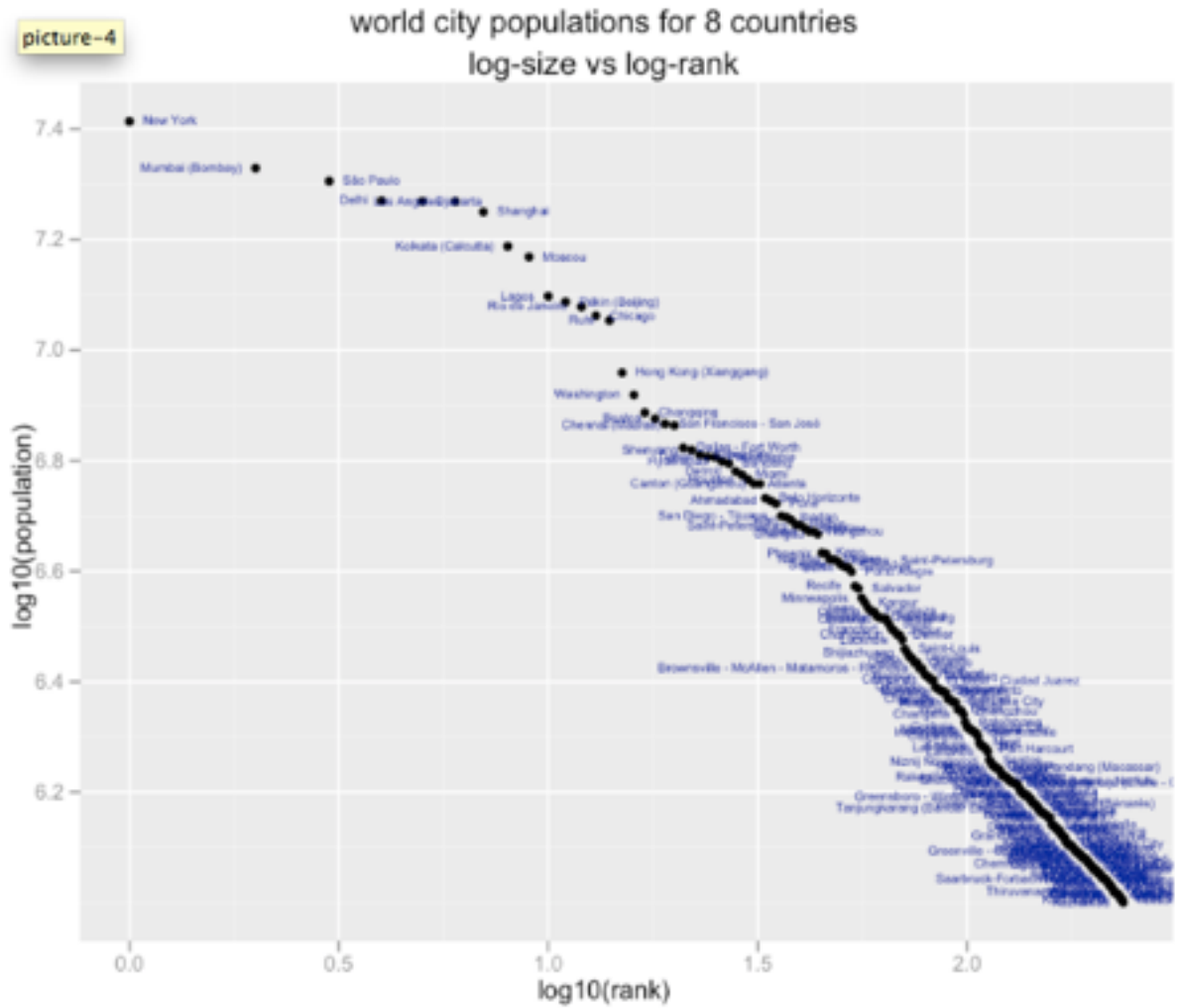

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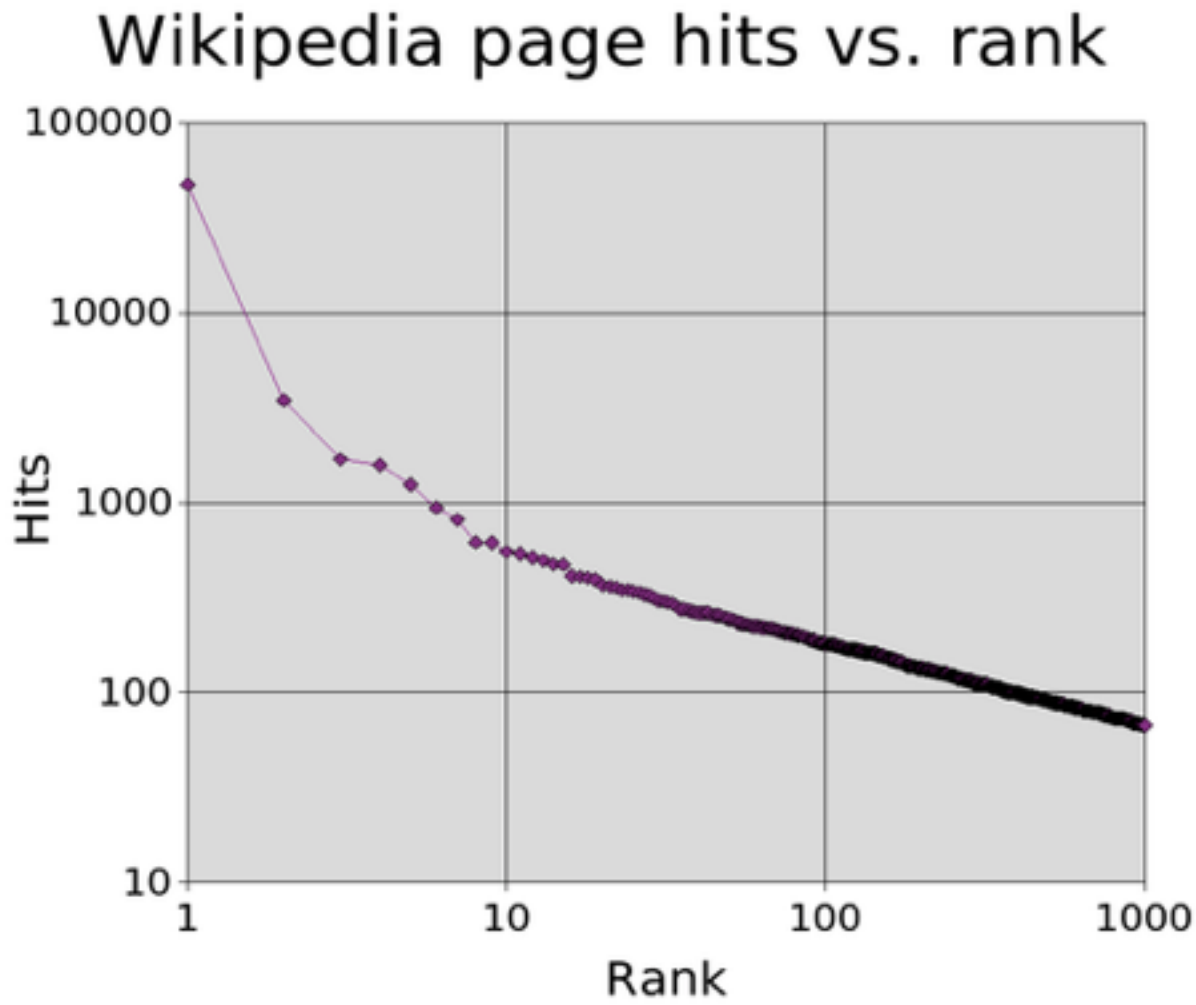
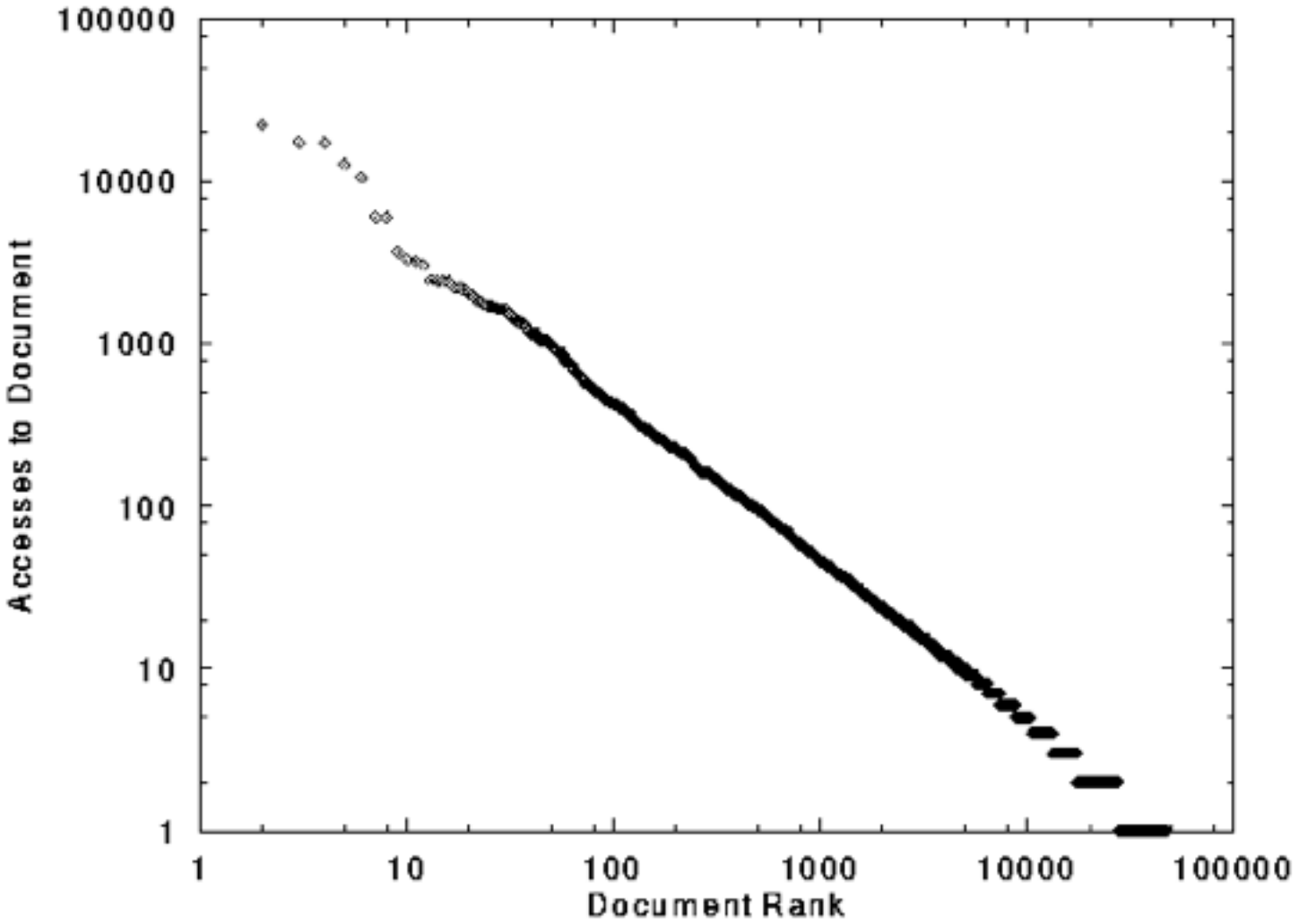
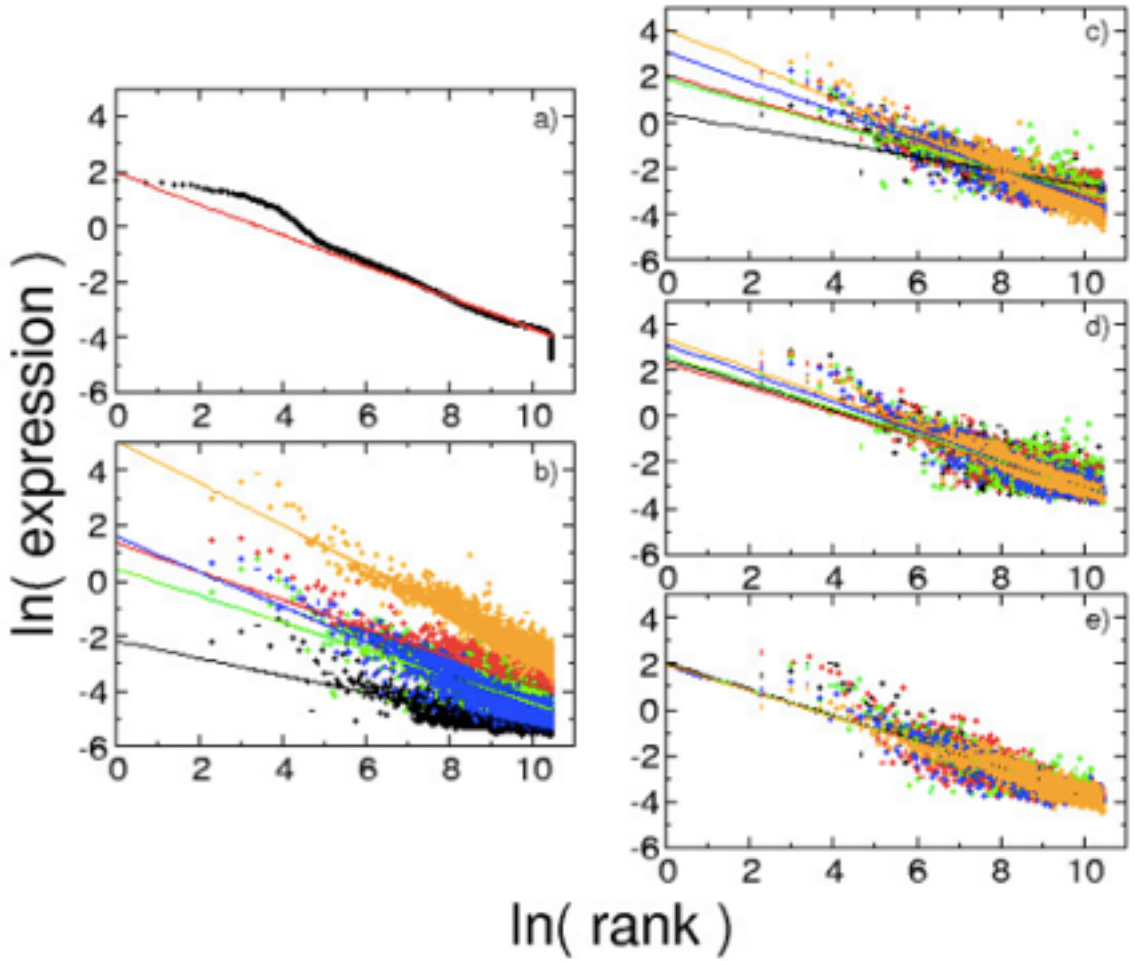


La legge di Zipf: EVERYWHERE !!!



La legge di Zipf: EVERYWHERE !!!



EVERYWHERE !!!

FORSE TROPPO.....

LEGGE PROFONDA O BANALE ??

LE NUOVE PAROLE…L'ANOMALIA DEI RECORDS

George Kingsley Zipf (/ˈzɪf/;[1] 1902–1950),

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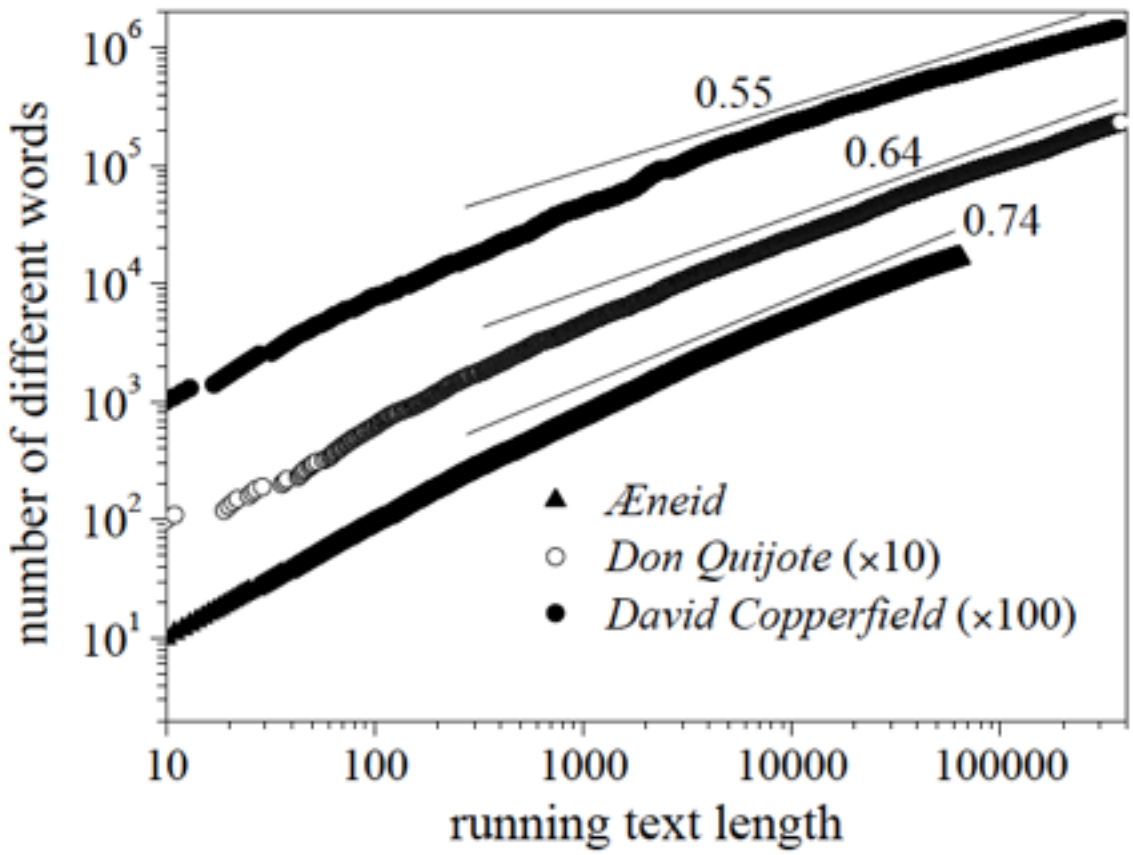

L'ULTIMO PASSO.....CREATIVITÀ & UNIVERSALITÀ

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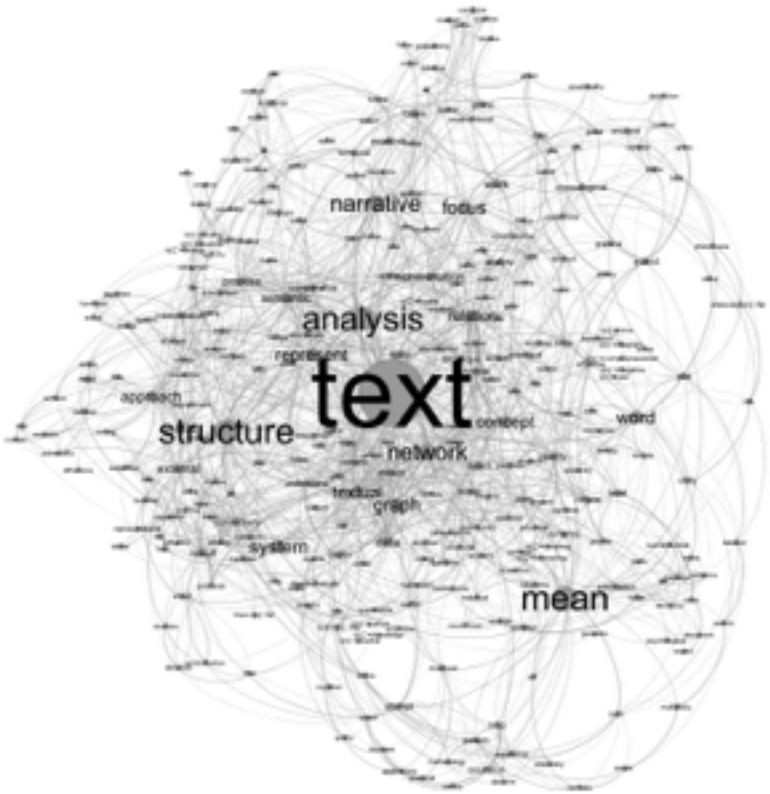
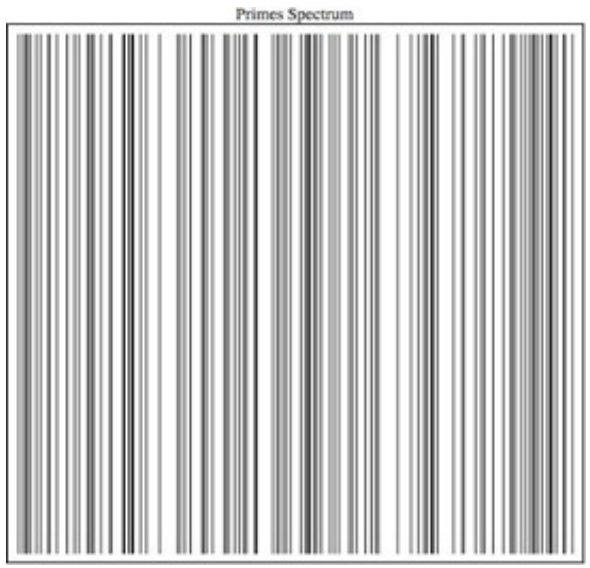


L'ULTIMO PASSO.....CREATIVITÀ & UNIVERSALITÀ

La Coscienza di Zeno
Italo Svevo (1923)

L'ULTIMO PASSO.....CREATIVITÀ & UNIVERSALITÀ



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ITALO SVEVO (1923)

CREATIVITÀ & UNIVERSALITÀ: AUTHORSHIP ATTRIBUTION

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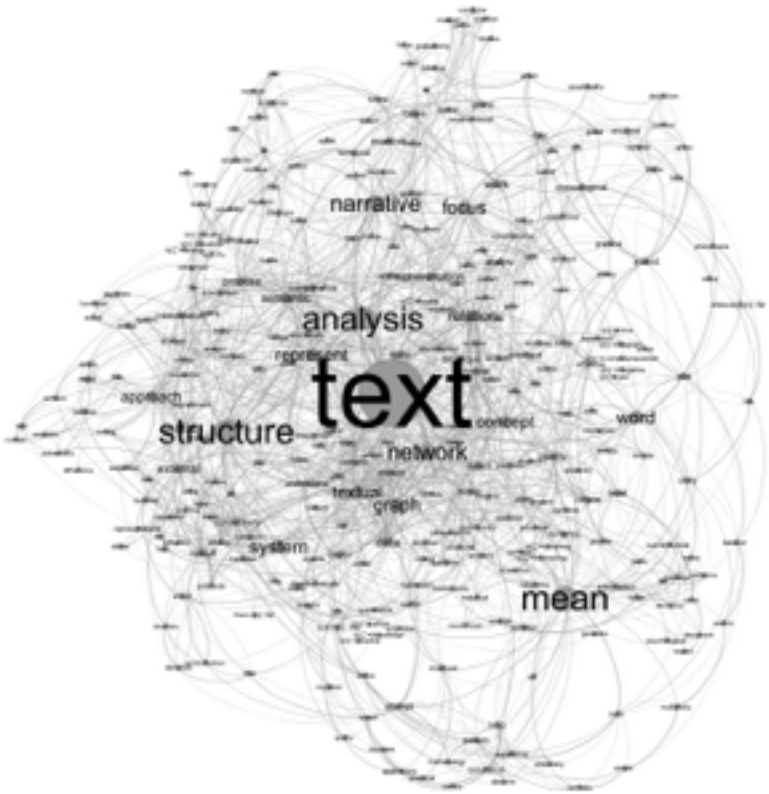

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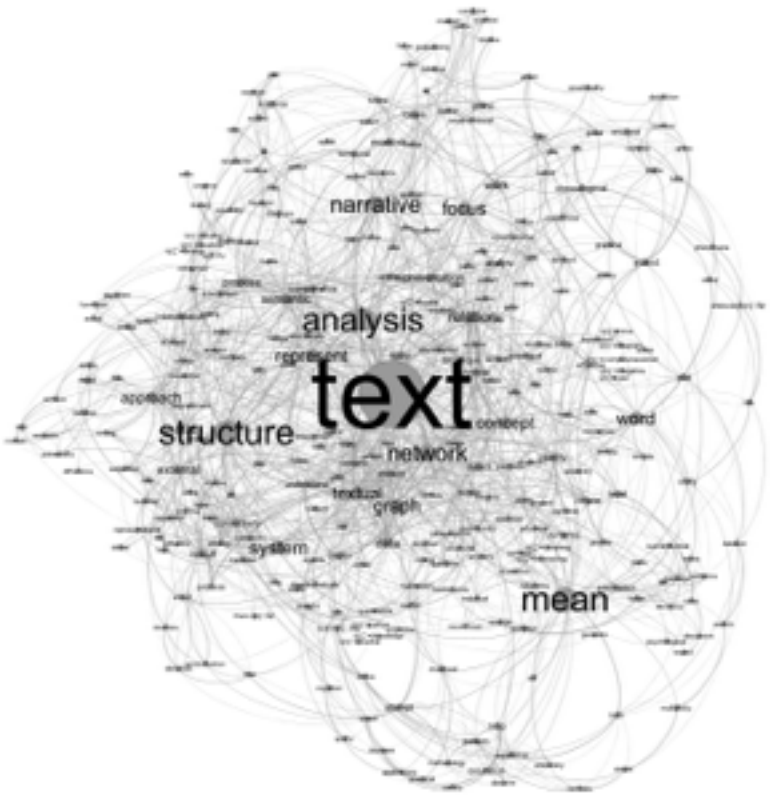

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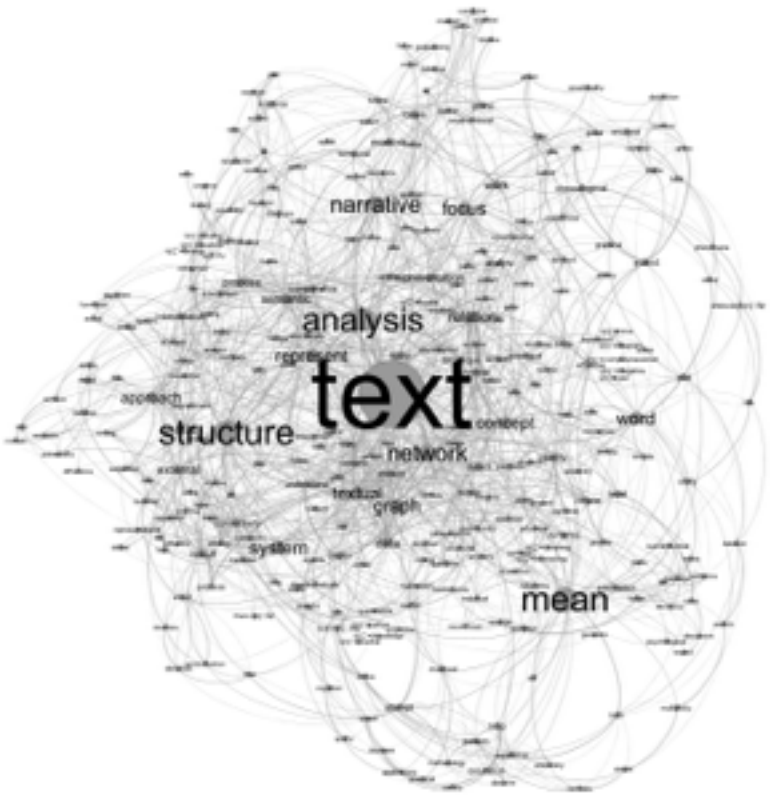

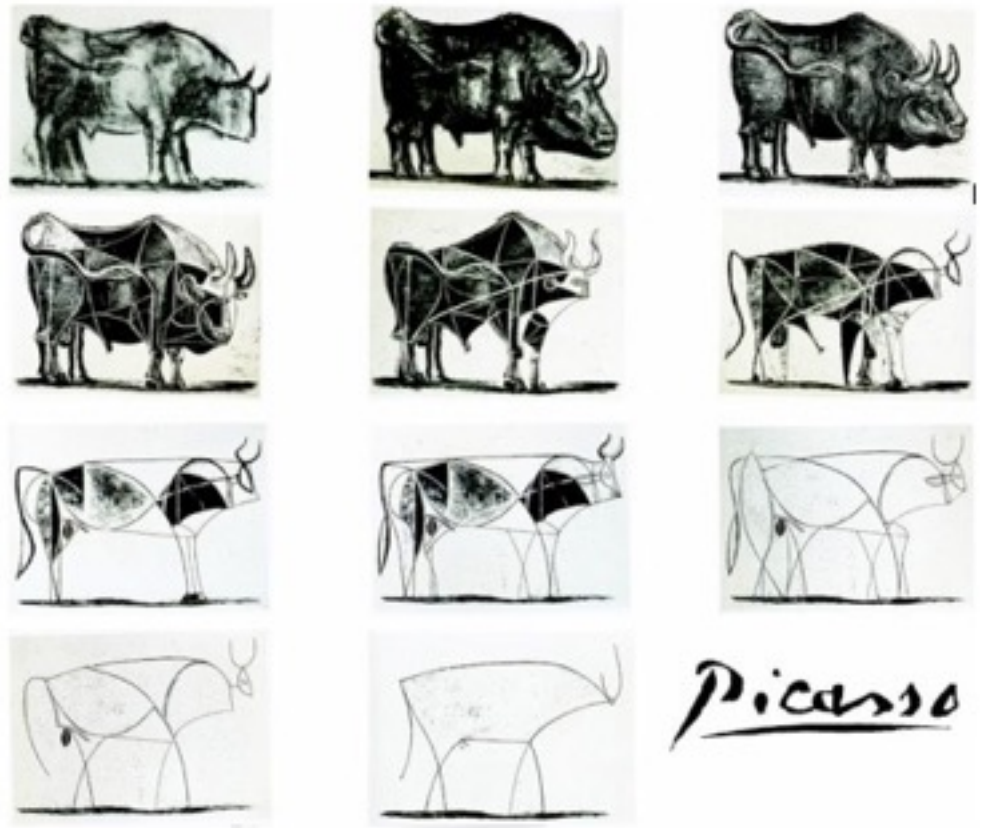

Pablo Picasso, Bull (plates I - XI) 1945

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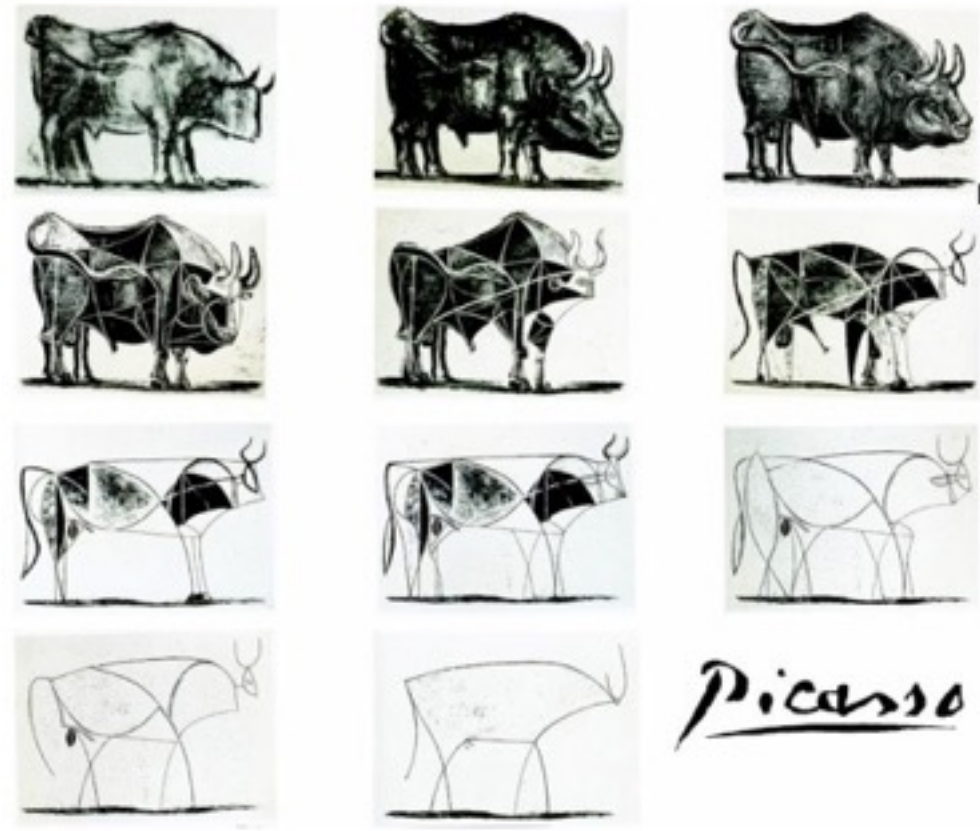

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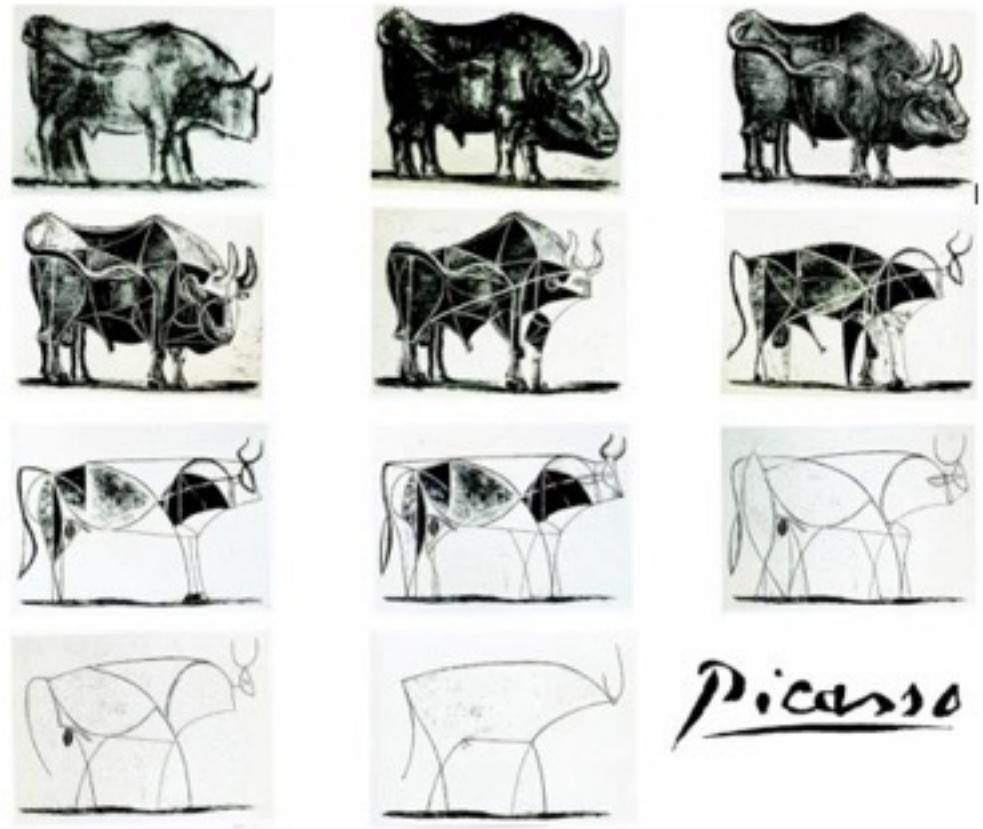

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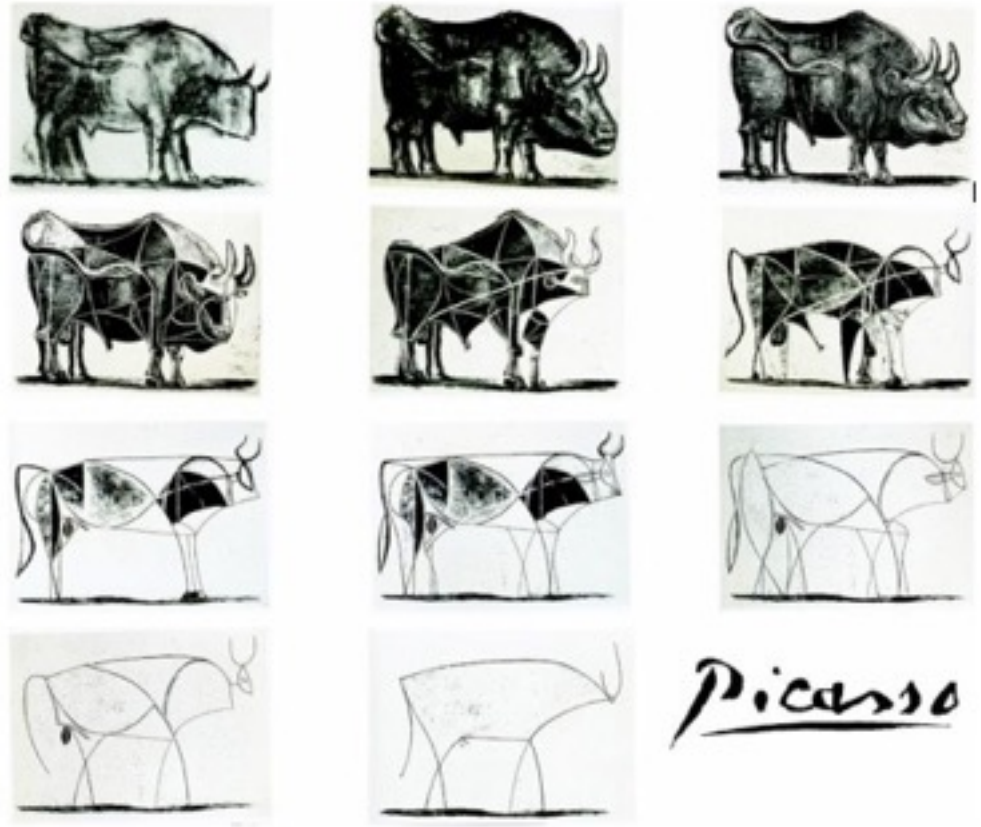

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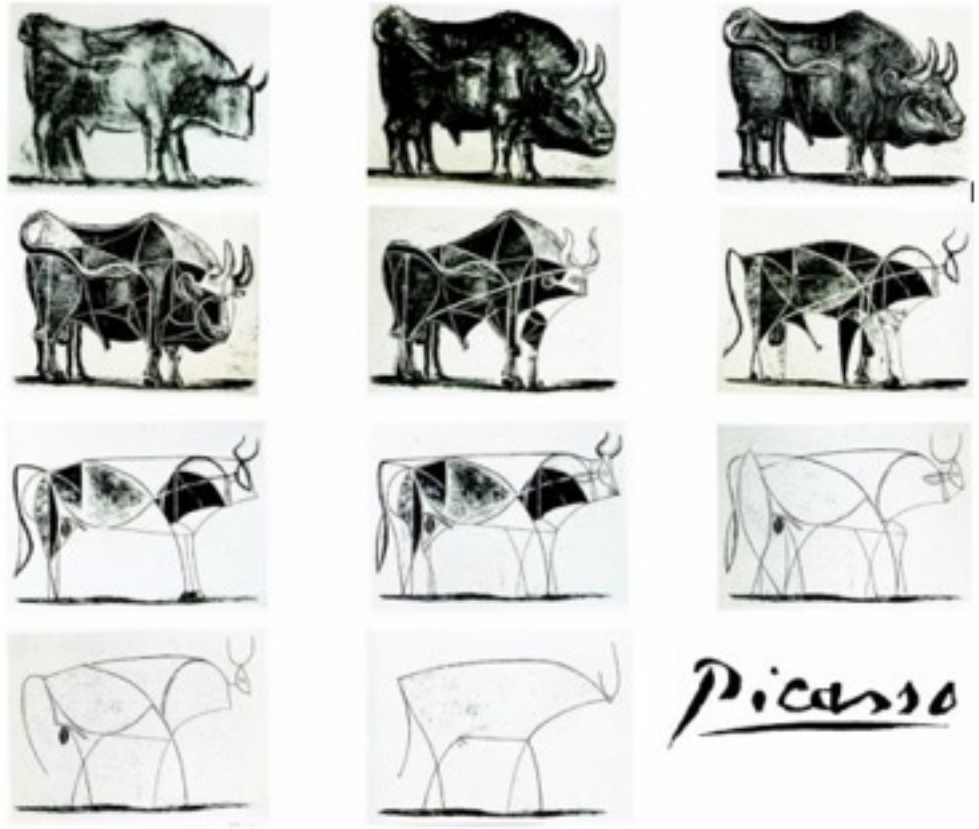

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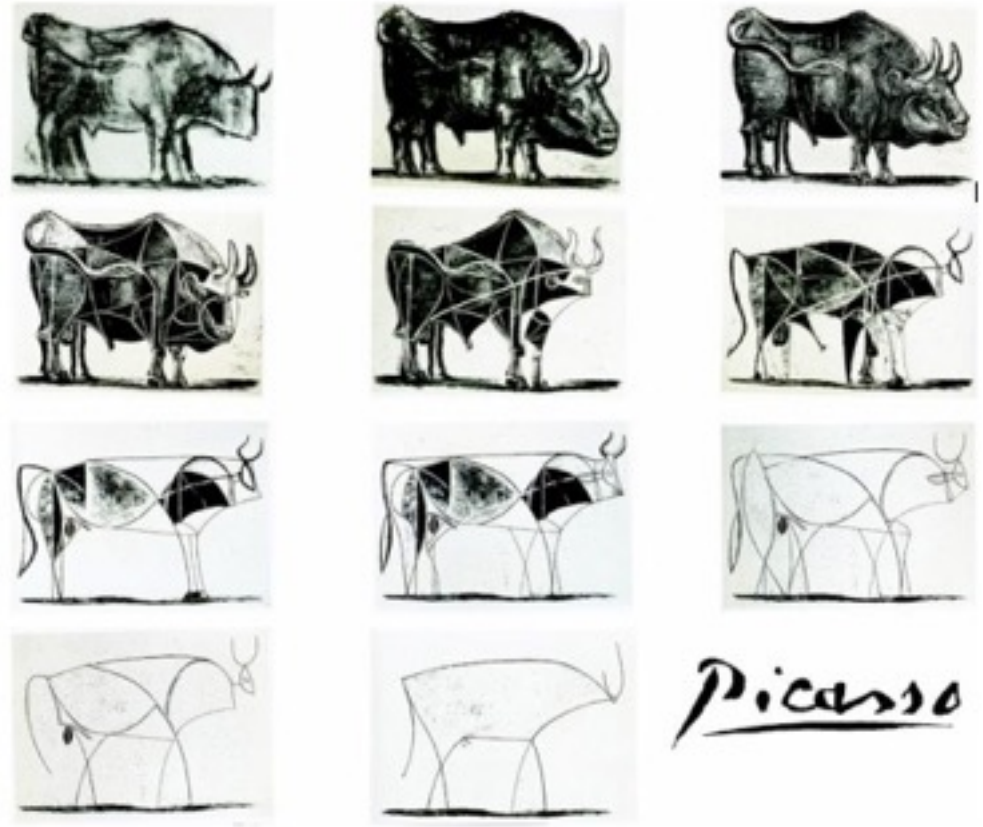


Pablo Picasso, Bull (plates I - XI) 1945

Voynich Manuscript (1404-1438)

Focus Storia, Numero Speciale, Agosto 2015

Voynich Manuscript (1404-1438)

Focus Storia, Numero Speciale, Agosto 2015

Some recent history

Wilfrid Voynich

- Found by Wilfrid Voynich in a monastery in Italy in 1912

- After his death in 1930, the manuscript passed to Wilfrid's wife

- In 1960 it was given to the antiquarian Hans Kraus, who

- In 1969 donated it to the Yale University where it was catalogue as MS 408

- Now it is at the Beinecke rare books library in Yale University

- In 2009 samples of the vellum were carbon dated at the Arizona University yielding a date between 1408 and 1438 with 95% confidence

a letter found inside the cover—written in 1666 to accompany the manuscript when it was

Not so recent history of the manuscript

Rudolf II
(1552-1612)

- A letter dated in 1669 found in the manuscript gave some hints on its previous history: sent by Johannes Marcus to Athanasius Kircher—which claims that the book once belonged to Emperor Rudolf II (1552–1612), who paid 600 gold ducats (~2.07 kg gold)
- In other related letters it is supposed that the author was Roger Bacon
- It was possibly sold to him by John Dee

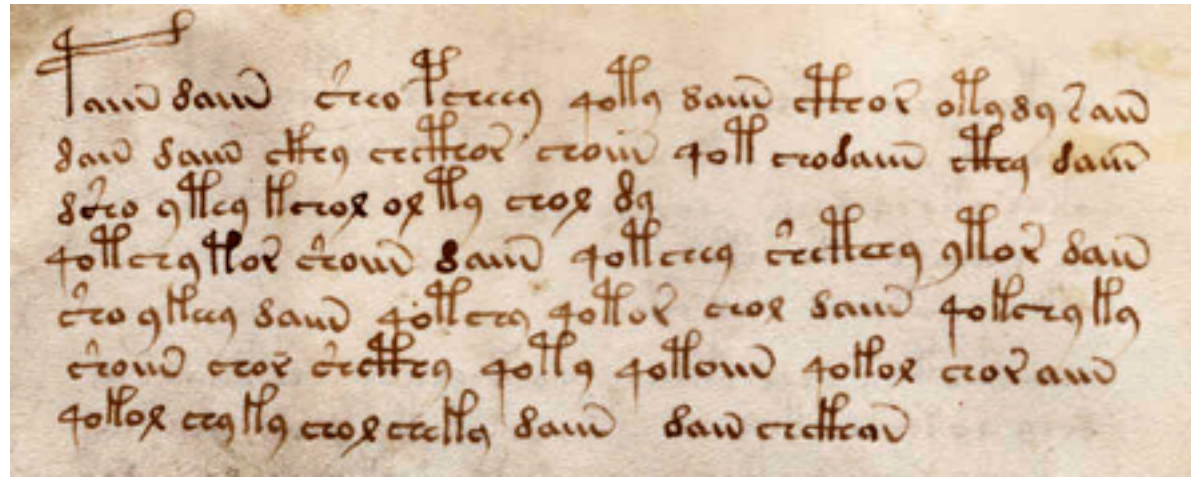
Edward Kelly
(1555-1597)

John Dee
(1527-1608)

Roger Bacon

Voynich Manuscript (1404-1438)

- 240 pages long
- Written on vellum
- Not identifiable language
- Unknown script
- No punctuation marks
- No corrections
- Lavishly illustrated



sections

- Herbal (12135 words)
- Astronomical (4877 words)
- Biological (6915 words)
- Pharmaceutical (2568 words)
- Recipes (11422 words)

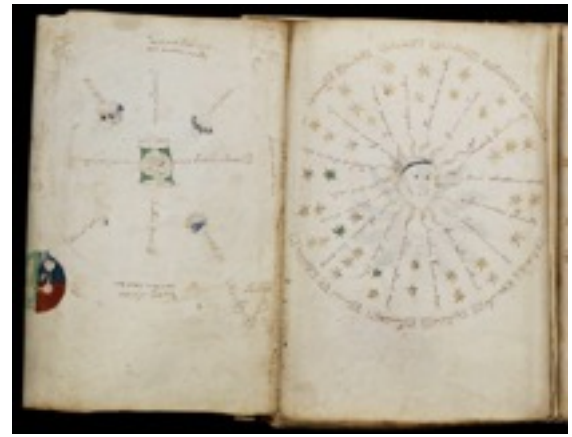
Herbal

- Longest section of the manuscript
- Plant drawings
- All but a few of the plants have been recognised with certainty
- Most of the plants incorporate parts from known species




Astronomical

- Drawings resembling the sun, moon, planets, and stars
- Indication to the 12 zodiac signs
- Circular diagrams with women holding stars



Voynich Manuscript (1404-1438)

Biological

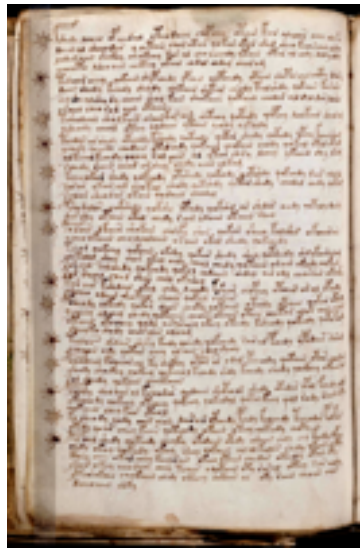
- Dominated by figures of bathing women
- Elaborate systems organic tubes and shapes

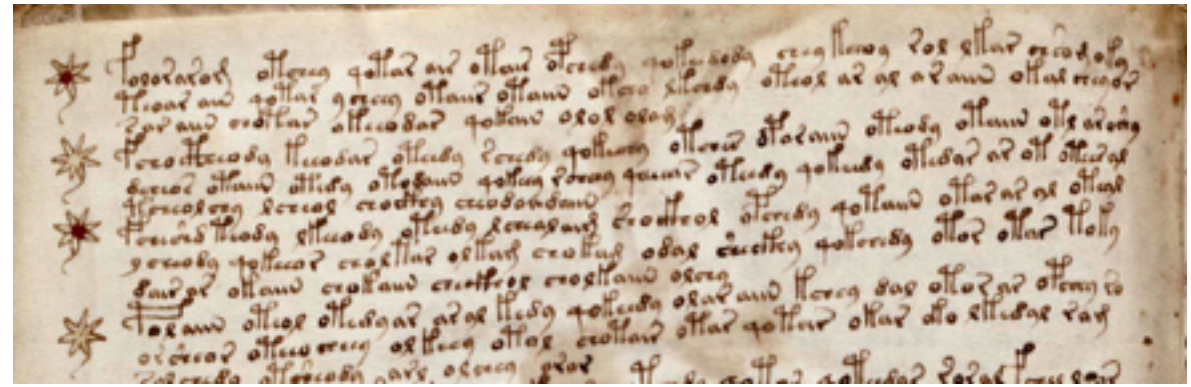
Pharmaceutical

- Labelled drawings of plants and plant parts
- Different types of flasks

Recipes

Short paragraphs preceded by stars





Some of the conjectures

Early attempts to decode the text date from the 17th century

Numerous attempts were made after the rediscovery of the manuscript. In particular, top WWII cryptographers.

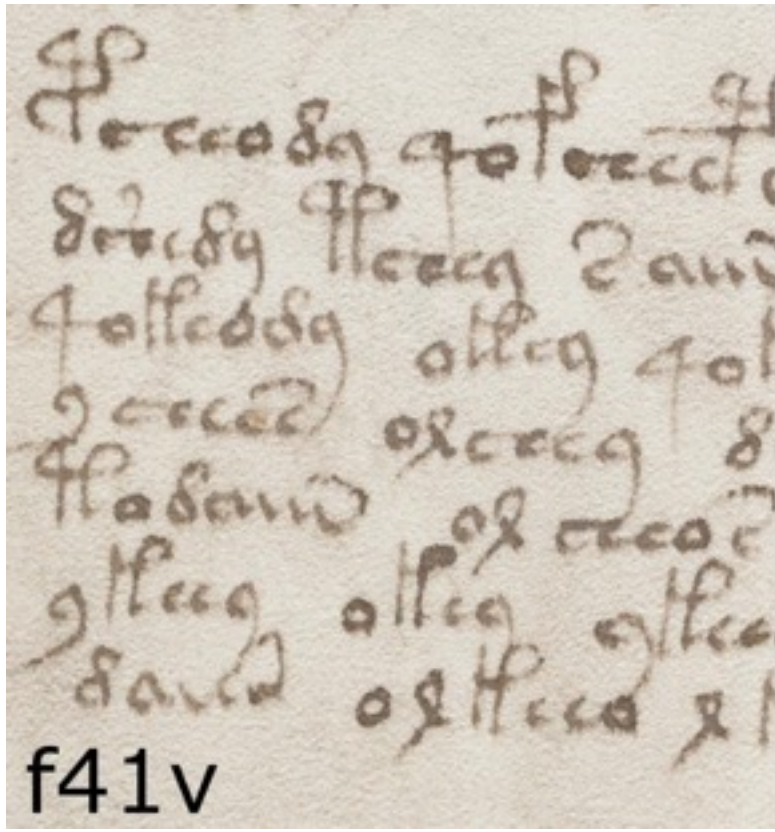
The proposed possibilities include:

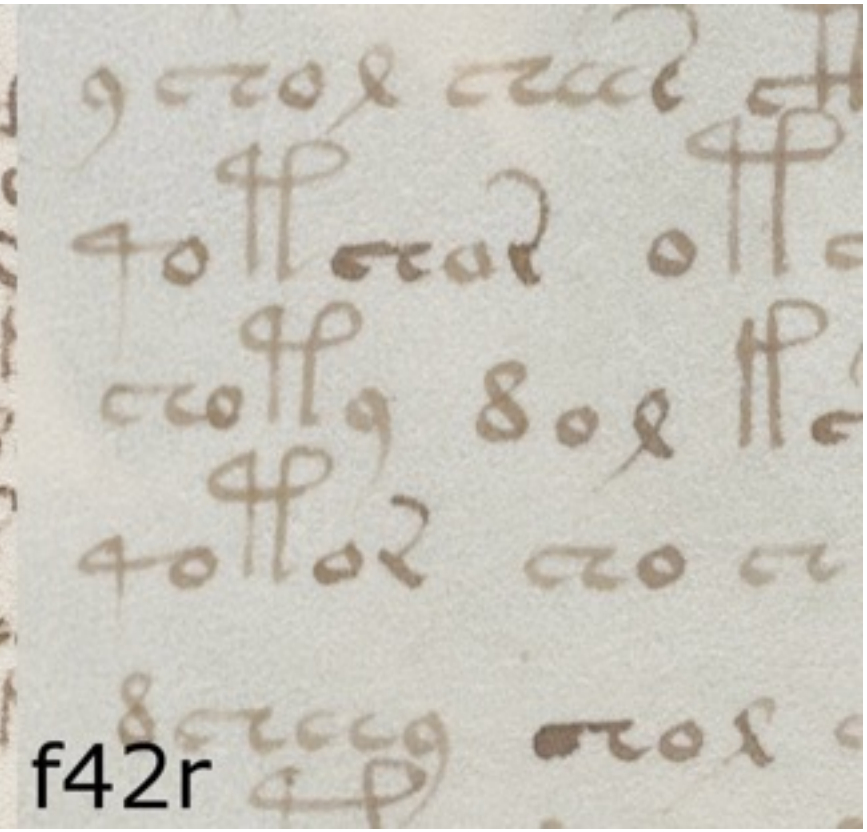
- A cipher
- A code
- "Exotic" natural language
- An artificial language
- A hoax

Voynich Manuscript (1404-1438)

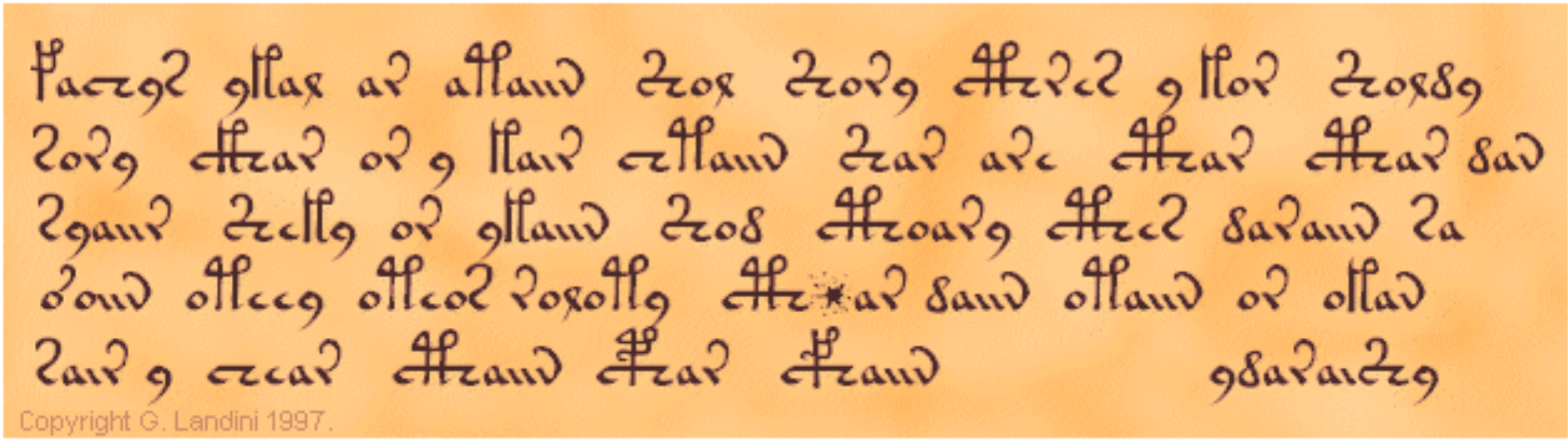
William Friedman

The TEXT.....and the Coding





The TEXT.....and the Coding

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fachys.ykal.ar.ataiin.shol.shory.cthres.ykor.sholdy
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syaiir.sheky.or.ykaiin.shod.cthoary.cthes.daraiin.sa
o'oiin.okeey.oteor.roloty.cth*ar.daiin.otaiin.or.okan
sair.y.chear.cthaiin.cphar.cfhaiin - ydaraishy
```

The TEXT.....and the Coding

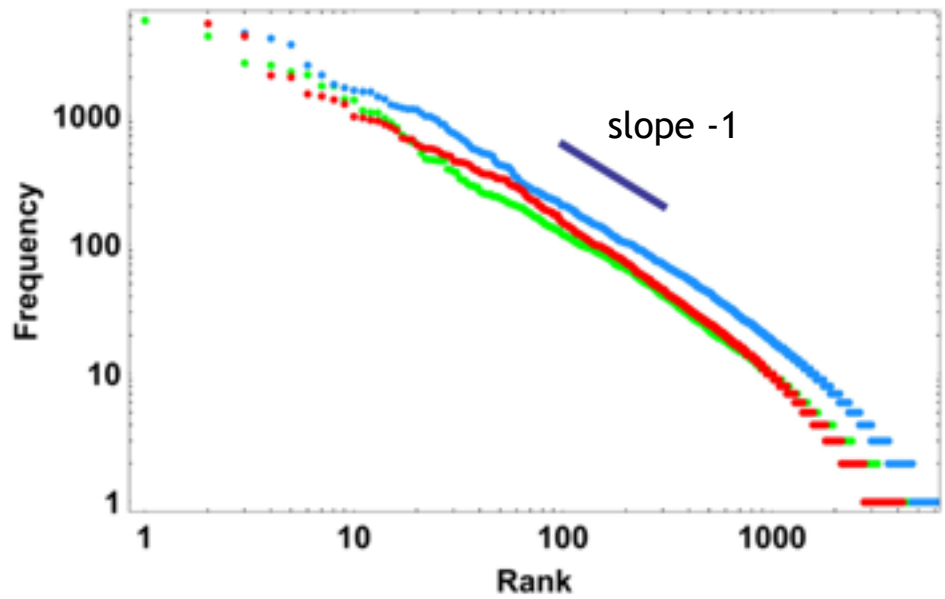
The TEXT.....and the Coding

....OEES OR QOKOR CHOL CTHOL TCHALODY CHOCKHY S OS CHY SAIN OR OCHY CTHAR CTHAR CTHY Y CHAIIR CKHAL CTHODAM DY YTCHO CTHOL CHES CTHOR OCHOLY KCHOS CHY DOR DCHOR CHOLDAR OKOL DAIIN YCHEOR CHOR OCTHAM FCHODAIIN SHOPCHEY QKO SHEY QOOS SHEEY CHOROCHY DCHEEY KEOR SHOR DOLD DCHEY KCHEY OTCHY CHEODY OEEEES CHEODAIIN SHEEY YTCHEEY QOTCHY CHALD QOKCHO CHO LOCHEY DAIIN YCHEY KCHOS ODAIIN OAIIR OTAIIN KSHOLOCHEY QOTOEES CHKOLDY OTCHOR CHOAIIN DSHOY CTHOL CHOL OTCHOL DAIN SHODY SHOL CHOTCHY OKCHEY DEEEESE CHOTY QOKCHY SHOL KEEY CHOTY DAIN QOKECHY OLCHOIIN CHOL CPHEY SHCKHY CHOCHY KCHOD SCHAIN CHOR DAIIN CHCKHY POLYSHY SHEY TCHODY QOPCHY OTSHOL DY DAIIN TSHODODY CHOCHY CTHY DAIIN QOKY CHCPHHY DAIIN CTHOL CTHY CTHD QOKCHY DYKCHY CHKEEY KSHY KY TY DOR CHEEY OL CHEOL DY CHOTEEEN OEEAR CHOSCHY DAIN SHO KSHY SHOL DEEES DOL DCHODAIIN QOTCHY CHEEY TCHEEY KCHOR SHEOD SHEODAIIN SHODAIIN OKSHOLSHOL DAIS QOS OKSHODEEEN CHOR CHEOR ODAIIN SHOTCHO DOL DOL DOR AIIN QOTEEEO RCHO CHEEODY QOTCHEY TEY OKCHOR DAIIN SHO KEEO DAIIR .....

Zipf's law

George K Zipf (1902-1950)

Word	rank	frequency
the	1	10 288
of	2	7851
and	3	4439
in	4	4016
to	5	3605
a	6	2491
that	7	2083
have	8	1762
be	9	1656
as	10	1591
on	11	1553
species	12	1542
is	13	1418
by	14	1356
which	15	1229
or	16	1190
for	17	1163
we	18	1157
are	19	1135
from	20	1132



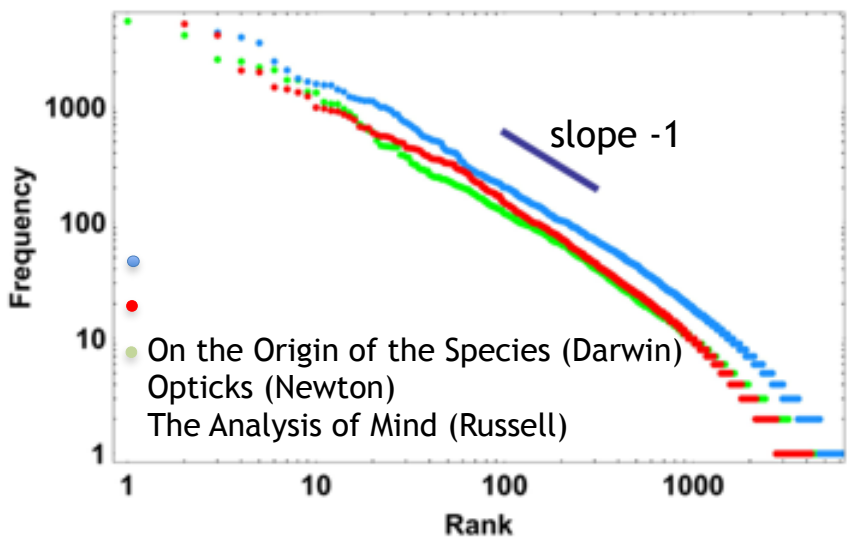
-George Zipf discovered the Law shortly before 1935

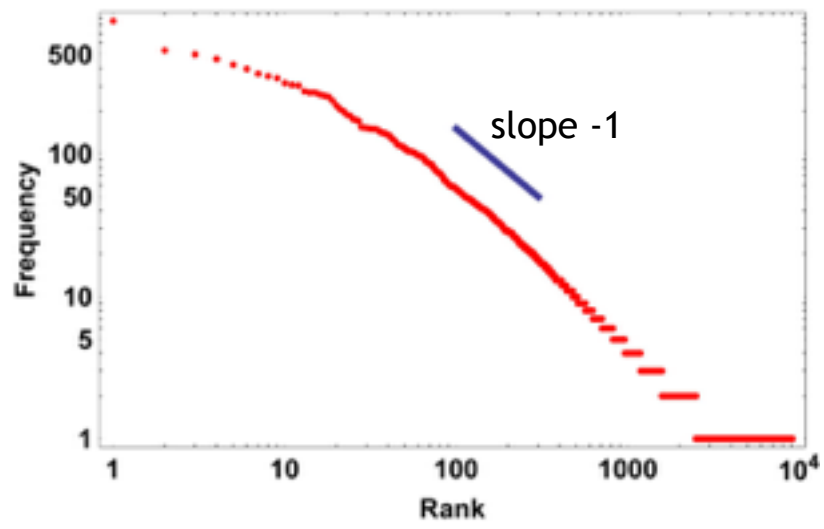
-All known languages obey it with small variations

-Similar distributions are found in other systems (internet links, city sizes, surname distributions)

Zipf's law

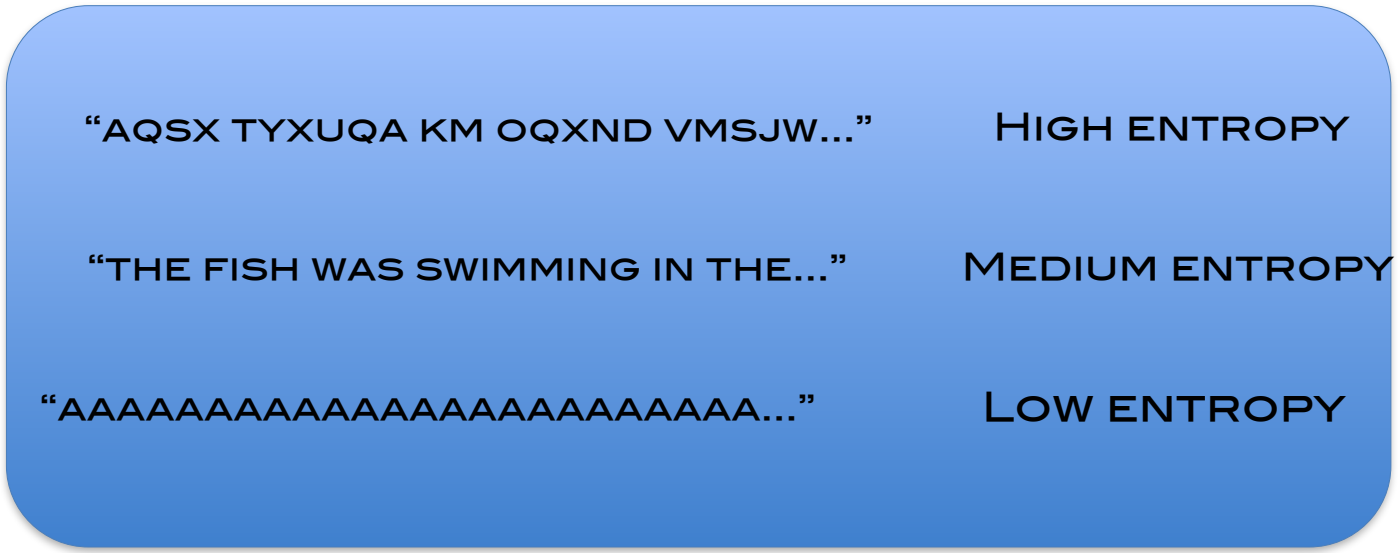
George K Zipf (1902-1950)





In 2001, Gabriel Landini reported that the Voynich text obeys Zipf's law

Entropy can be interpreted as a measure of disorder

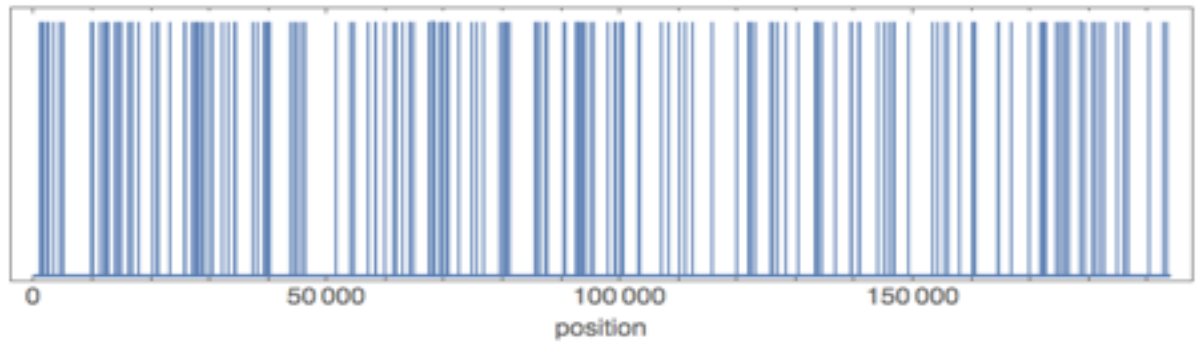


It can also be interpreted as a measure of predictability in the sequence:

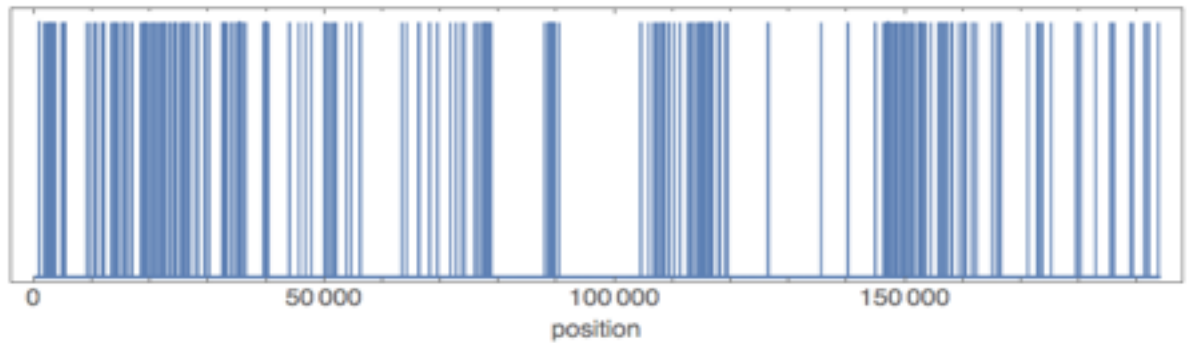
The lower the entropy, the more predictable are future occurrences in the sequence

Complexity in word distribution

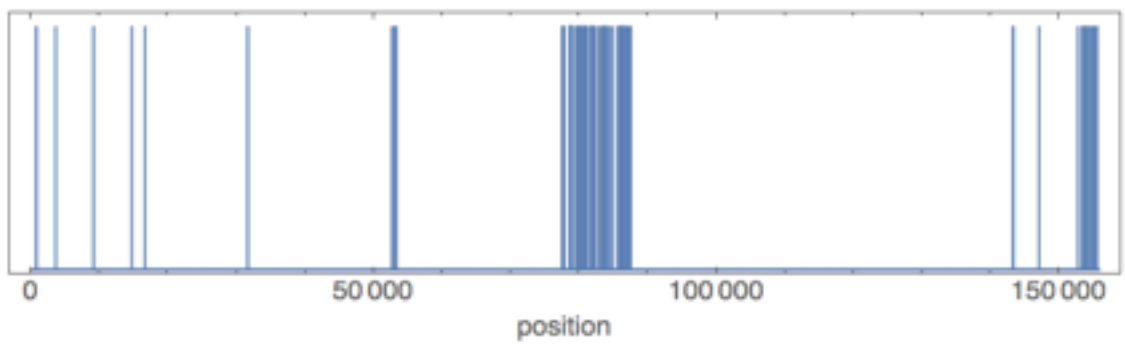
what



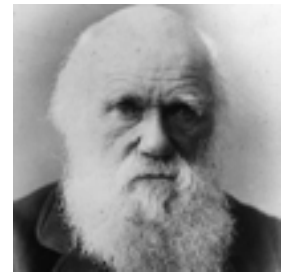
plants



hybrids

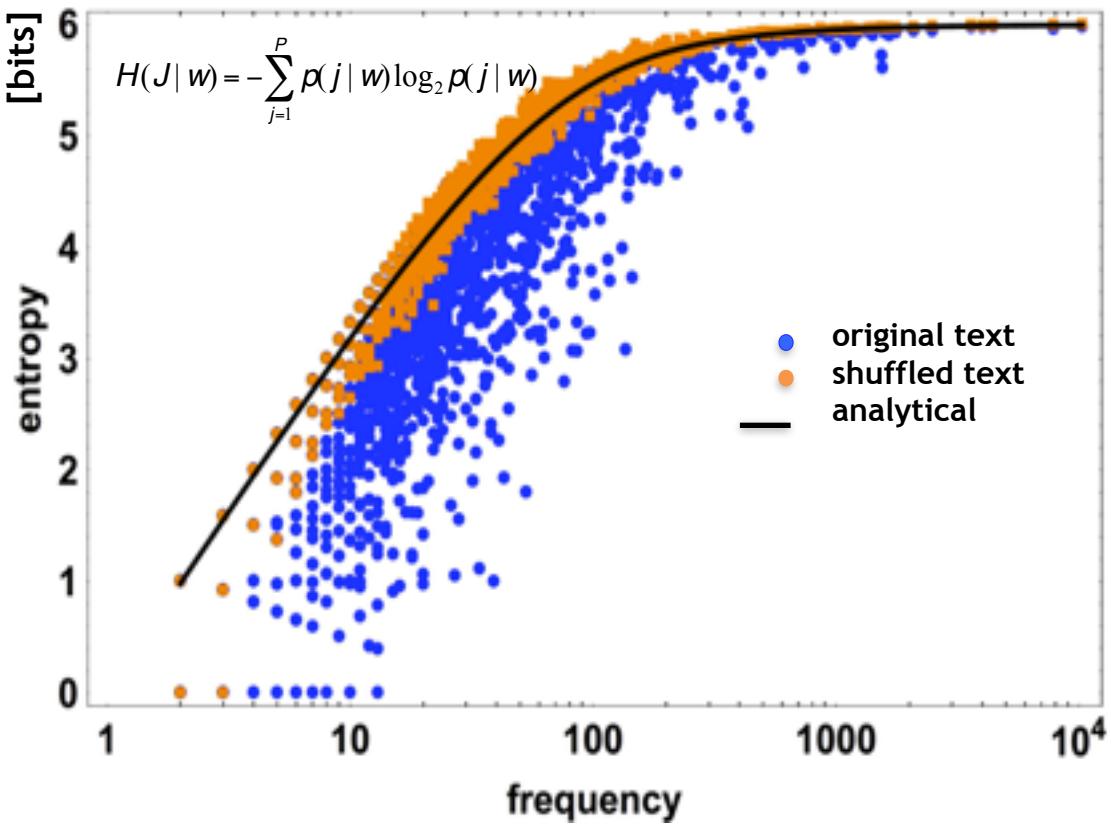


Words that convey semantic information are more clustered



Example: On the origin of species by C Darwin

155000 words in length, 7000 different words



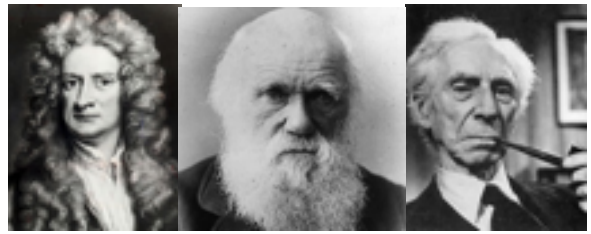
$\log_2 P \quad (P = 64)$

$$H(J|w) = -\sum_{j=1}^{P} \frac{n_j}{n} \log_2 \frac{n_j}{n}$$

$$\left\langle \hat{H}(J|w) \right\rangle = -\sum_{m=1}^{\min\{n,s\}} \frac{\binom{m}{n}\binom{N-m}{s-m}}{\binom{N}{s}} \frac{m}{n} \log_2 \frac{m}{n}$$

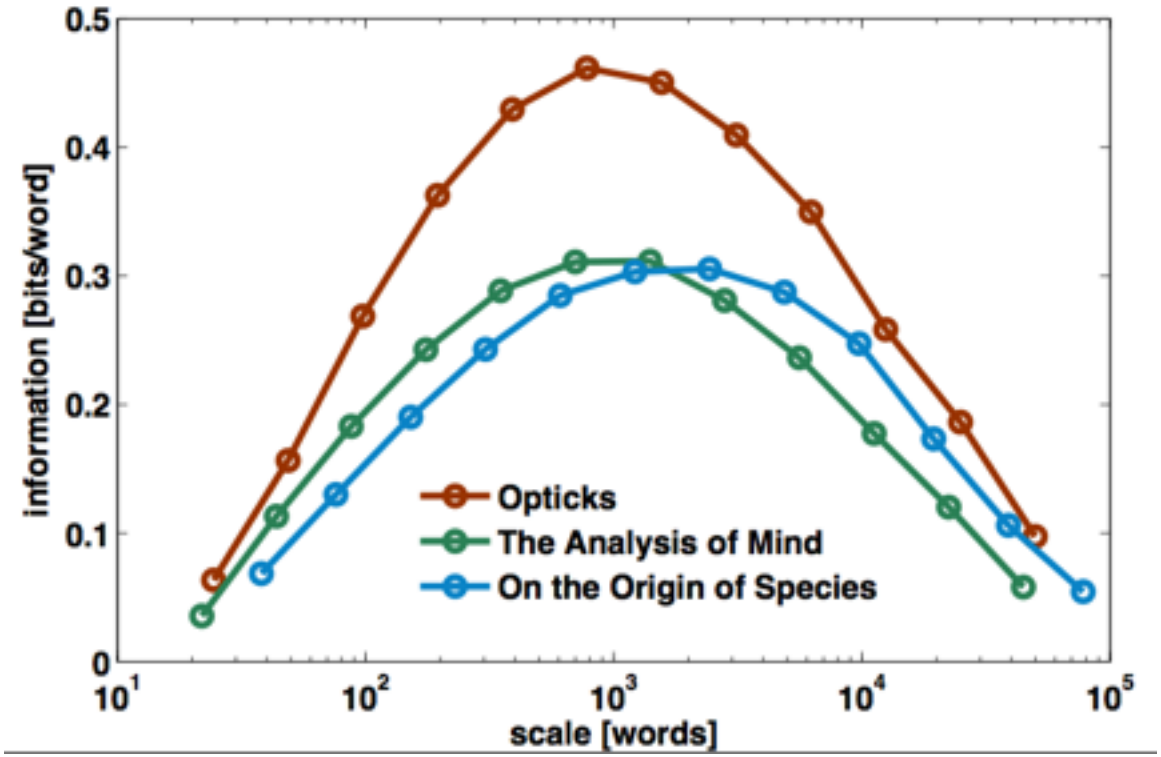
Departures from random behaviour is due to the linguistic function of words; thus, quantifying that departure may in turn quantify linguistic function.

THE INFORMATION AS A FUNCTION OF THE SCALE



$$\Delta I(s) = \sum_{w=1}^{K} p(w)\left[\left\langle \hat{H}(J \mid w)\right\rangle - H(J \mid w)\right]$$

$s = N / P$



s

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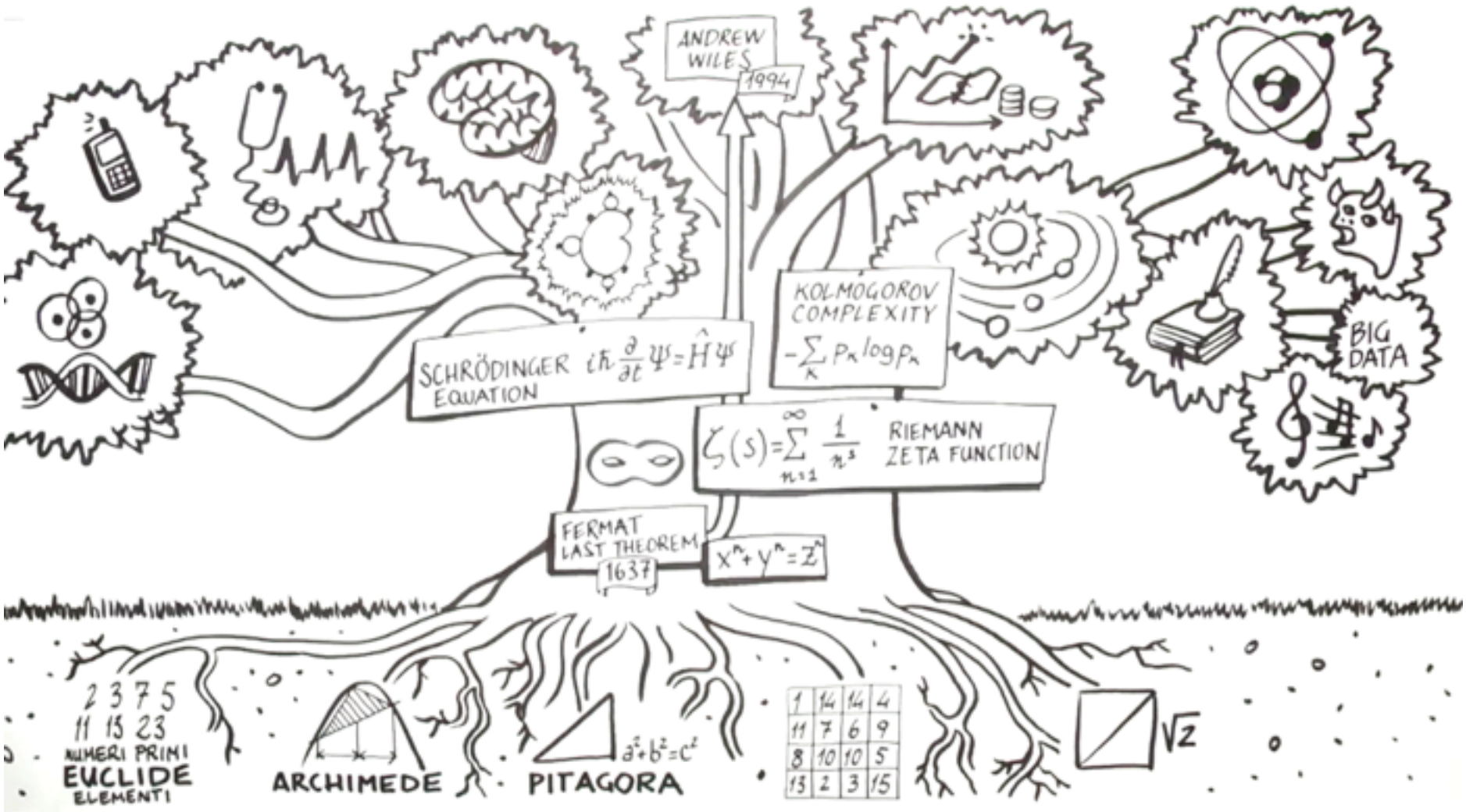

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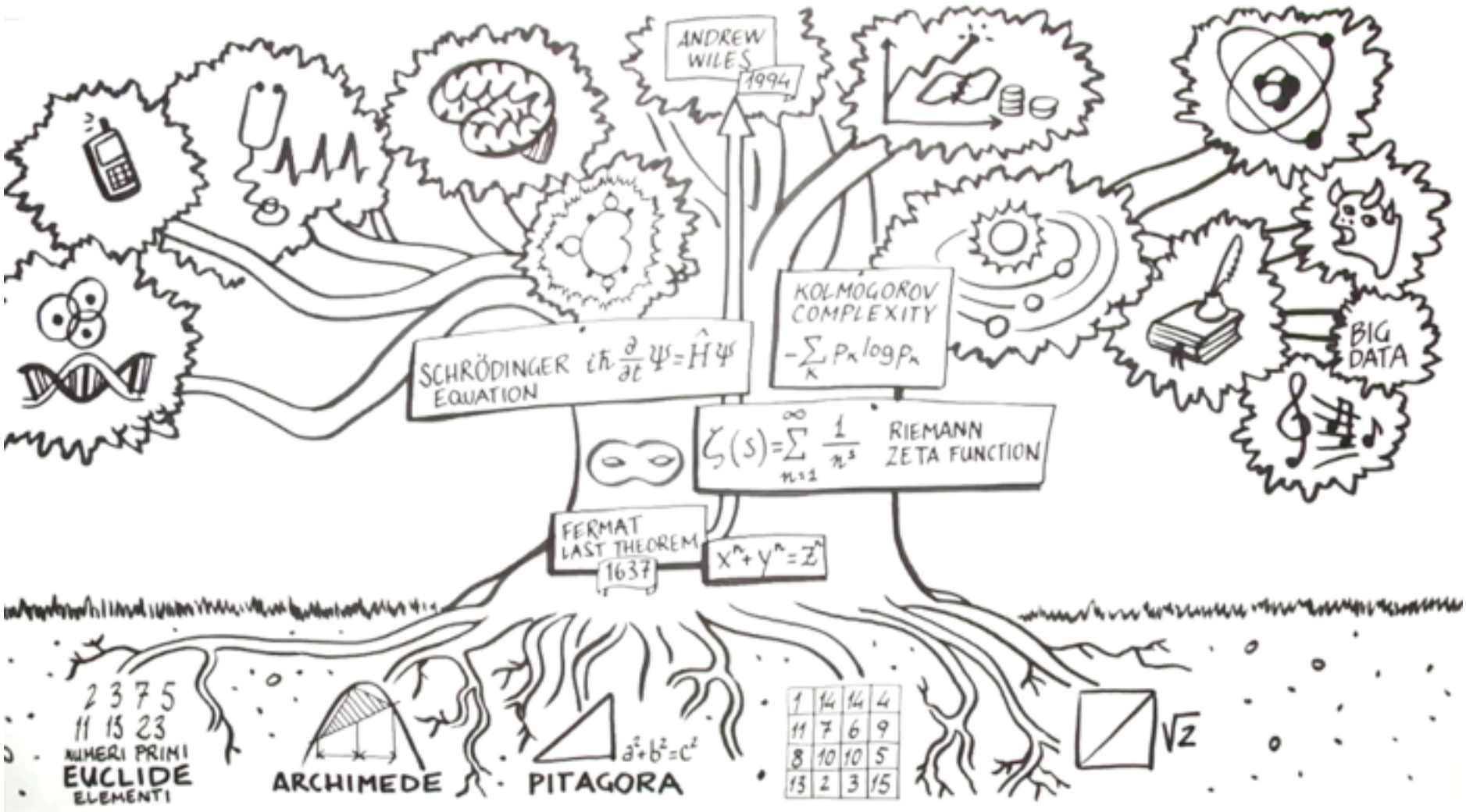

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