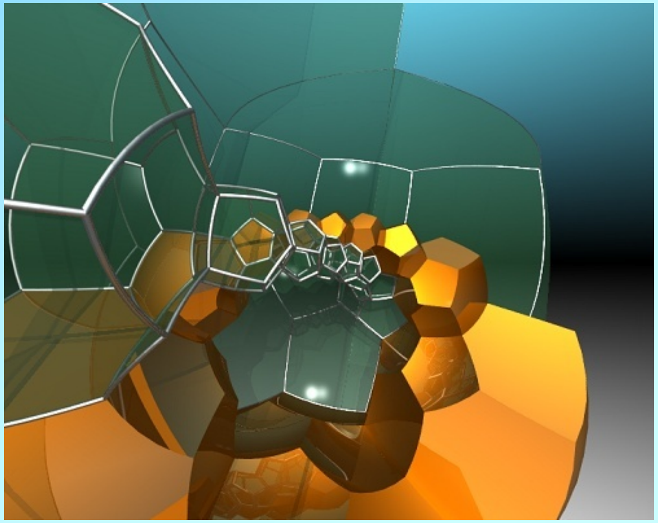
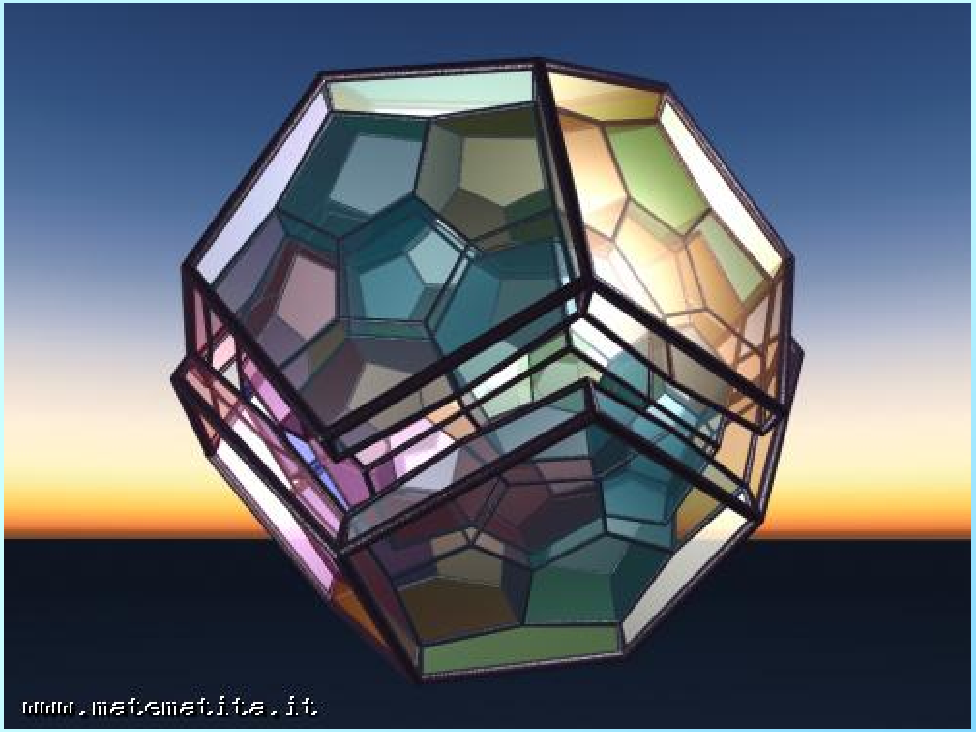
Le immagini della matematica: esempi a quattro dimensioni





Summer School: La matematica incontra le altre Scienze

San Pellegrino Terme, 08-09-2014

M. Dedò

Da anni il Centro ***matematita*** riserva una particolare attenzione all’uso delle immagini nella comunicazione (informale) della matematica.

Perché?

Immaginazione, visualizzazione, associazione di idee, comunicazione informale…

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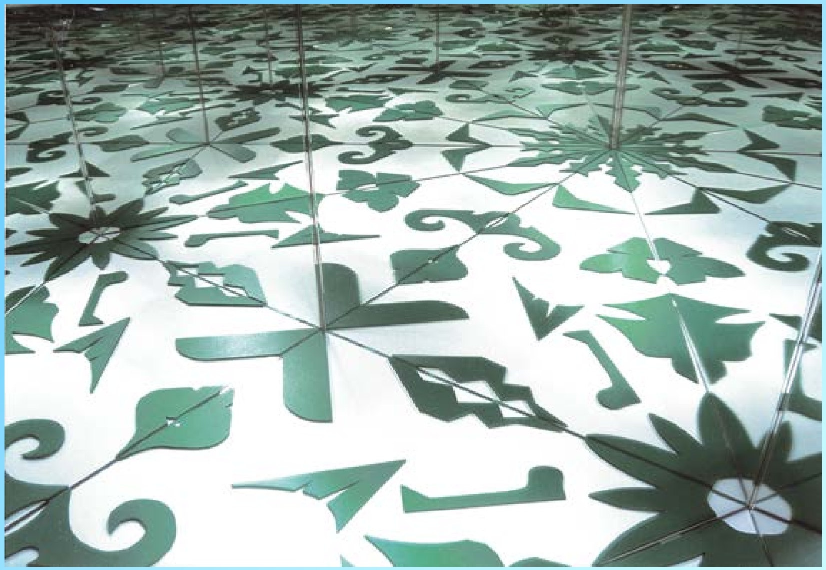

infinito…

spirali

frattali

prospettiva

Attenzione!

Immagini molto simili possono essere l'una adatta e l'altra no per rappresentare lo stesso concetto matematico

Immagini e modelli sono **SEMPRE** e **necessariamente** «un po' falsi»

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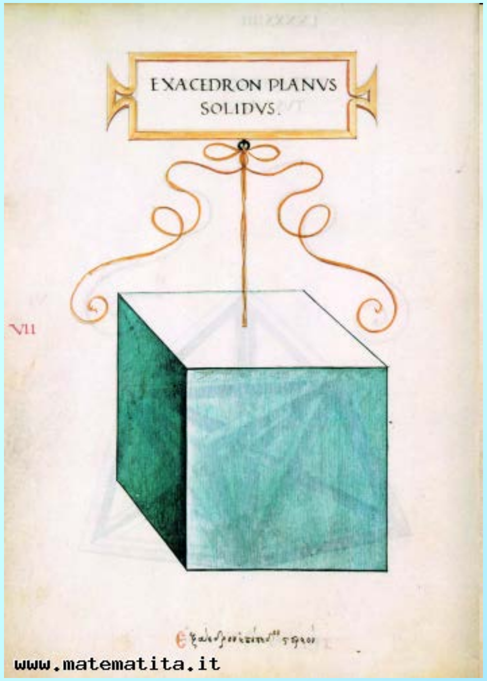
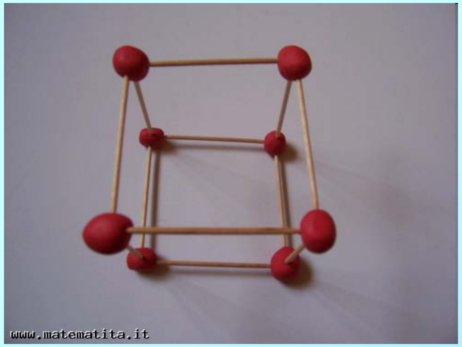
L’immagine di un cubo **non è un cubo**

Il modello di un cubo **non è un cubo**





Ma allora **che cos’è** il modello di un cubo**?**

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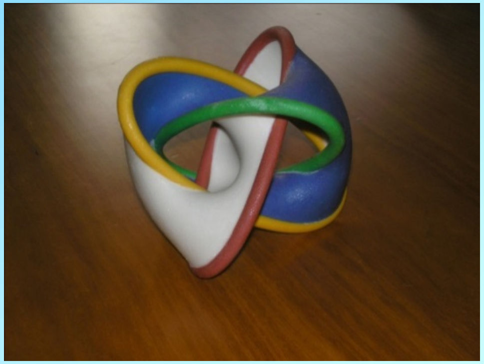
Riconoscere un'idea astratta in un'immagine o in un modello aiuta la comprensione del concetto, soprattutto quando va di pari passo con il piano formale.



Quando diciamo «***Io vedo!***», a proposito di un concetto astratto, significa che ci siamo costruiti un'immagine mentale.

... ma allora si può provare anche a...

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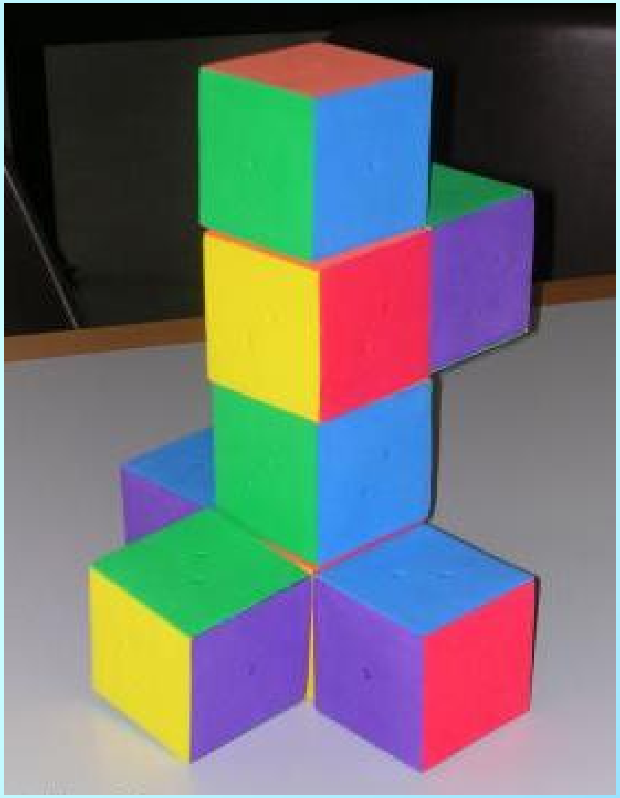
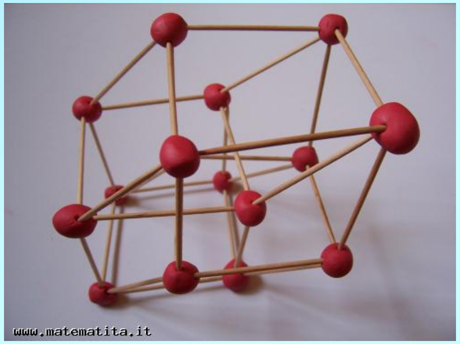
... vedere un ipercubo!

(un cubo a 4 dimensioni)!





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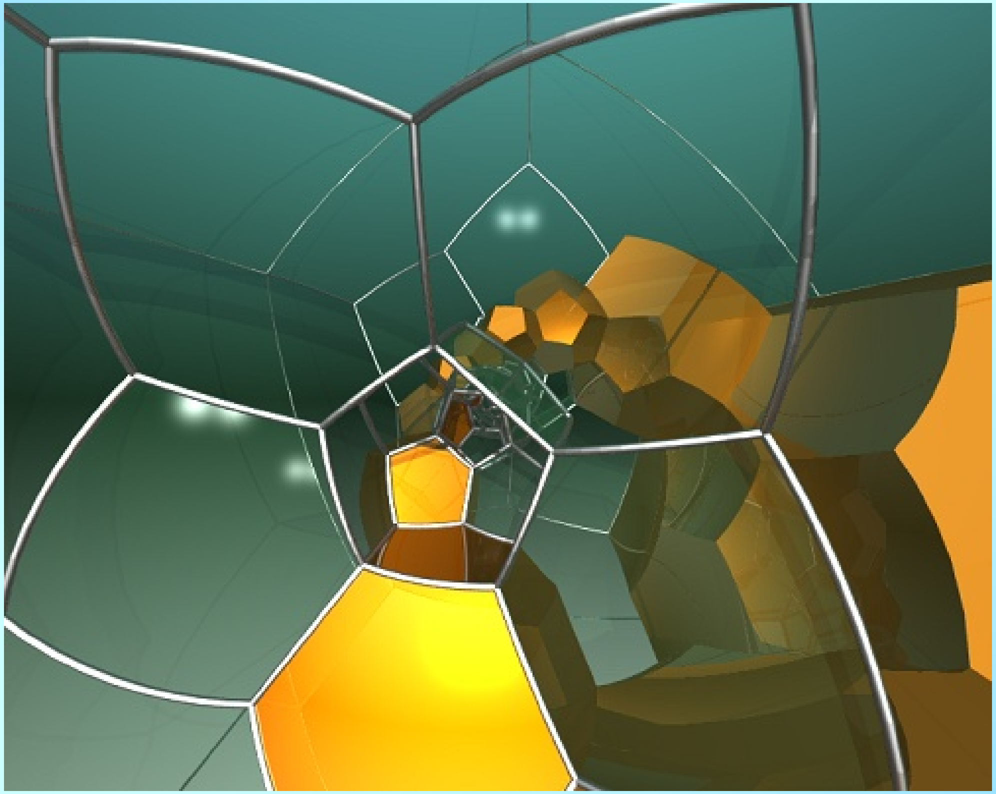
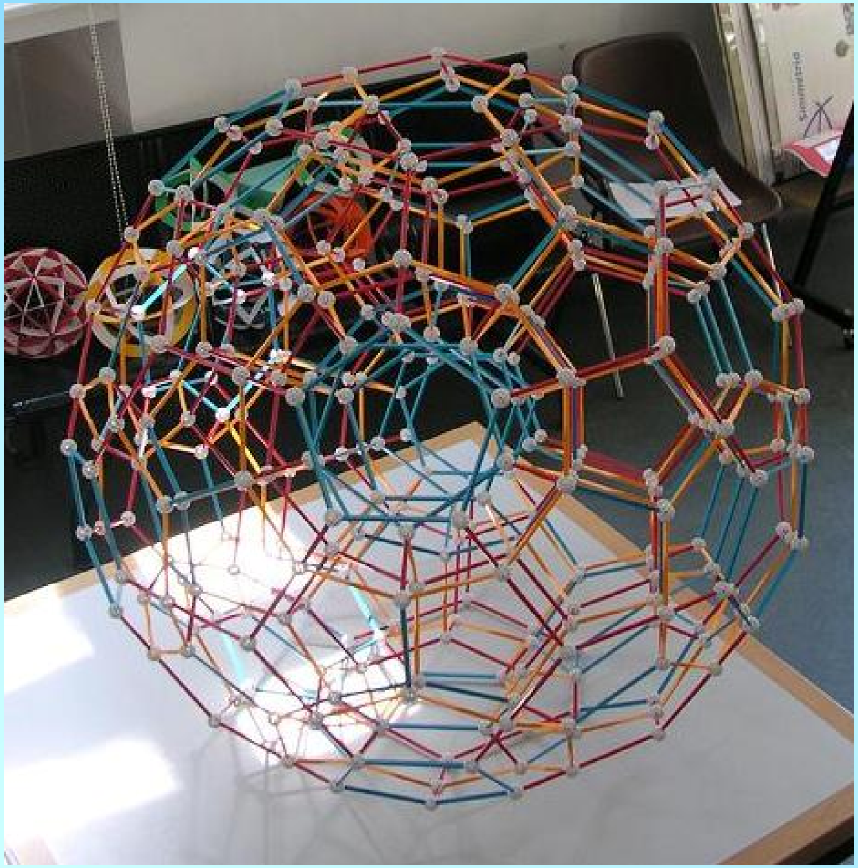
Ma **che cos'è** la "quarta dimensione"?





Esiste? A cosa serve?
Come possiamo immaginarla?
Ha senso immaginarla?

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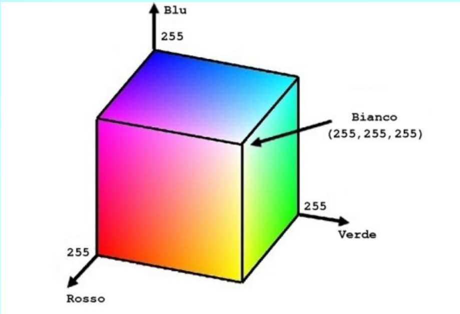
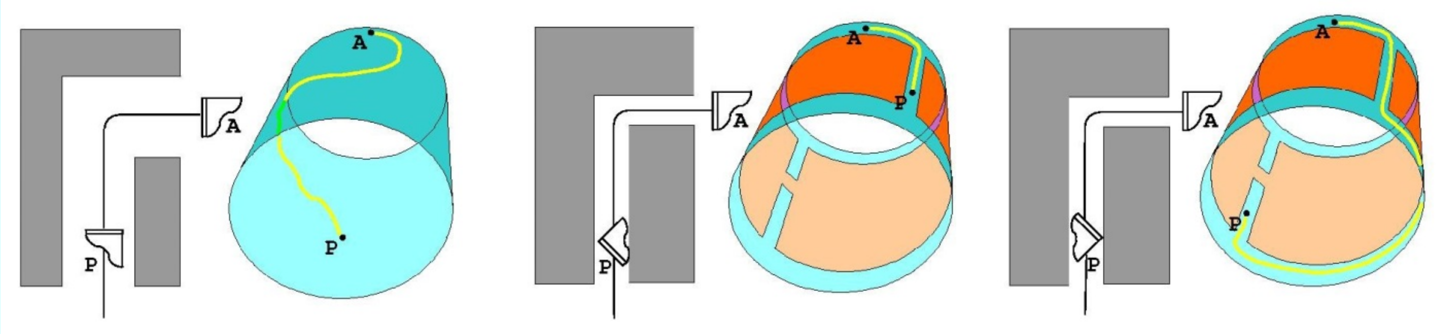
Una qualsiasi situazione in cui un problema può essere descritto da 4 (o 5, o 6, o ...) parametri può essere descritta con un opportuno sottoinsieme dello spazio a 4 (o 5, o 6, o ...) dimensioni



Parametri non spaziali possono essere

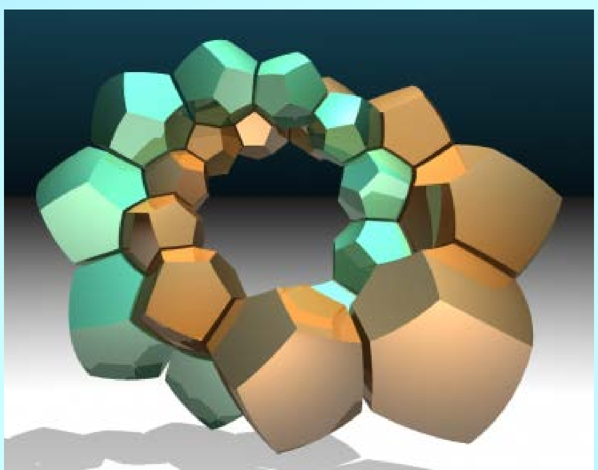
- il tempo
- una gradazione di colore
- la temperatura
- ...

Astrazione = prescindere dal possibile significato



E come si possono immaginare le DUE dimensioni?

... **e un punto?**



... ma allora possiamo anche immaginare le QUATTRO (o 5,6,...) dimensioni

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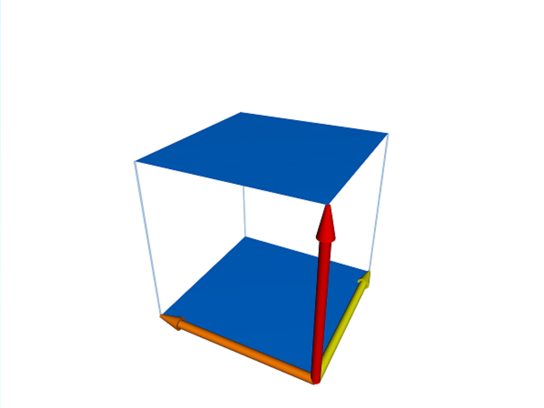
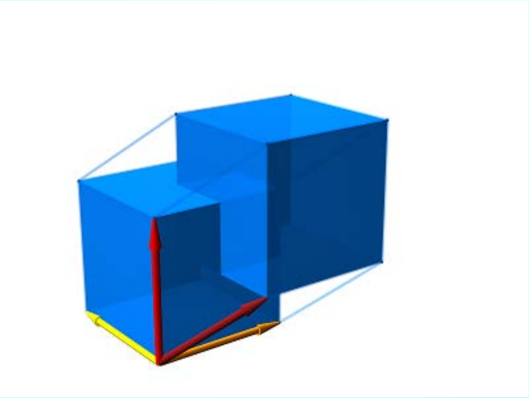
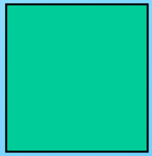
Ragionare per analogia



Se si capisce **come** si passa da un quadrato a un cubo...



... si può immaginare **come** passare da un cubo a un ipercubo



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un ipercubo con un modello (3d)

Maniere per rappresentare un cubo con una figura (2d)

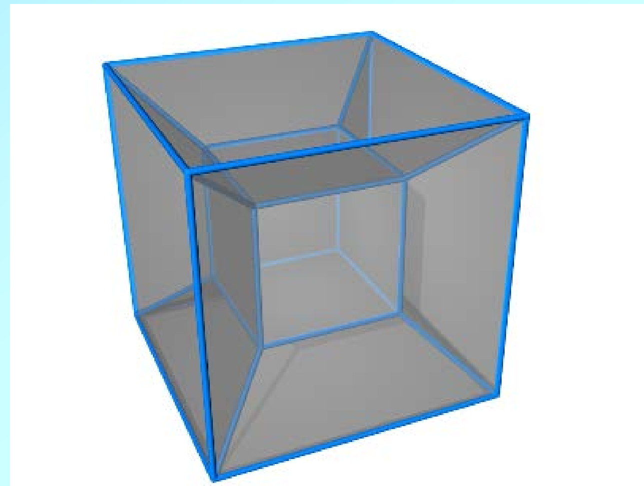
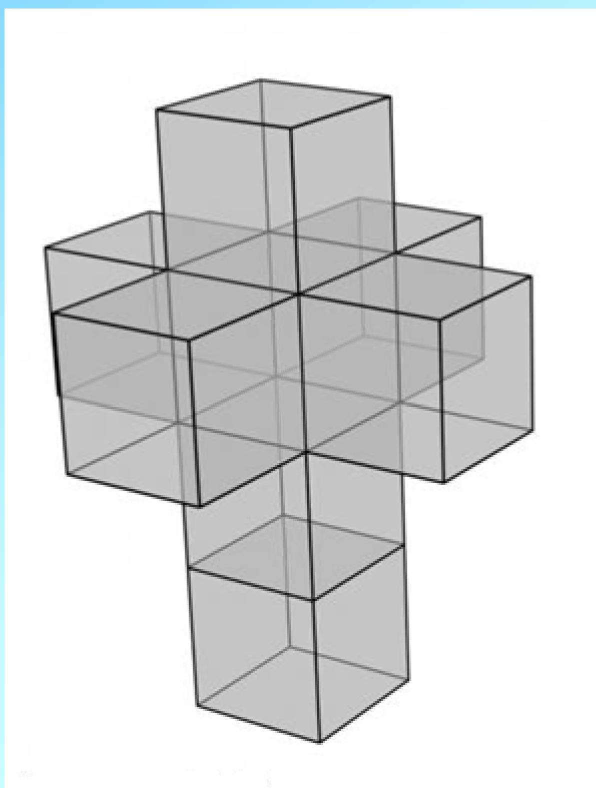
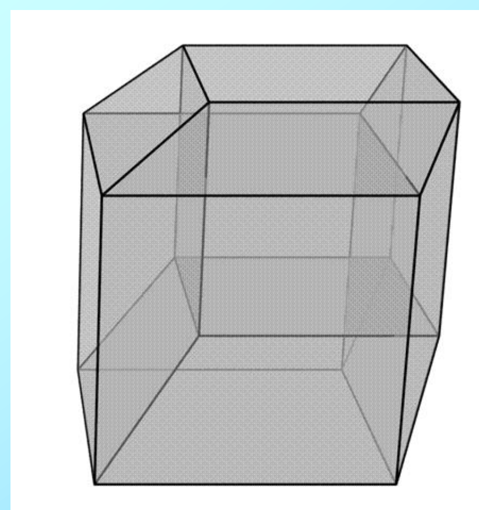


Diagramma di Schlegel



Sviluppo



Proiezione

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Salvador Dalì
Corpus hypercubicus

La Grande Arche (Parigi)

Attilio Pierelli,
Viterbo

rappresentazioni di
un ipercubo nell'arte
e nell'architettura

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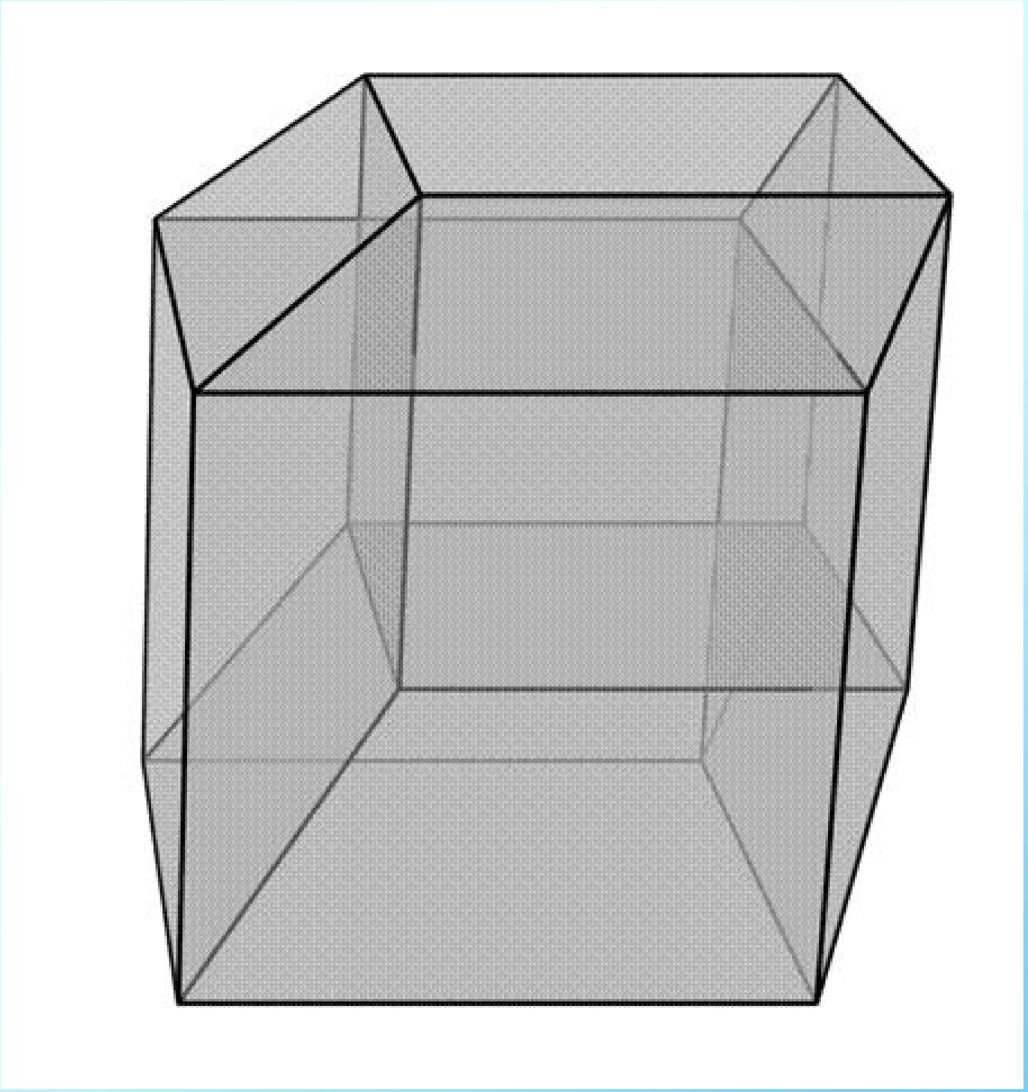
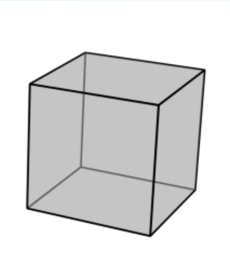
Le 2-facce di un cubo
sono sei quadrati.
Le 3-facce di un ipercubo
sono otto cubi.

Ma dove sono gli 8 cubi?



una proiezione



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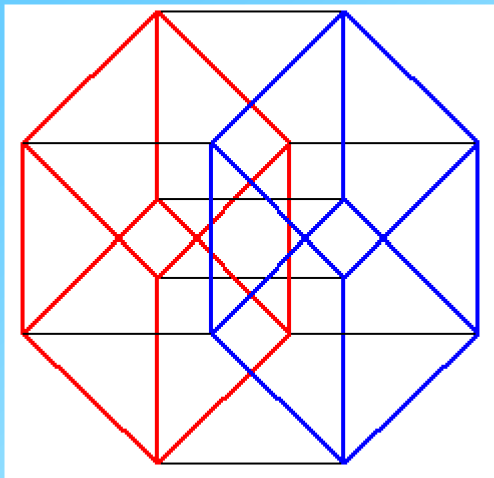

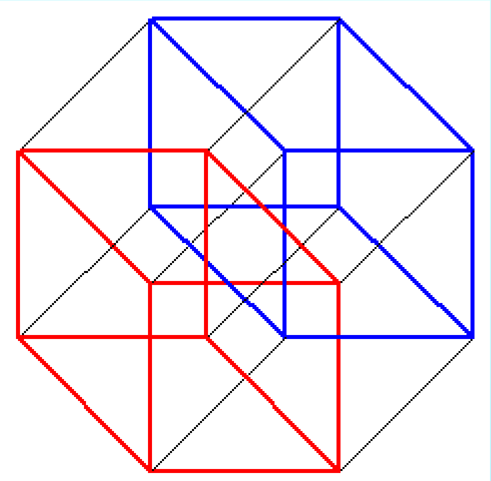

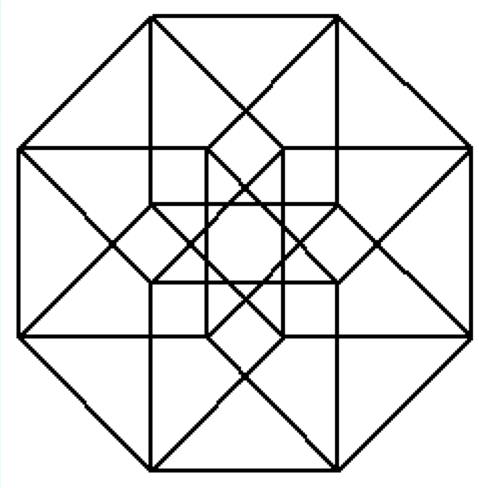

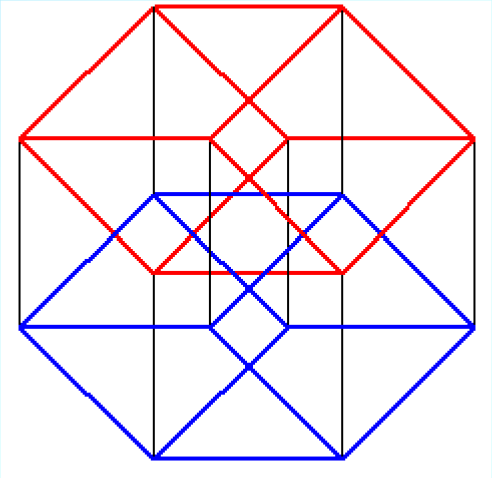

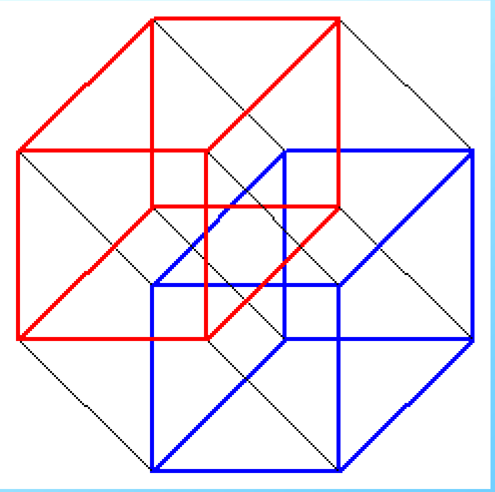

Le otto facce cubiche
di un ipercubo

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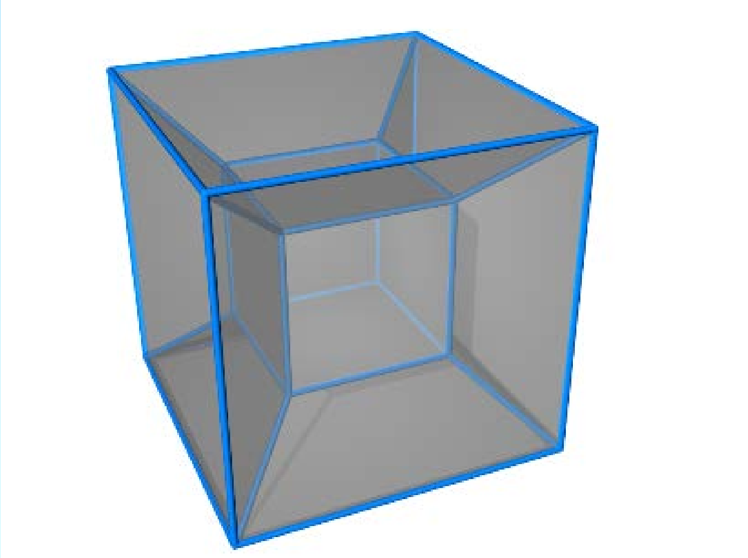
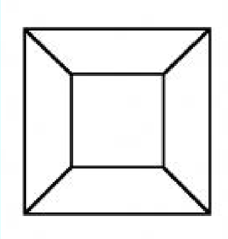
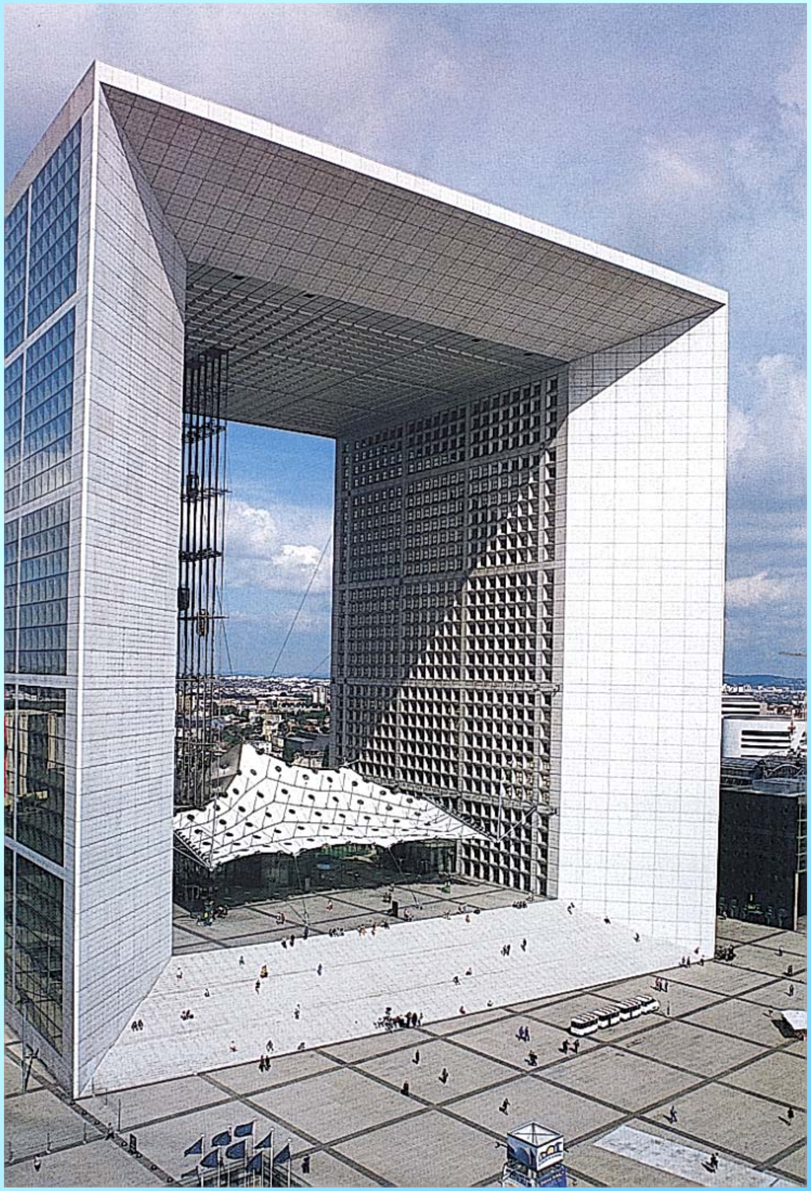
un diagramma di Schlegel

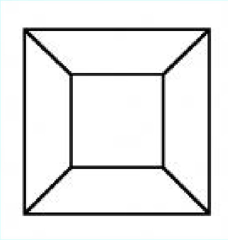


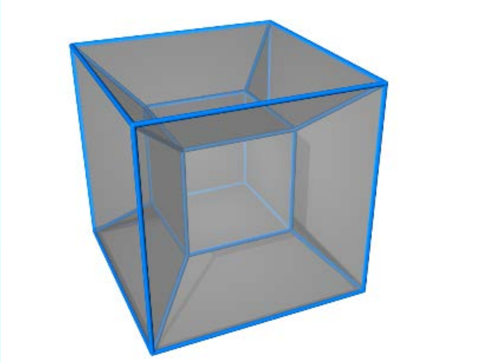




Un po' di conti…

Punto	1				
Segmento	2	1			
Quadrato	4	4	1		
Cubo	8	12	6	1	
Ipercubo	**16**	**32**	**24**	**8**	**1**





$12 = 4\times2 + 4$ **$12\times2 + 8 = 32$**

$6 = 1\times2 + 4$ **$6\times2 + 12 = 24$**

- 16 vertici
- 32 spigoli
- 24 facce di dimensione 2 (quadrati)
- 8 celle di dimensione 3 (cubi)

Ma… ha senso?

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Usando le coordinate...

	Cubo	Ipercubo
i **vertici**	$(\pm1,\pm1,\pm1)$ sono **8**	$(\pm1,\pm1,\pm1,\pm1)$ sono **16**
i punti medi degli **spigoli**	$(0,\pm1,\pm1)$ sono 3×4 =**12**	$(0,\pm1,\pm1,\pm1)$ sono 4×8 = **32**
i centri delle **facce**	$(0,0,\pm1)$ sono 3×2 = **6**	$(0,0,\pm1,\pm1)$ sono 6×4 **= 24**
i centri delle **3-facce**	$(0,0,0)$ è **1**	$(0,0,0,\pm1)$ sono 4×2 = **8**

... e si potrebbe continuare...

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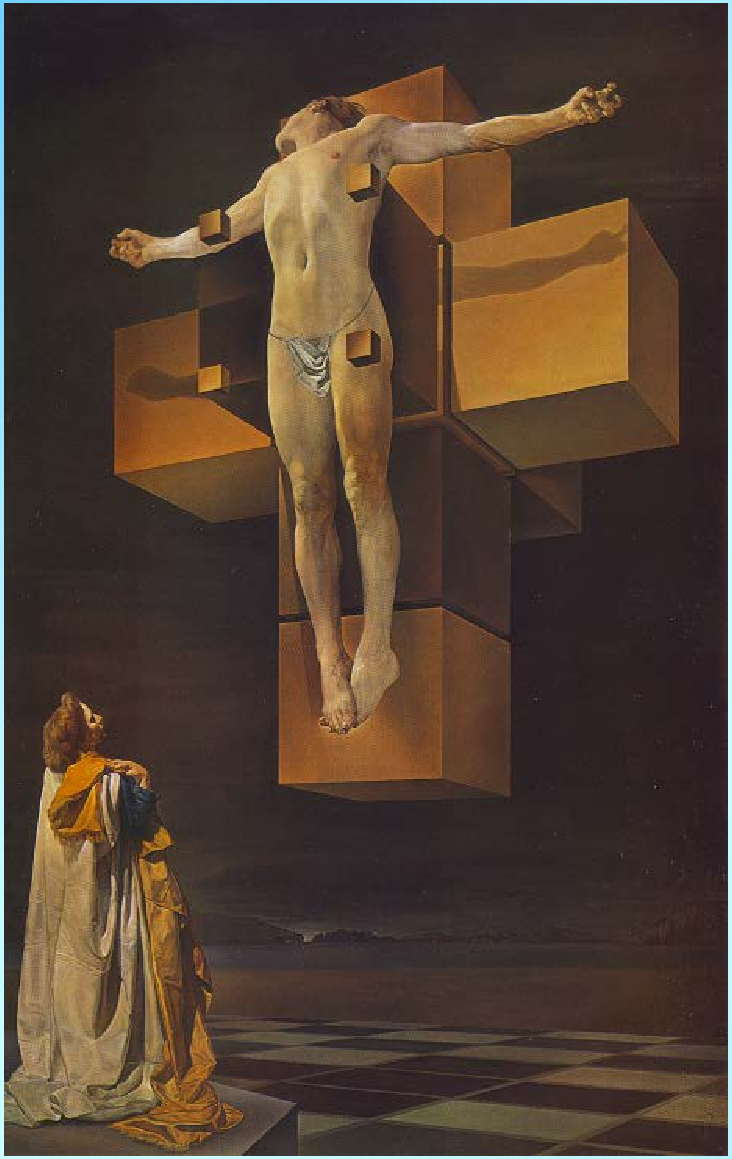
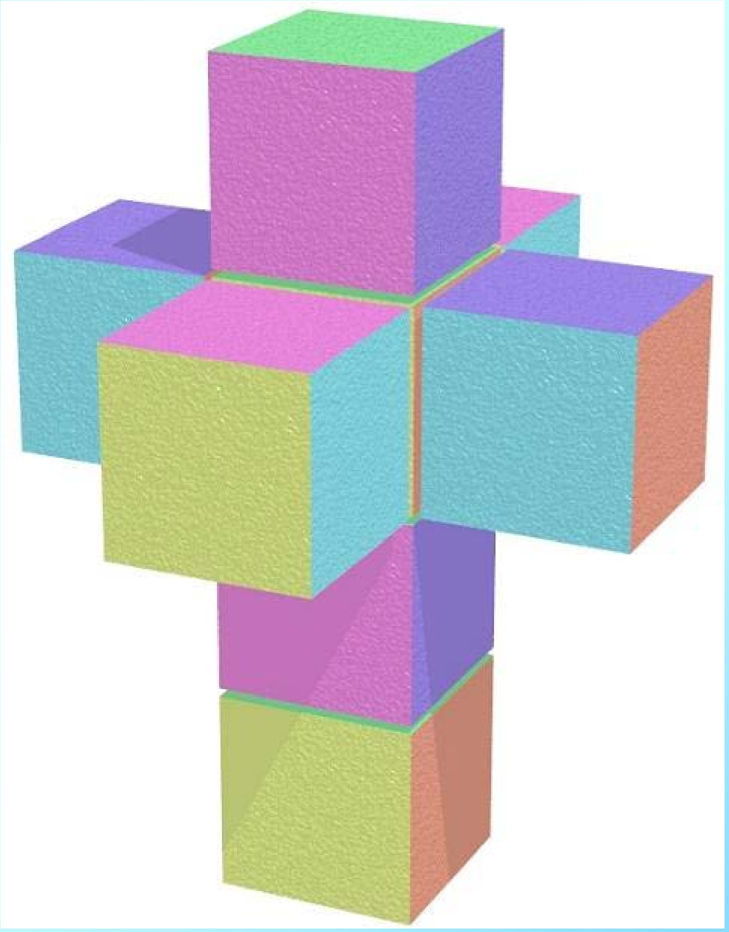
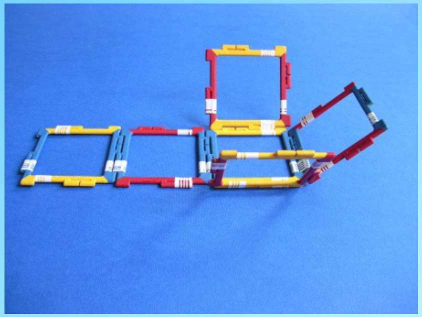
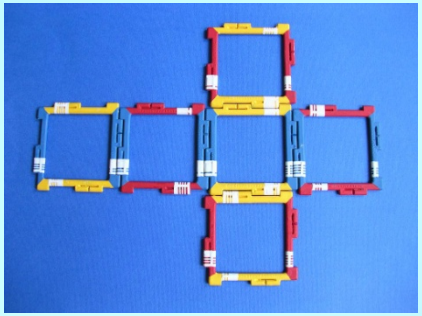
uno sviluppo

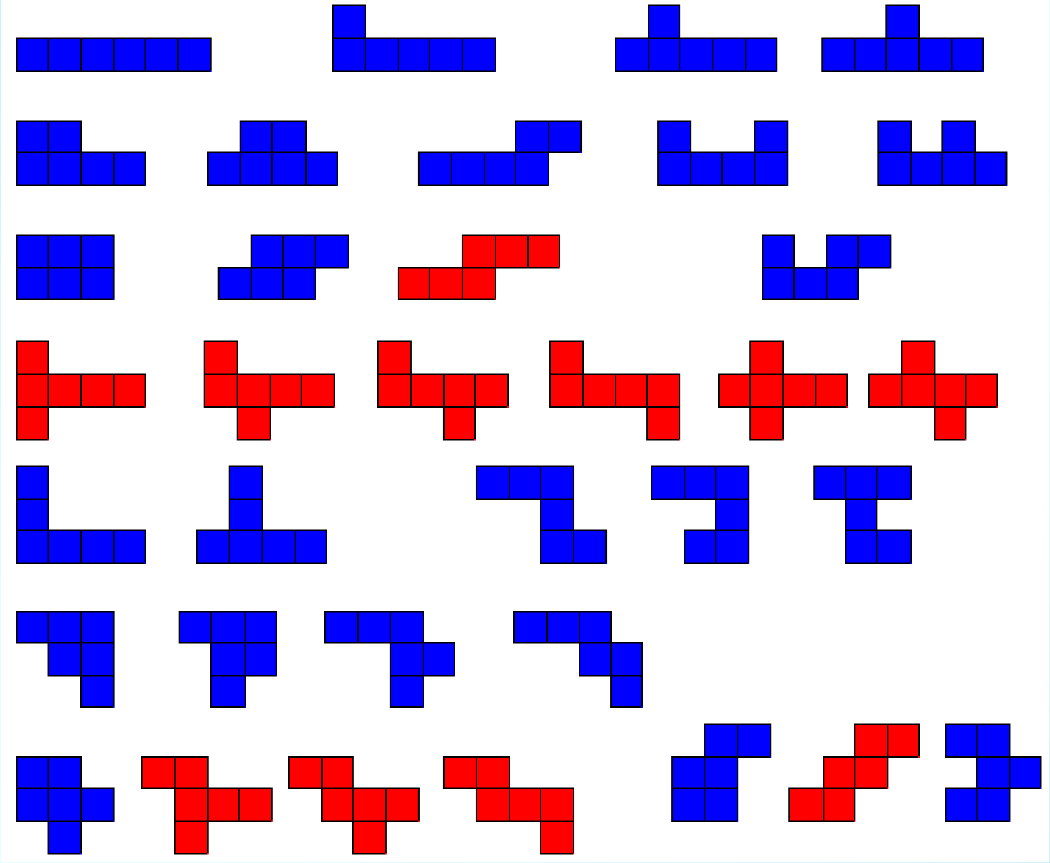






Sviluppi di un cubo





i 35 esamini e gli 11 sviluppi di un cubo

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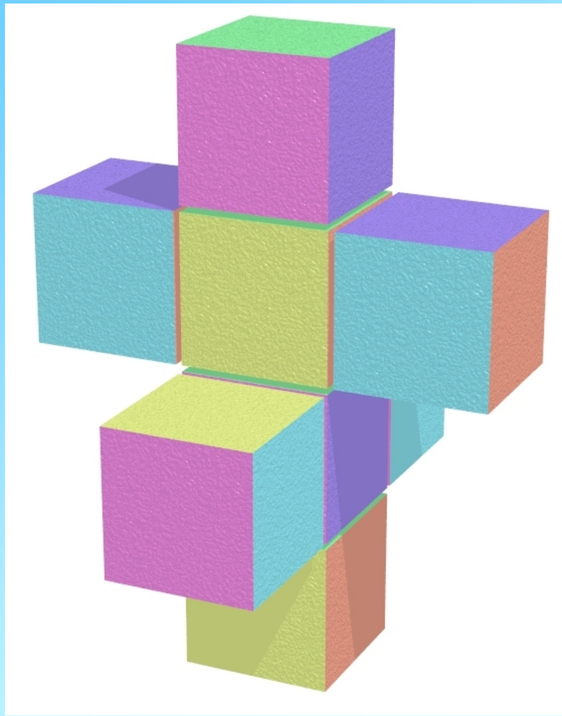
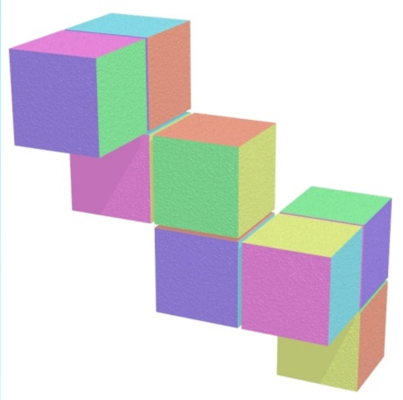
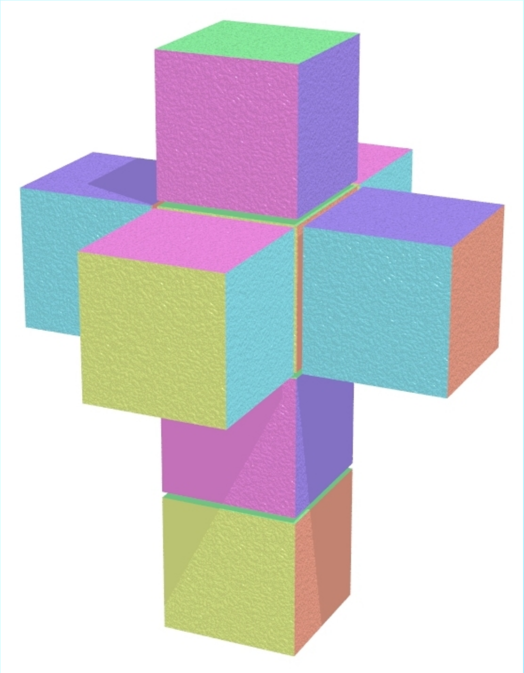
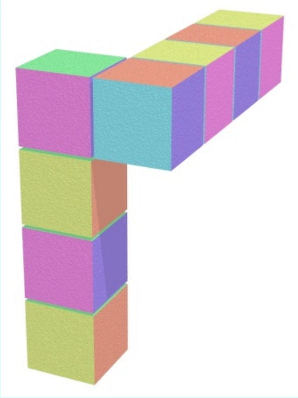
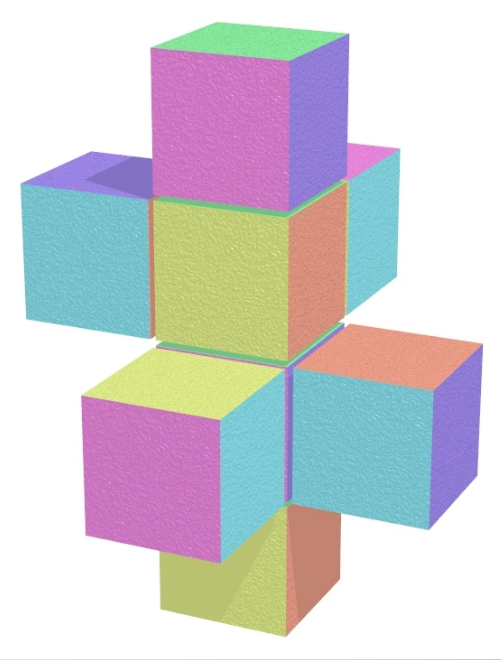
alcuni sviluppi di un ipercubo...

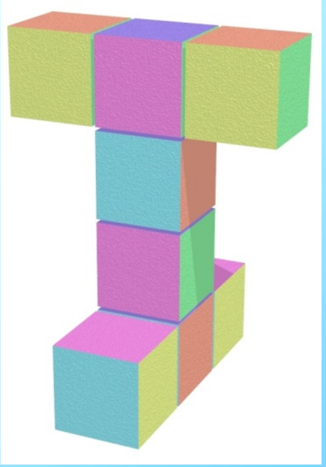












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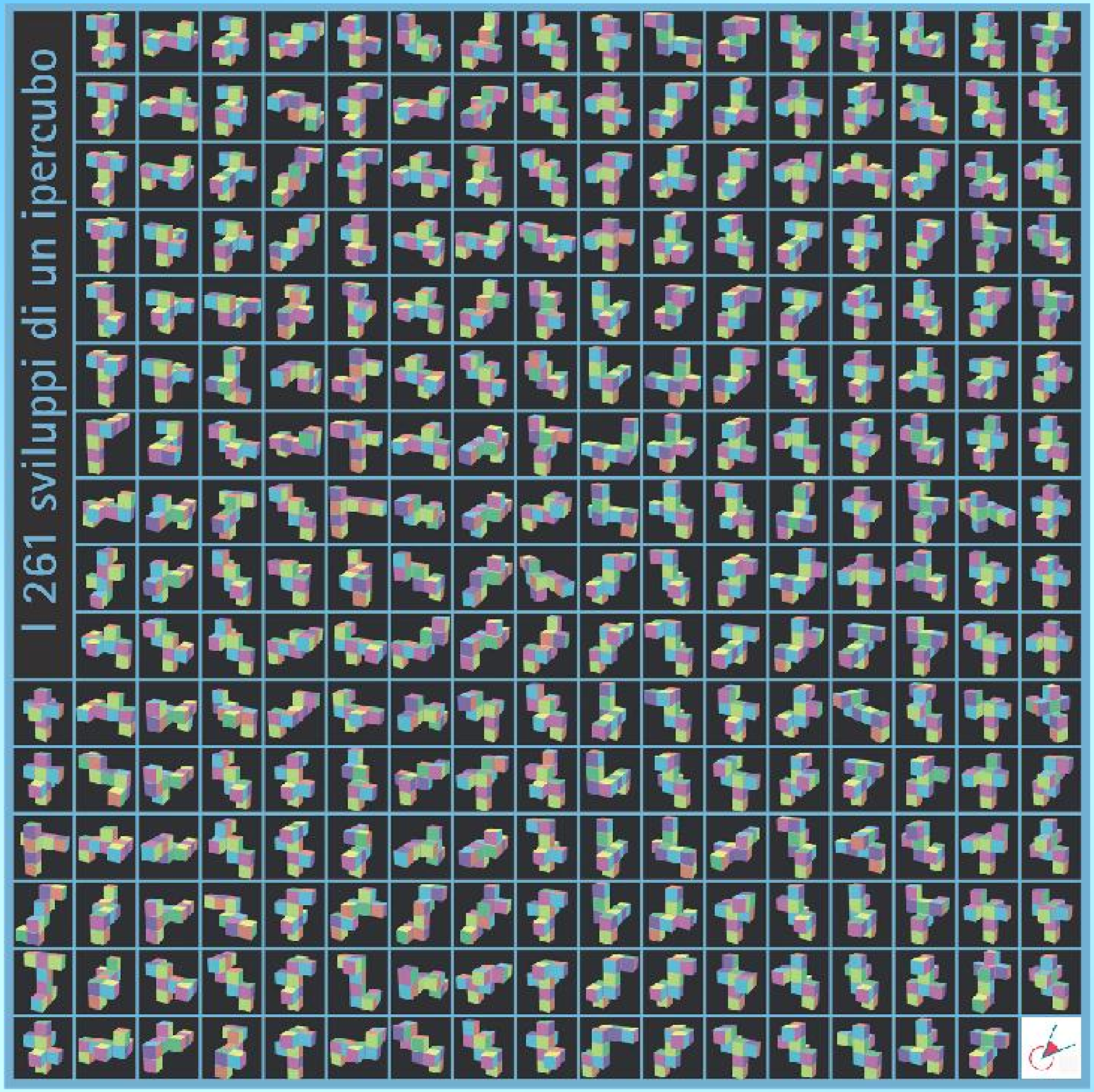

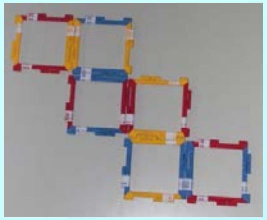

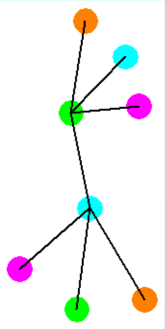

in tutto sono 261

... tanti?

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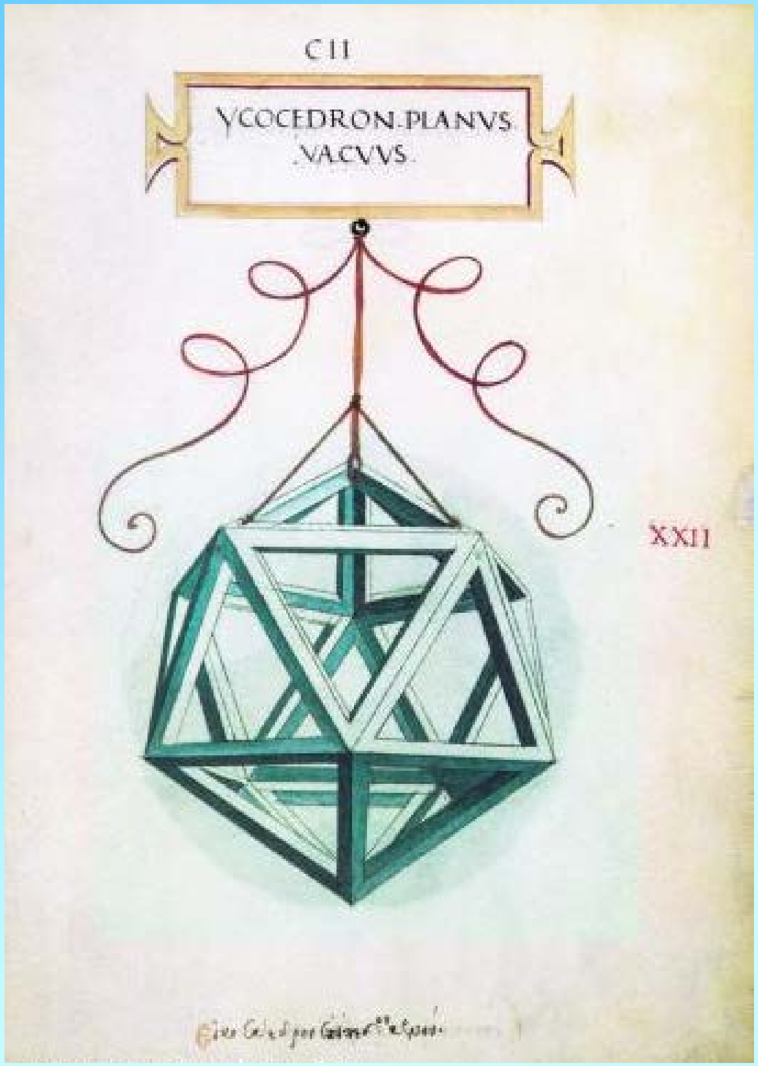
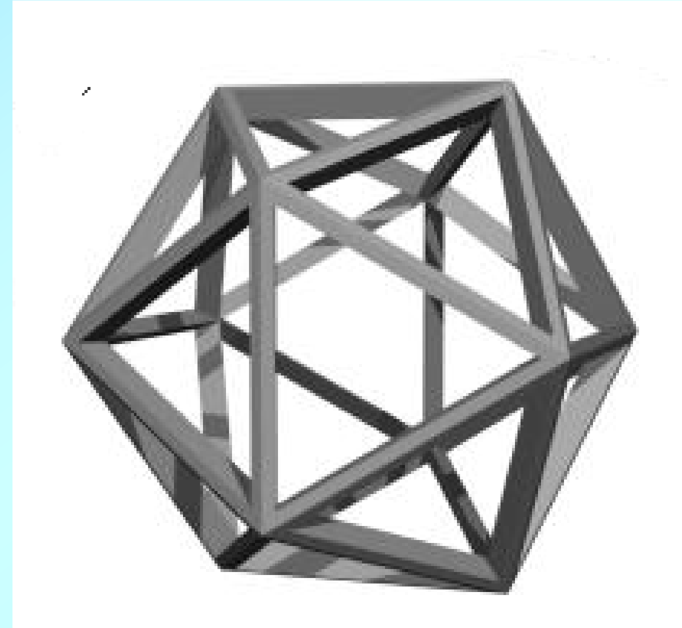
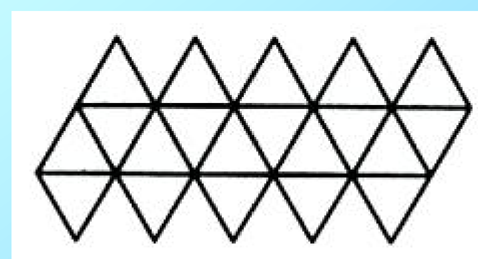
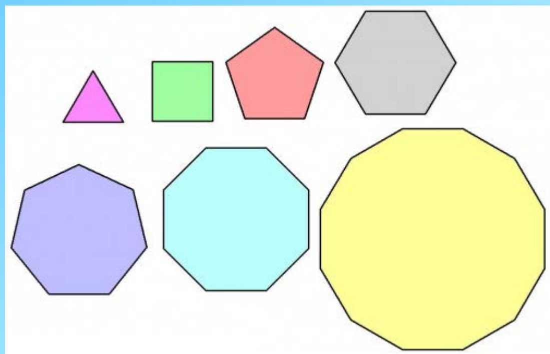




... ma gli sviluppi di un icosaedro sono... **43380**



Ci sono altri oggetti regolari oltre all’ipercubo?



In dimensione due: infiniti poligoni regolari, e 3 tassellazioni regolari

triangoli (3,6)

quadrati (4,4)

esagoni (6,3)

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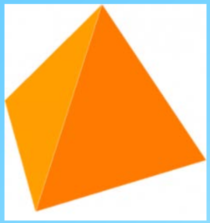
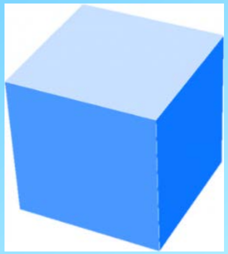
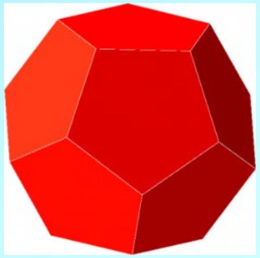


(3,3)
tetraedro
triangoli
3 a 3

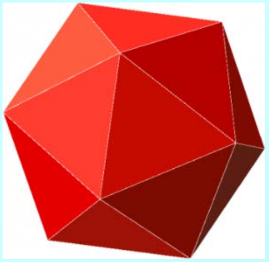


(4,3)
cubo
quadrati
3 a 3

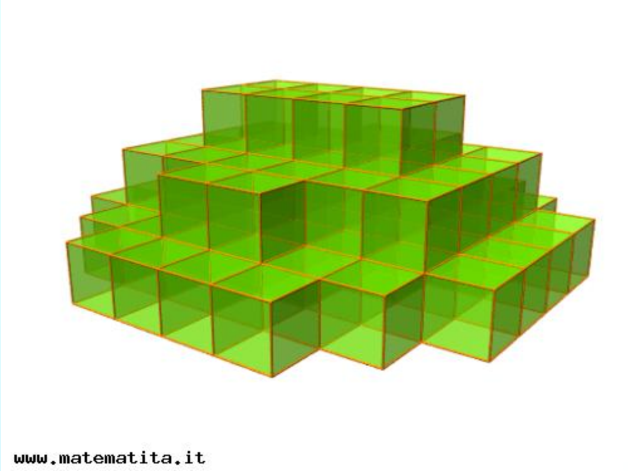
(3,4)
ottaedro
triangoli
4 a 4

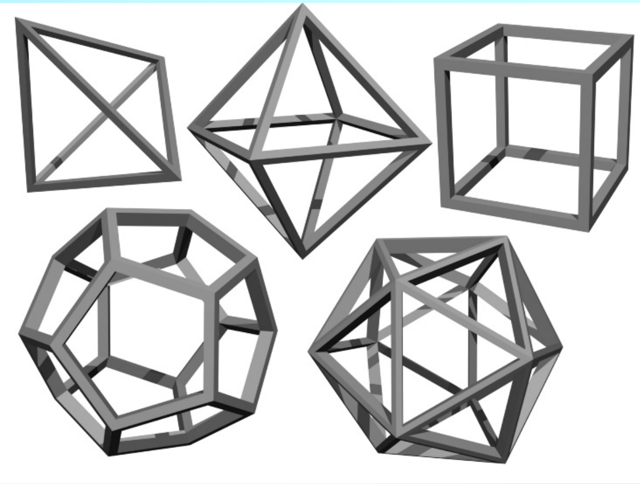


(5,3)
dodecaedro
pentagoni
3 a 3



(3,5)
icosaedro
triangoli
5 a 5

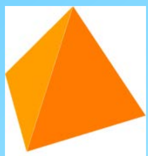




In dimensione tre: 5 poliedri regolari, ma una sola tassellazione regolare

Perché solo questi 5?

intorno a un vertice la somma degli angoli delle diverse facce deve essere **minore di 360°**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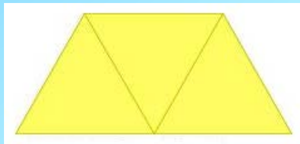
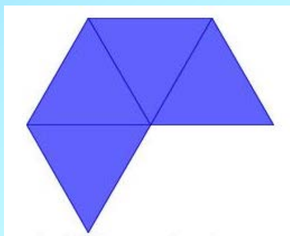
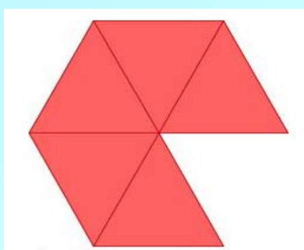
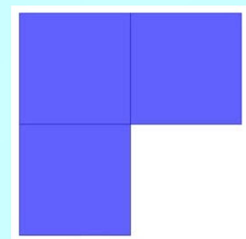
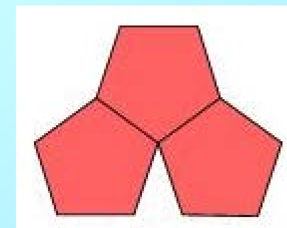
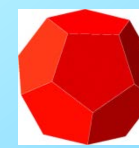
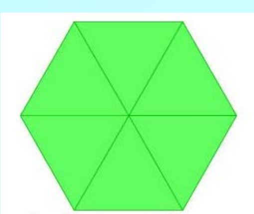


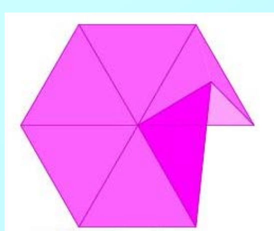

(3,4)


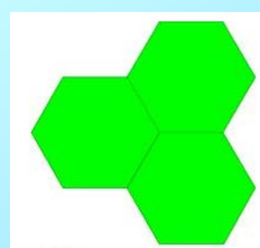

(3,5)



(4,3)



(5,3)



NO!

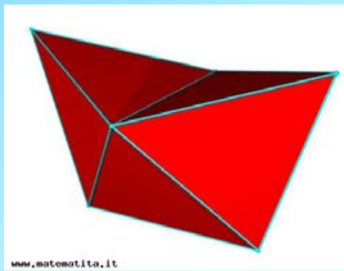

NO!


NO!

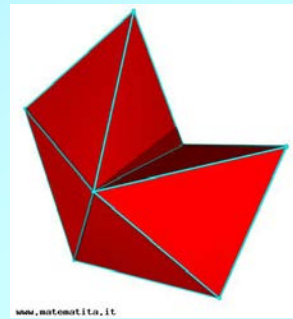
Analogia:

intorno a uno spigolo la somma degli angoli diedri deve essere **minore di 360°**

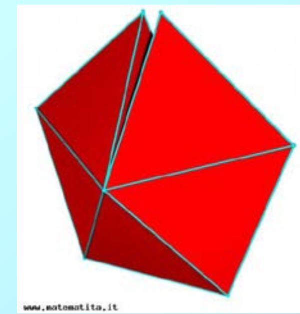
tetraedri
a 3 a 3

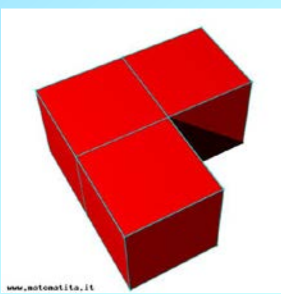


a 4 a 4



a 5 a 5

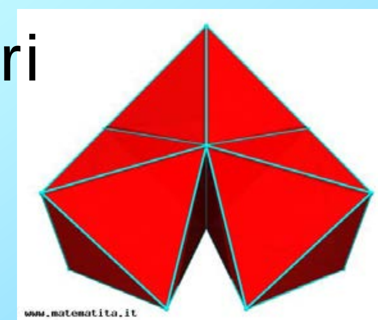




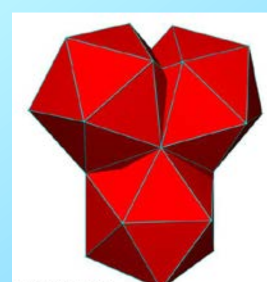
cubi
3 a 3

dodecaedri
3 a 3

ottaedri
3 a 3

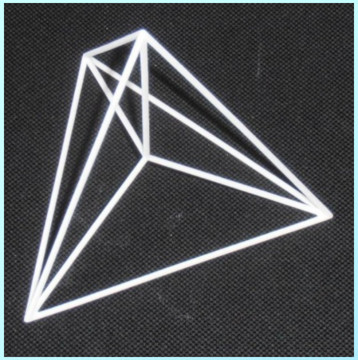


con gli icosaedri NON si può!
l’angolo diedro è troppo grosso.

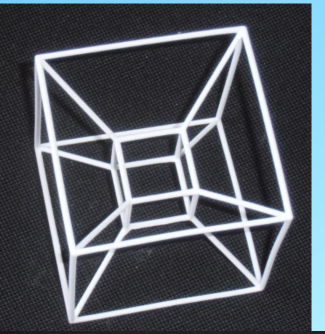


Oggetti regolari in dimensione 4

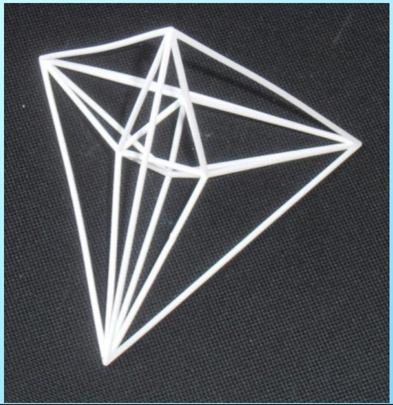
I politopi regolari sono **sei**:



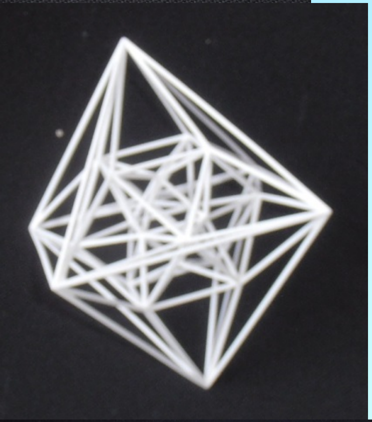
- ipertetraedro (3,3,3)



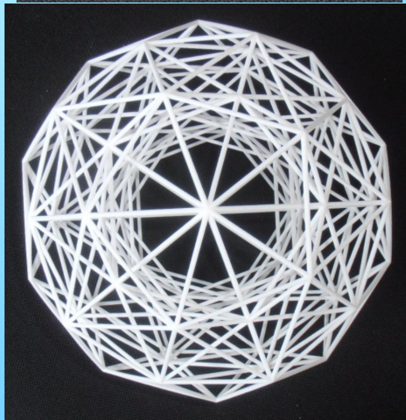
- (4,3,3) ipercubo



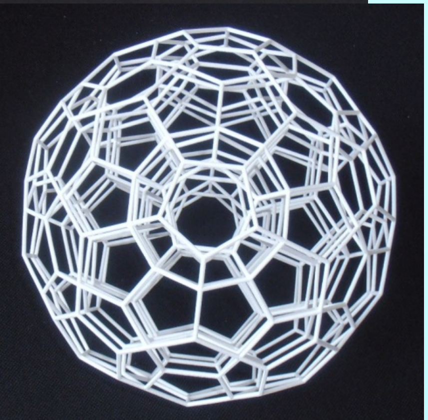
- iperottaedro (3,3,4)



- (3,4,3) 24-celle



- 600-celle (3,3,5)



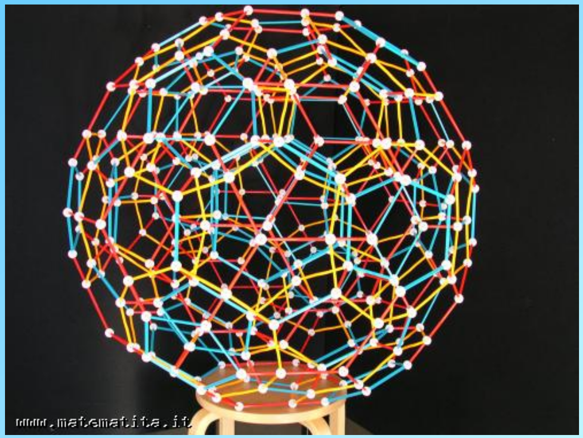
- (5,3,3) 120-celle

Oggetti regolari in dimensione n

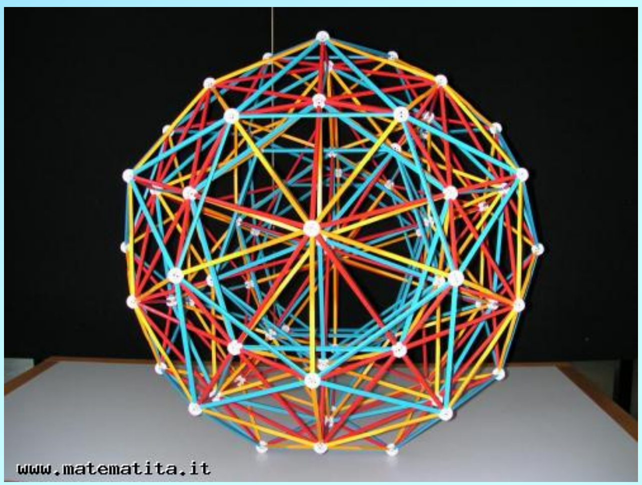
dimensione	poli...	tassellazioni
2	infiniti	3
3	5	1
4	**6**	**3**
5	3	1
6	3	1
...	...	...
n	3	1

... la dimensione 4 è proprio strana!

120-celle e 600-celle



modelli di una proiezione 3d



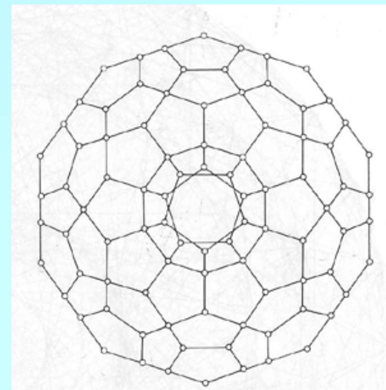


Figure 4·6E: {5, 3, 3}

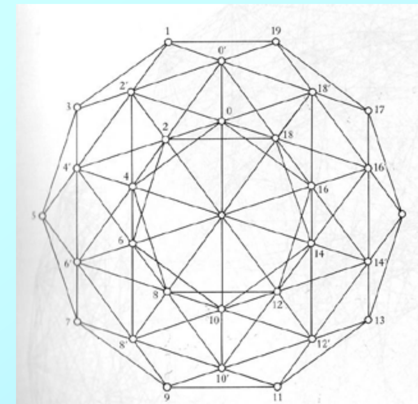


Figure 4·6D: {3, 3, 5}

figure dal libro *Complex regular polytopes* di H.S.M. Coxeter

come è possibile che le figure siano così semplici quando i modelli sono così complicati?

Ombre dal 4d...!

il 120-celle

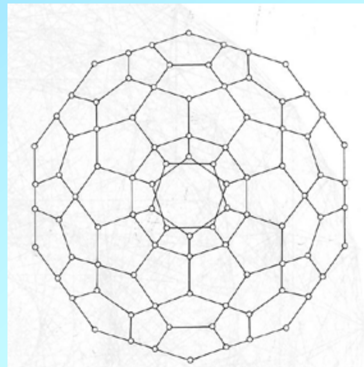
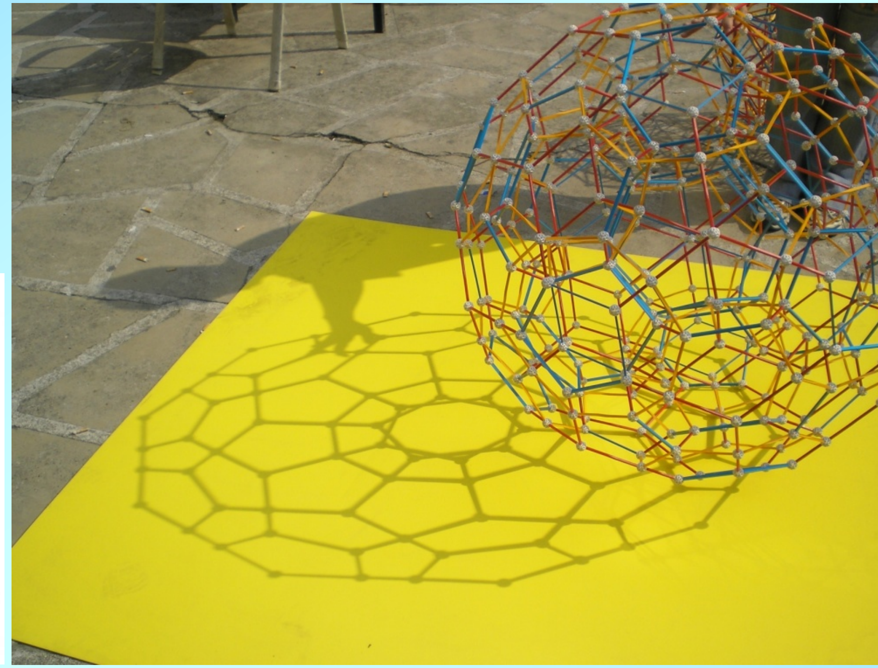
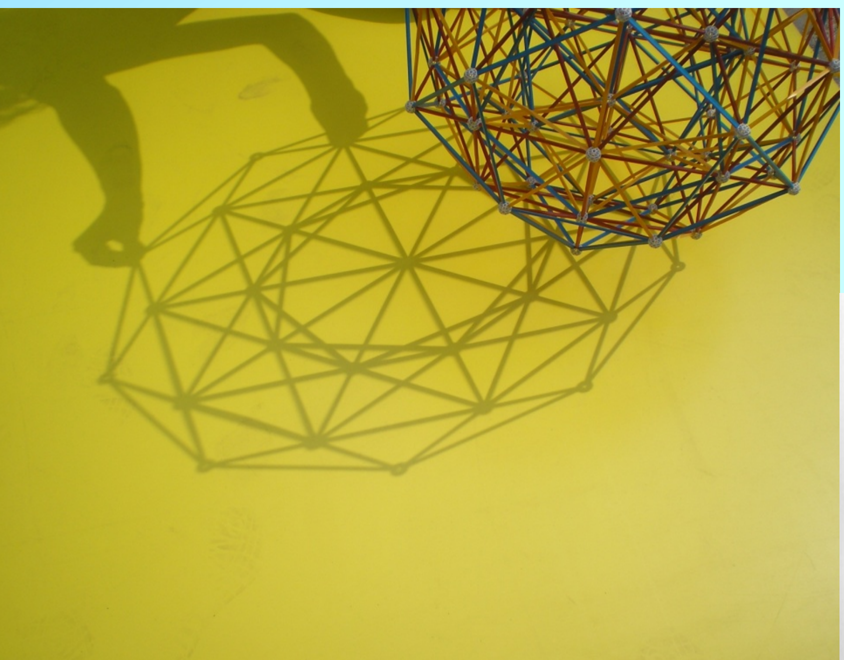


Figure 4·6E: {5, 3, 3}





il 600-celle

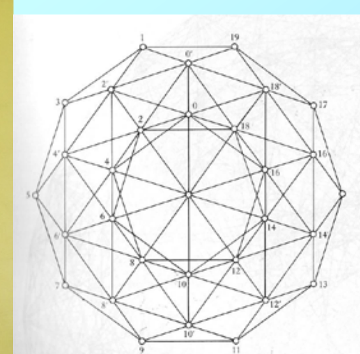


Figure 4·6D: {3, 3, 5}

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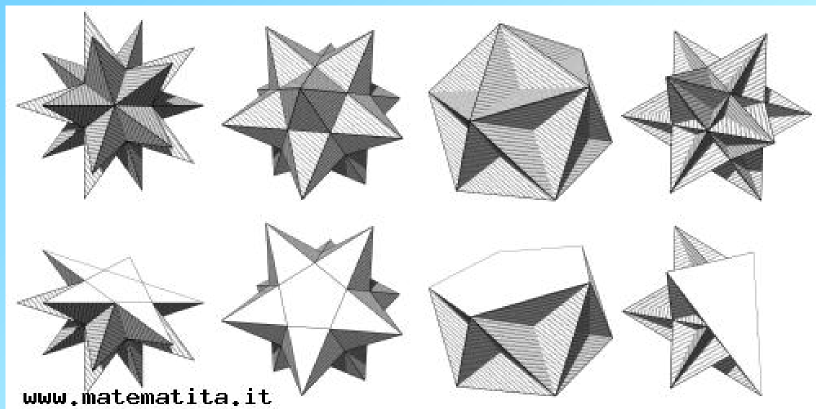
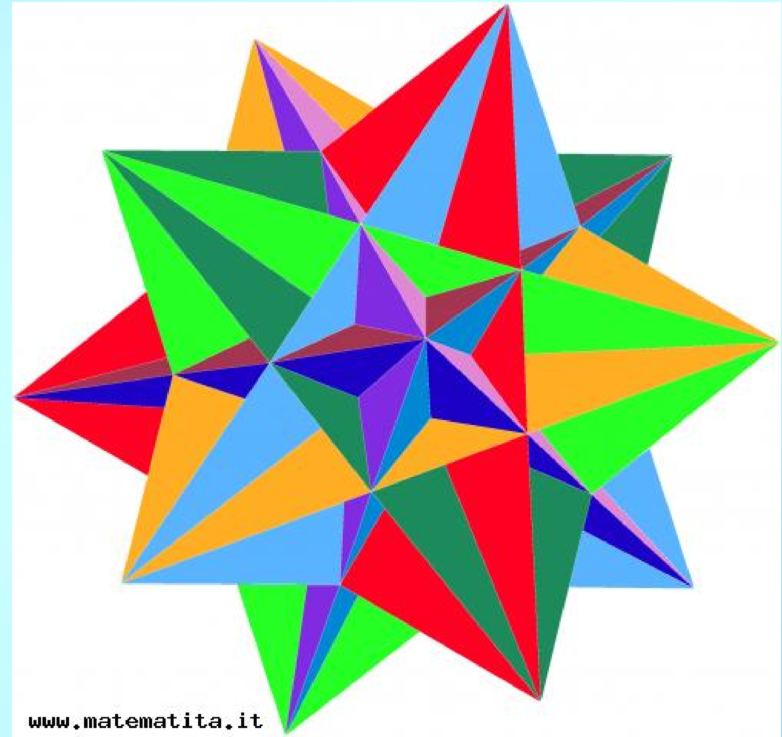
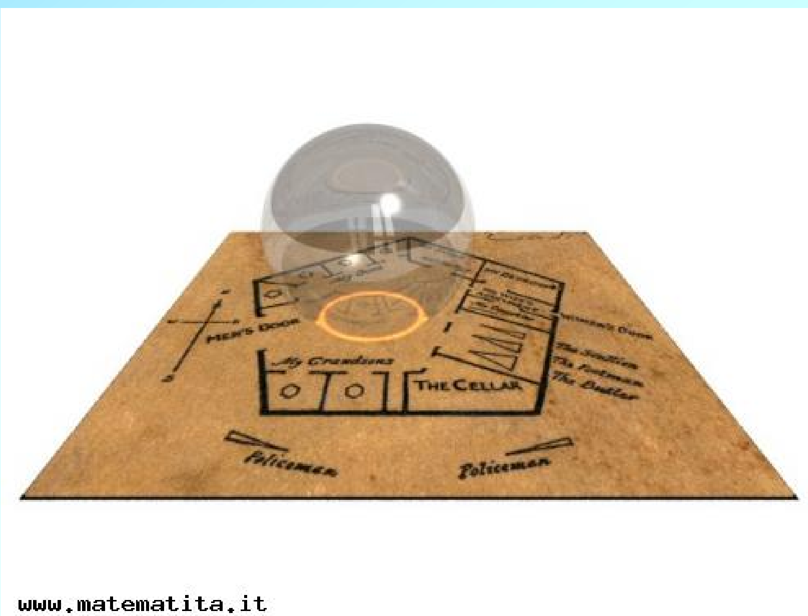
E non ci sono solo gli oggetti regolari…

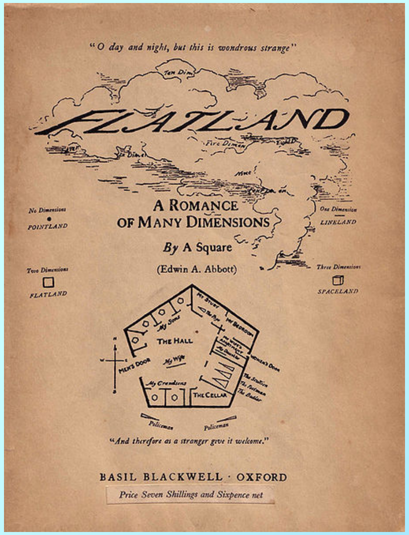






Suggerimenti di lettura

- Abbott, *Flatlandia*, ed. Adelphi
- Heinlein, *La casa nuova,* in *Racconti matematici*, a cura di Bartocci, ed. Einaudi
- Dossier 4d su *XlaTangente* (n. 4-5 e n.6)
- Siobhan Roberts, *Il re dello spazio infinito,* Rizzoli



Molte immagini, animazioni, approfondimenti, FAQ si possono trovare nel sito **http://www.matematita.it/materiale/**

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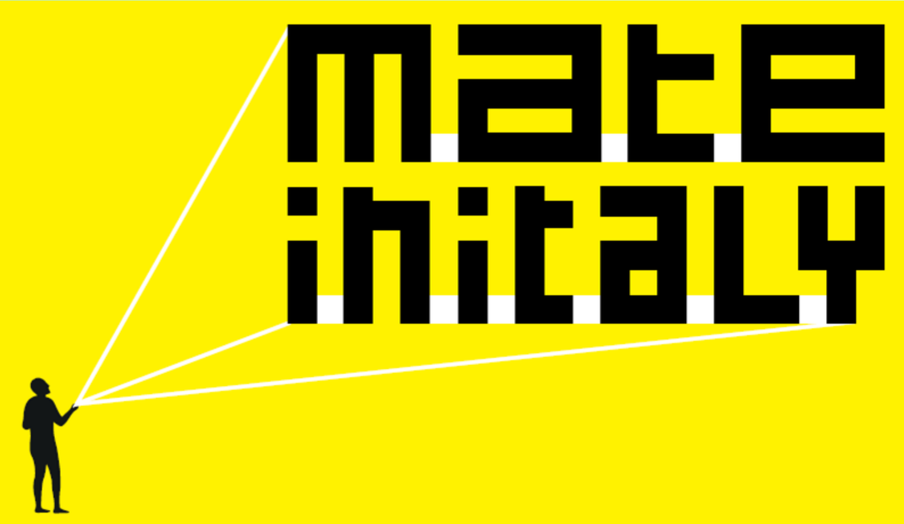
Milano, Palazzo della Triennale,
Via Alemagna 6
dal 13 settembre
al 23 novembre 2014



Organizzata da:
Centro “matematita” dell’Università degli Studi di Milano
Centro PRISTEM dell’Università Bocconi di Milano.

Una intera sezione della mostra MaTeinItaly sarà dedicata proprio allo spazio a 4 dimensioni!

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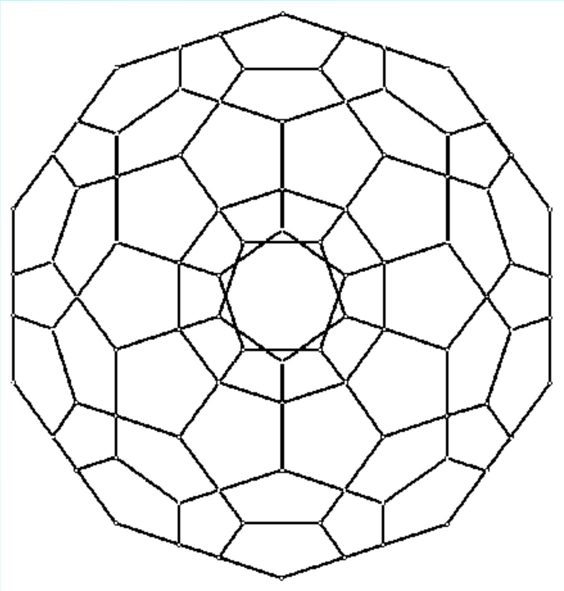

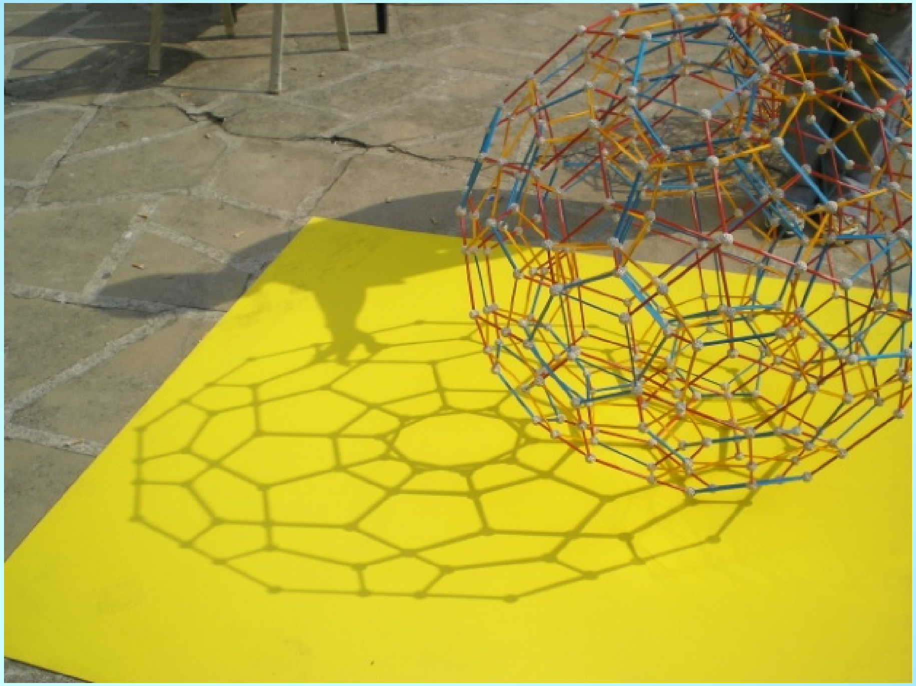

Grazie dell'attenzione!

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