

IL BUON TEMPERAMENTO

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UNIVERSITÀ DI PADOVA

Numeri naturali primi: 2, 3, 5, 7, 11..

1, 2, 3, 4, ..., 1995, ...

Numeri razionali

$\frac{5}{1}$ $\frac{43}{4}$ $\frac{1}{2}$ $\frac{22}{3}$, ... $\frac{121}{100}$ $\frac{215}{99}$

decimale
↓

" " " " " "

5 10.75 0.5 7.333... 1.21 2, $\overline{17}$

hanno sviluppo decimale finito o periodico

Numeri reali

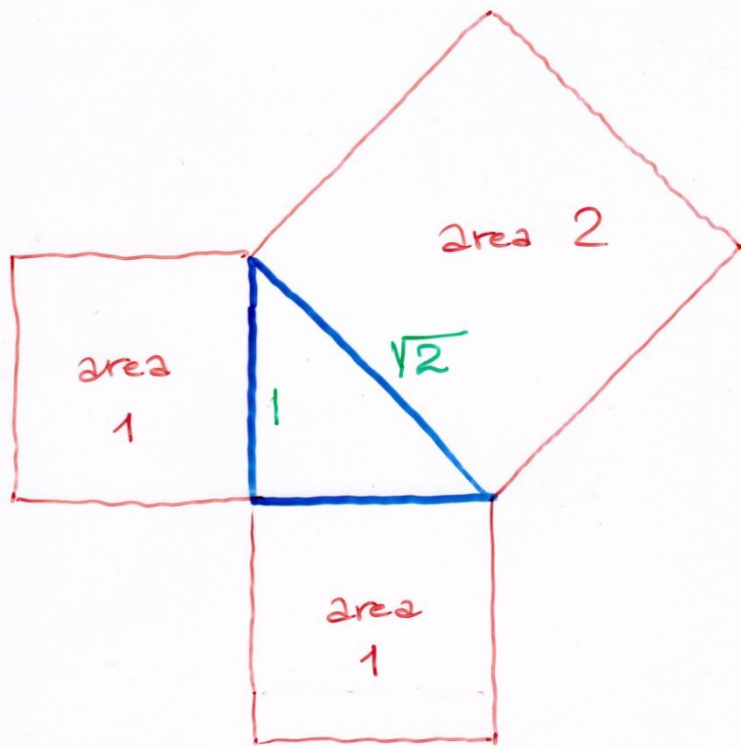
0.123456789101112....

$\pi = 3.14....$

$\sqrt{2} = 1.41....$

$\sqrt[12]{2} = 1.06....$

hanno sviluppo decimale qualsiasi



per assurdo:

$$\sqrt{2} = \frac{m}{n} \Rightarrow 2 = \frac{m^2}{n^2}$$

$$2m^2 = m^2$$

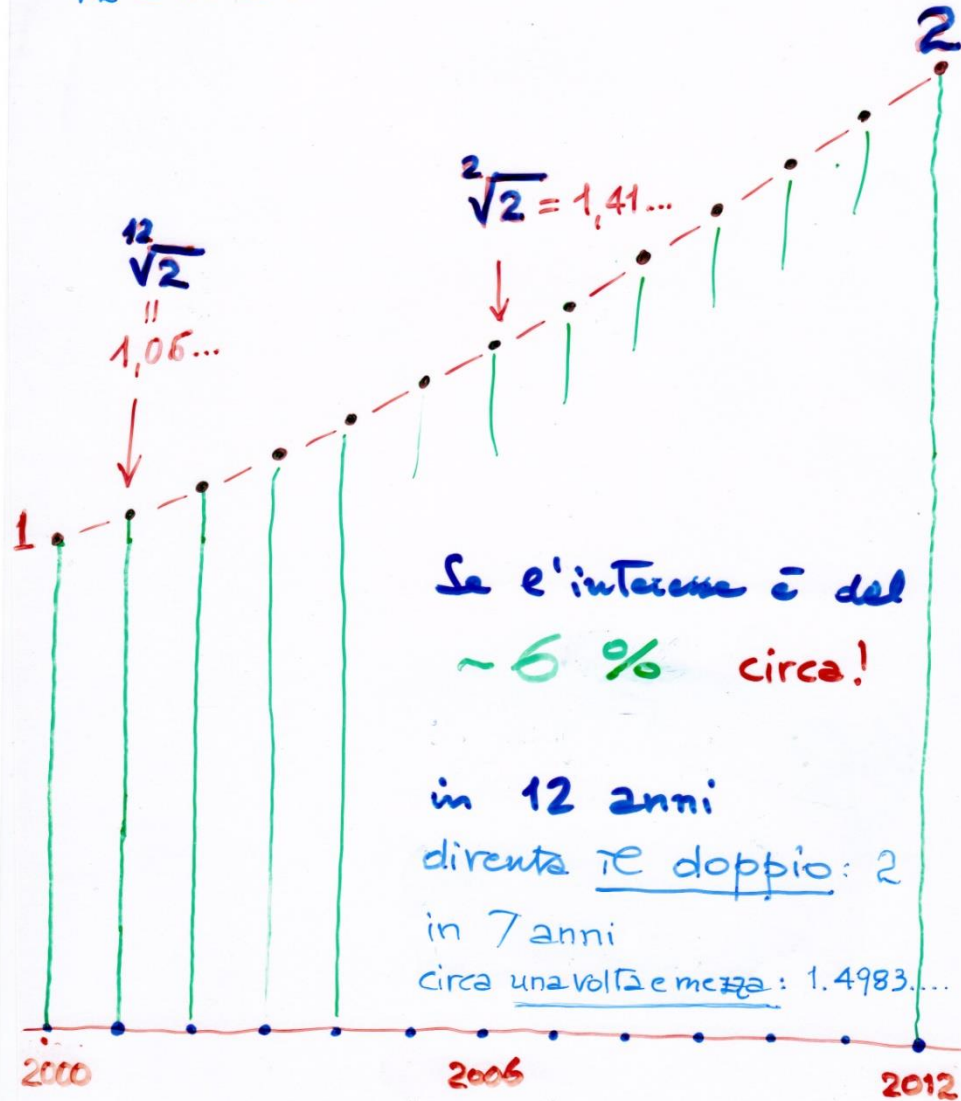
il numero
dei fattori primi
è dispari

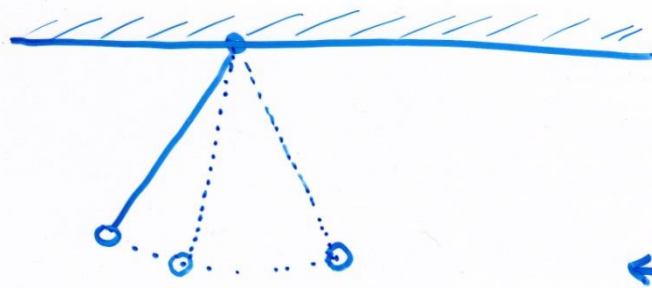
il numero
dei fattori primi
è pari

IMPOSSIBILE

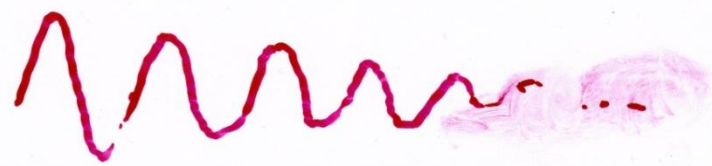
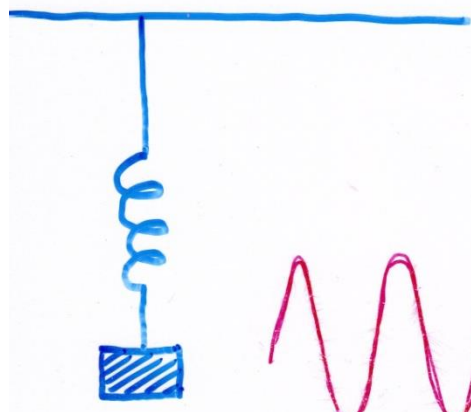
Interesse Composto

$$\sqrt[12]{2} = 1.05946\dots$$

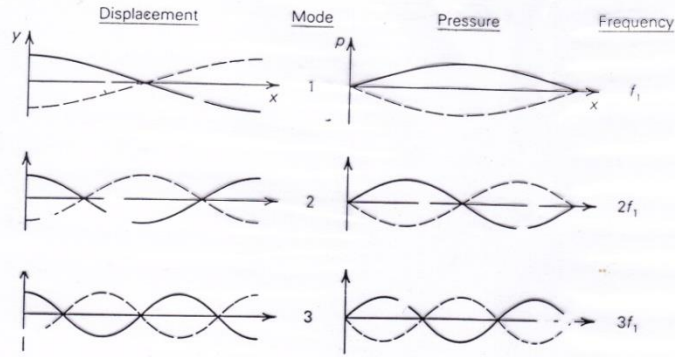




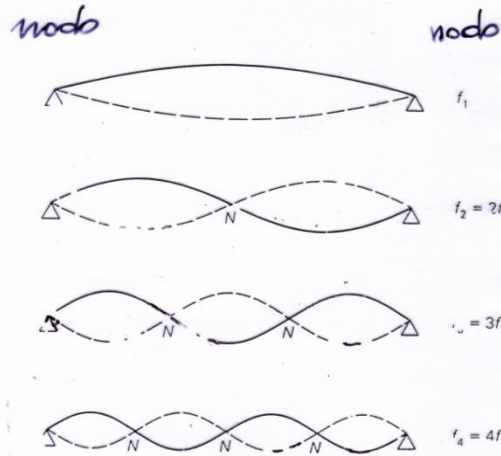
↓
tempo



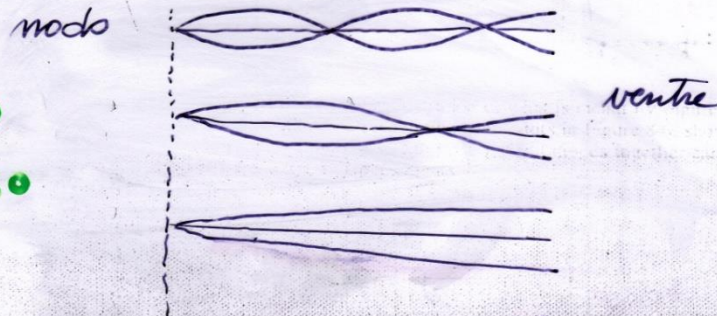
tubo
aperto



Corda
(ideale)

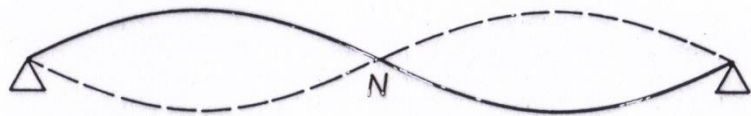


Tubo
chiuso





f_1



$f_2 = 2f_1$



$f_3 = 3f_1$



$f_4 = 4f_1$

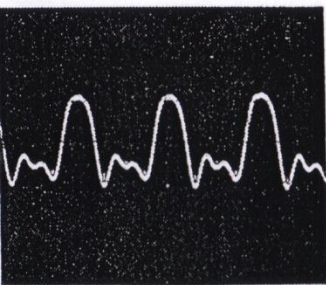
FLAUTO

TROMBA

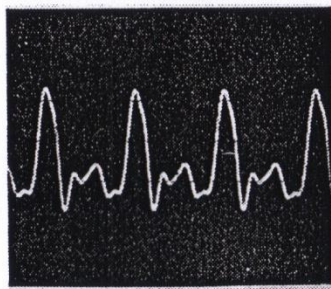
SASSOFONO

VIOLINO

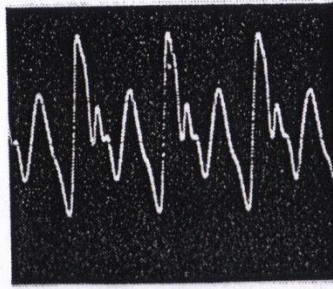
FAGOTTO



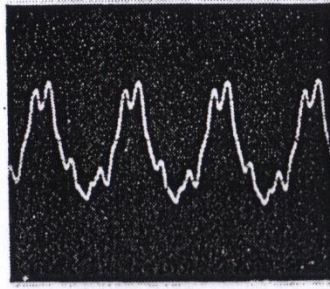
(a)



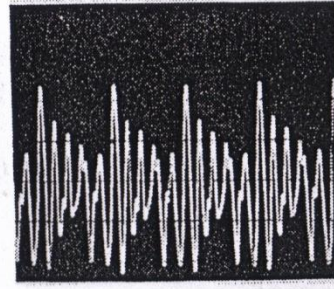
(b)



(c)



(d)



(e)

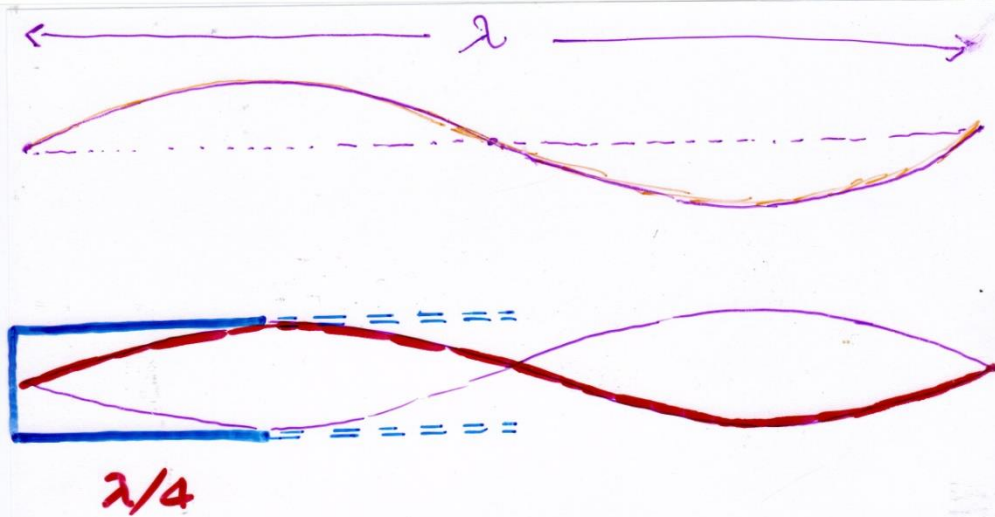
TUTTI

SUONANO

UN

LA

440 Hz



$$v = f \cdot \lambda$$

velocità = frequenza · lungh. d'onda

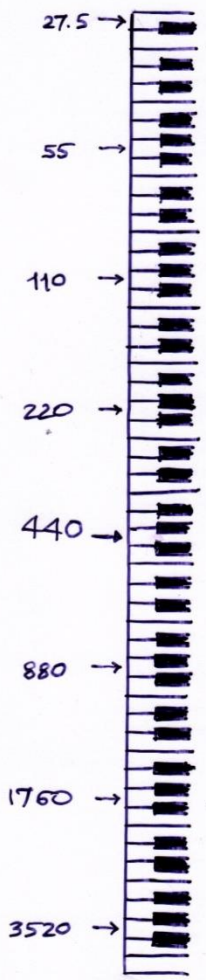
$$\lambda/4 = 9.8 \text{ cm}$$

$$v = 340 \text{ m/sec}$$

$$f = \frac{340}{4 \cdot 9.8 \cdot 10^{-2}} = 870 \text{ Herz}$$

La

Hz



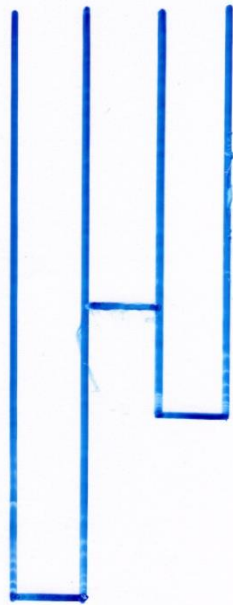
ottava = 12 semitoni



Soglia

→ ultrasuoni

- arpa, organo
- flauto piccolo
- flauto
- oboe
- stardini (= piccolo)
- viola
- violino
- viola
- contrabbasso
- basso - tuba
- corni, fagotti



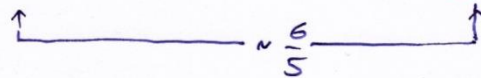
$\lambda \sim 12 \quad 6 \quad 8$
 $f_n \sim 2 \quad 4 \quad 3$
 $1 \quad 2 \quad \frac{3}{2}$

$\sim 2 \quad 4.05 \quad 3$
 \uparrow

"stonata"

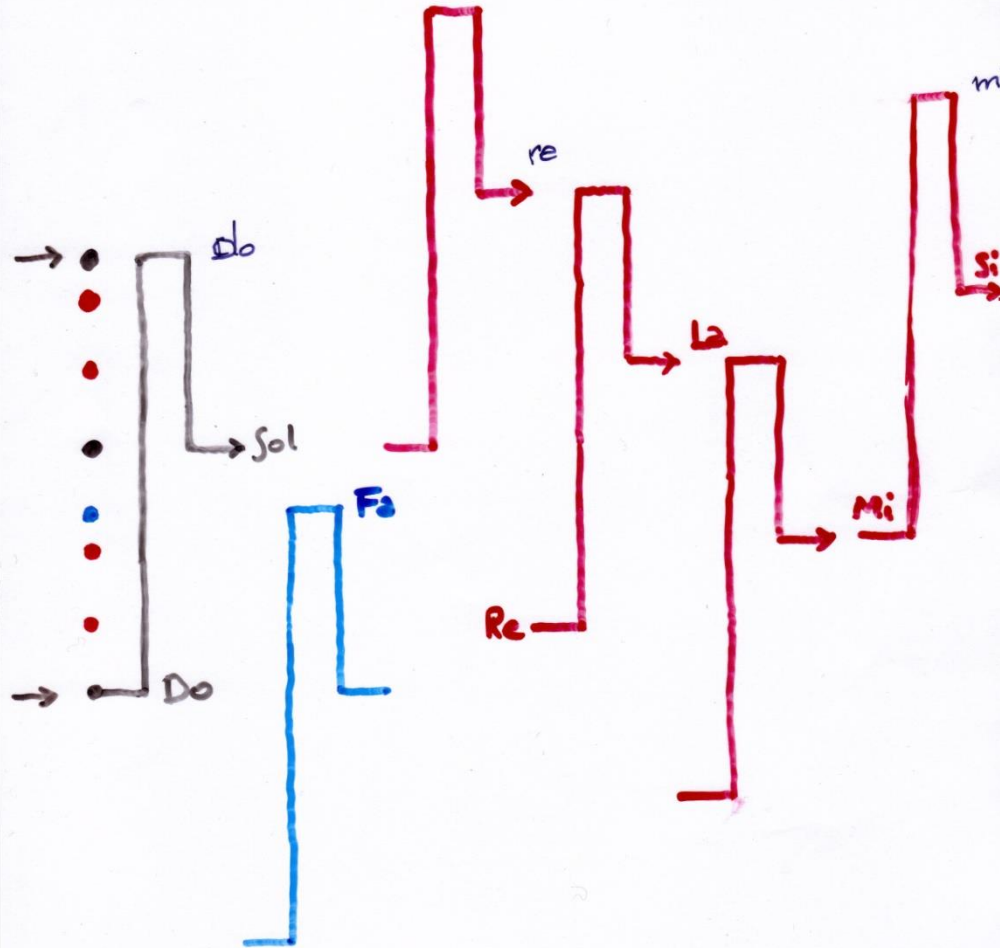
Mi mi si

Sol sol re



(N.B. sentiamo la stessa "melodia")

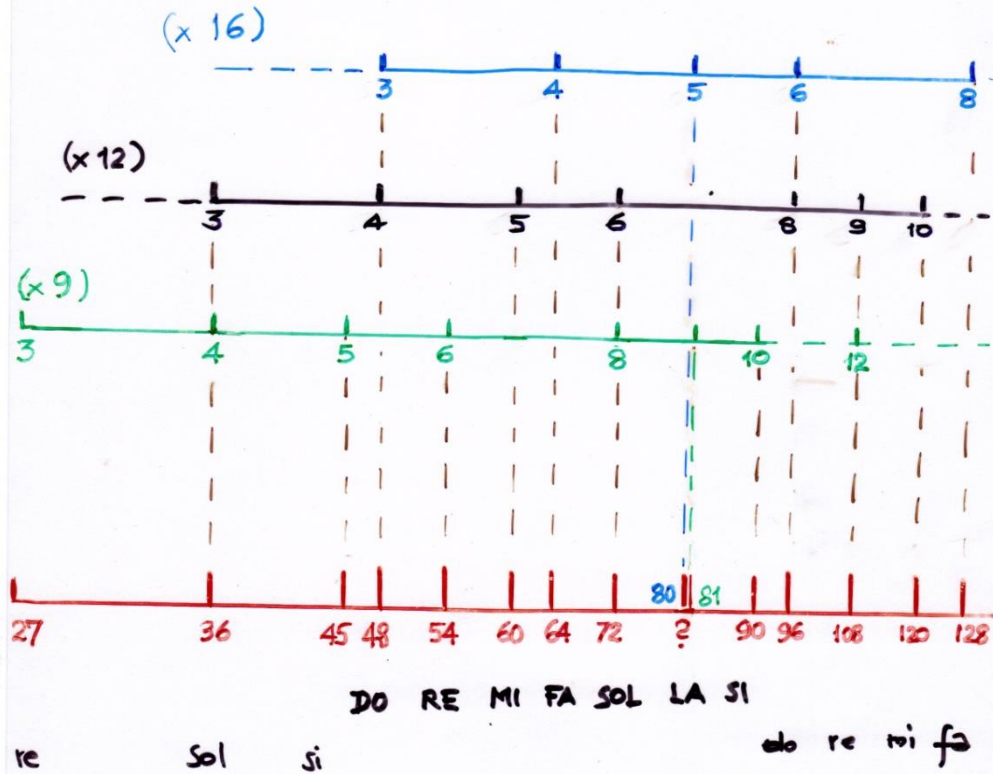
Genesi pitagorica della scala naturale
(= tasti bianchi)



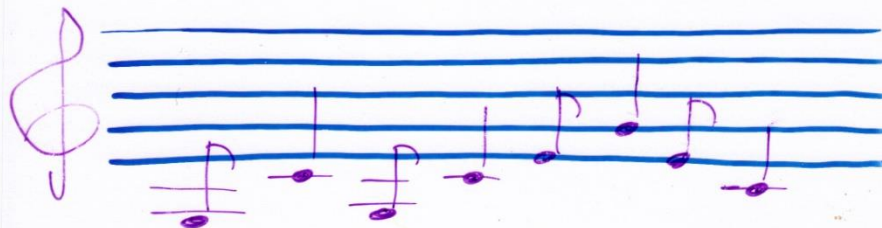
Genesis della scala delle 7 note "bianche"

(teorica):

prime armoniche su tubi di varie lunghezze

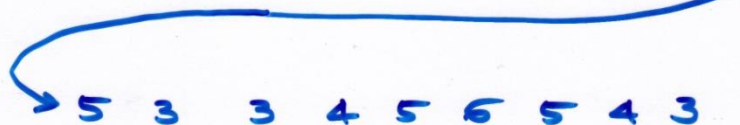


Nella tromba, per esempio, i pistoni (= valvole) variano il percorso dell'aria e quindi la lunghezza del tubo



Hz: 198 264 . . 330 396 . .

3 4 3 4 5 6 5 4



Hz: 264 297 330 352 396 . . .

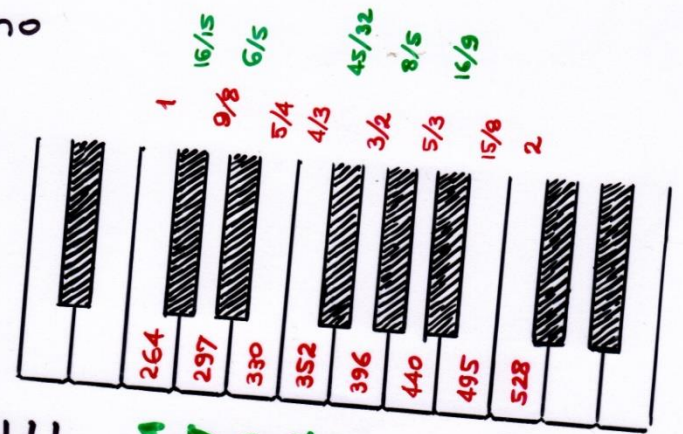
24 27 30

32 36

trasporti! #

(5)

Zarlino



INTERVALLI

"quinta" = $\frac{3}{2}$

cinque giuste

una stonata: $\frac{40}{27}$

una stonatissima: $\frac{64}{45}$

"terza maggiore" = $\frac{5}{4}$

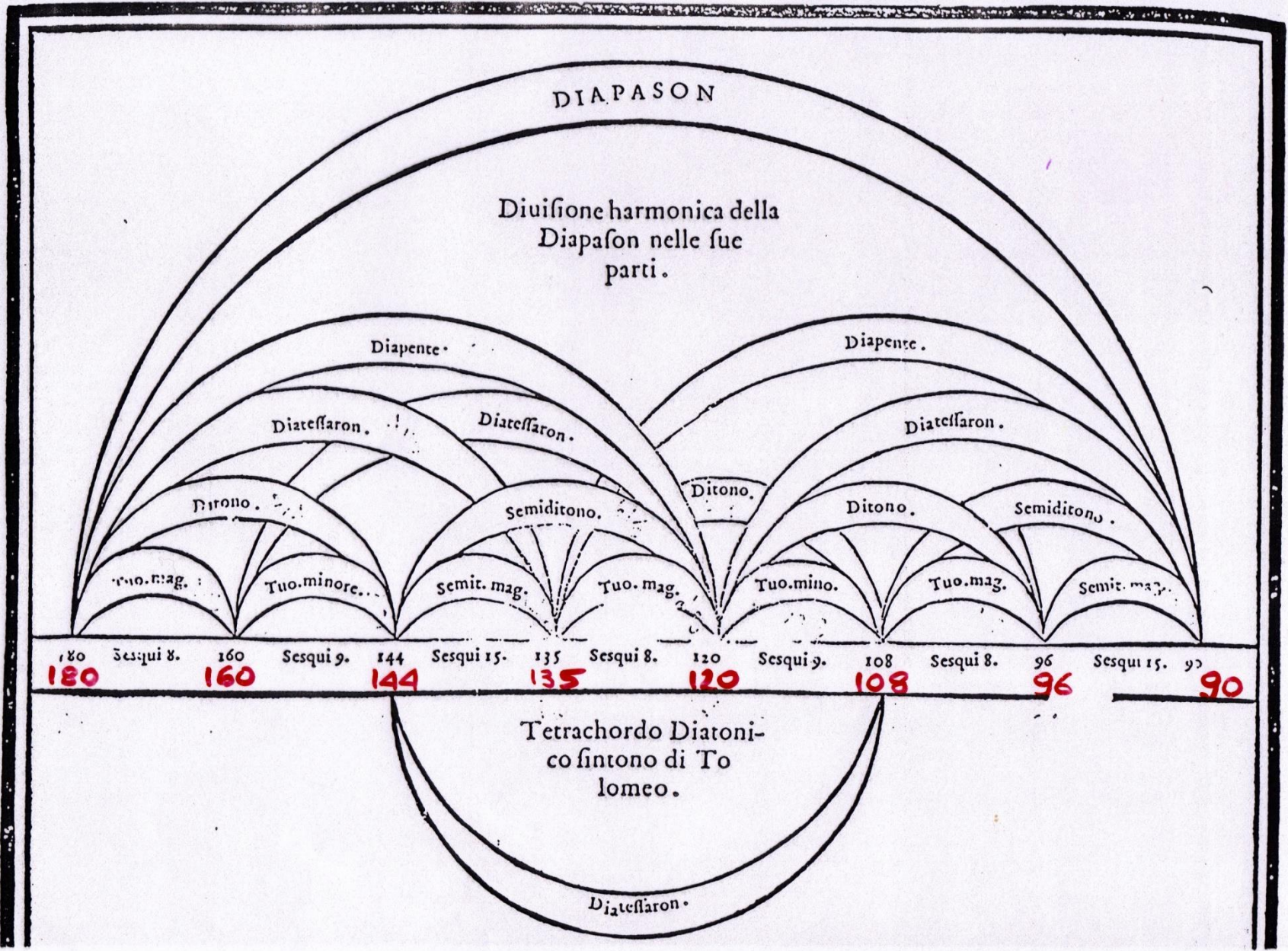
tre giuste

"terza minore" = $\frac{6}{5}$

una stonata: $\frac{32}{27}$



Fazlino 1558

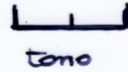


INTERVALLI

$$2^{\frac{1}{12}} = 1.059..$$



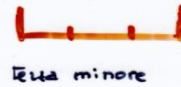
$$(2^{\frac{1}{12}})^2 = 1.122...$$



suona

$$12 \quad \frac{9}{10}$$

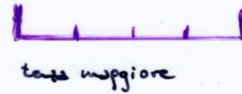
$$(2^{\frac{1}{12}})^3 = 1.189...$$



discreta

$$12 \quad \frac{6}{5}$$

$$(2^{\frac{1}{12}})^4 = 1.259...$$



$$12 \quad \frac{4}{3}$$

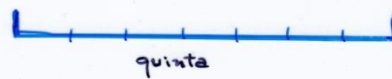
$$(2^{\frac{1}{12}})^5 = 1.334...$$



ottima approssimazione

$$12 \quad \frac{4}{3}$$

$$(2^{\frac{1}{12}})^7 = 1.498...$$



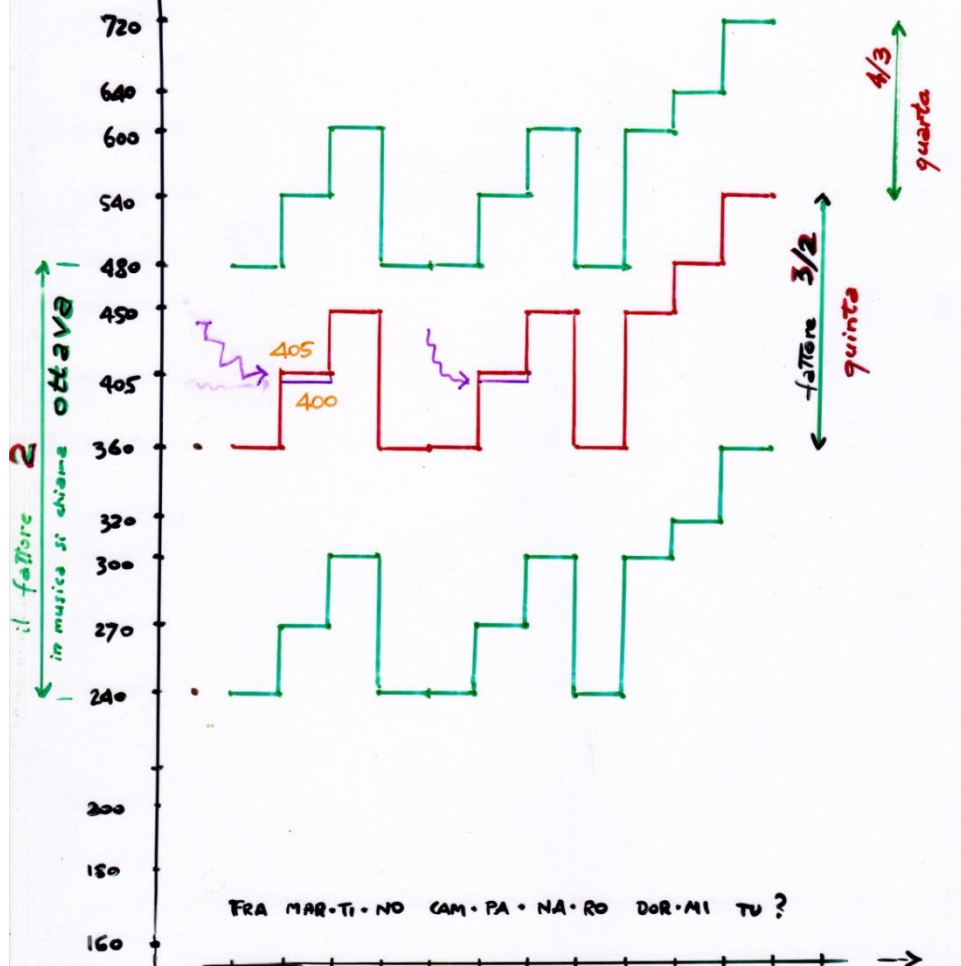
$$12 \quad \frac{3}{2}$$

$$(2^{\frac{1}{12}})^{12} = 2$$



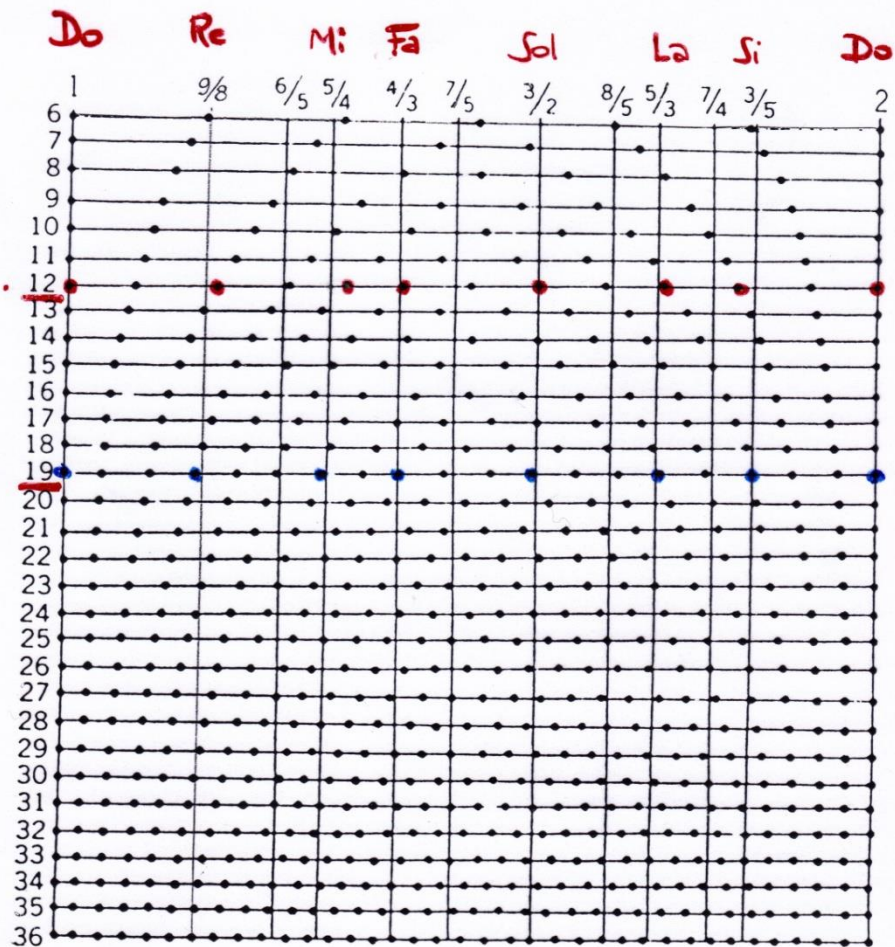
$$2$$

frequenza (scala logaritmica: le traslazioni sono moltiplicazioni)



FRA MAR-TI-NO CAM-PA-NA-RO DOR-MI TU?

tempo



suddivisione dell'ottava

secondo intervalli' eguali ... ● ... ●

" " razionali ... | ... | ...

Siamo alla definitiva assunzione del concetto di fisicizzazione del suono in luogo della matematizzazione; un cammino iniziato oltre due secoli prima, alle origini della rivoluzione scientifica. Galileo (fors'anche contraddittoriamente negli esiti) aveva scritto nel 1638 che:

«Consonanti, e con diletto ricevute, saranno quelle coppie di suoni che verranno a percuotere con qualche ordine sopra il timpano; il qual ordine ricerca che le percosse fatte dentro all'istesso tempo siano commensurabili di numero, acciò che la cartilagine del timpano non abbia a star in un perpetuo tormento d'inflattersi in due diverse maniere per acconsentire ed ubbidire alle sempre discordi battiture».



66 132 198 264 330 396 462 528

f 2f 3f 4f 5f 6f 7f 8f

do do sol do mi sol [si b] do

armoniche



f $\frac{3}{2}f$ $\frac{3}{4}f$ $\frac{27}{16}f$

do sol re la

65,1... 97.7 146.6 220

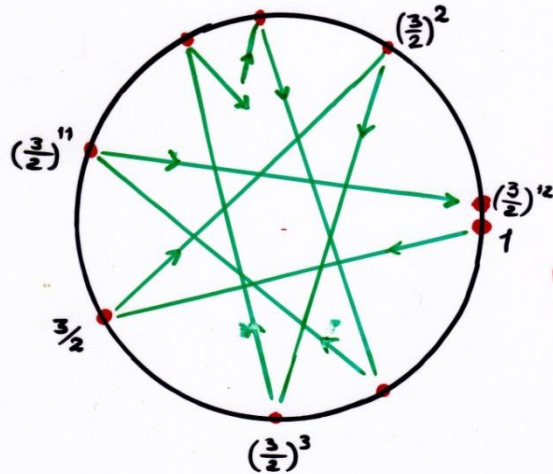
} violoncello

$$2^{1/12} = 1.059\dots$$

$$2^{7/12} = 1.498\dots$$

scala
Pitagorica

	scala naturale		scala pitagorica
do	1	1 = 1	
sol	$3/2$	$\frac{3}{2} = \frac{3}{2}$	$3/2$
re	$9/8$	$\frac{9}{8} = \frac{9}{8}$	$(3/2)^2 \cdot \frac{1}{2}$
fa	$5/3$	$\frac{80}{48} \sim \frac{81}{48}$	$(3/2)^3 \cdot \frac{1}{2}$
mi	$5/4$	$\frac{80}{64} \sim \frac{81}{64}$	$(3/2)^4 \cdot \frac{1}{4}$
si	$15/8$	$\frac{240}{128} \sim \frac{243}{128}$	$(3/2)^5 \cdot \frac{1}{4}$
fa #			
do #			



$$3^{12} = 531.441$$

$$2^{19} = 524.288$$

errore 1.3%

