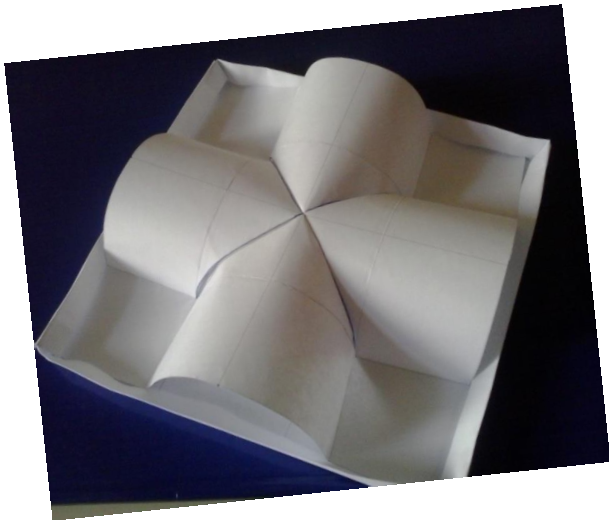
Modellando le volte: dai rilievi laser all'origami



Maria Luisa Spreafico
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maria.spreafico@polito.it

Summer School: la matematica incontra l'arte e la tecnologia
San Pellegrino Terme 7-8-9 Settembre 2015

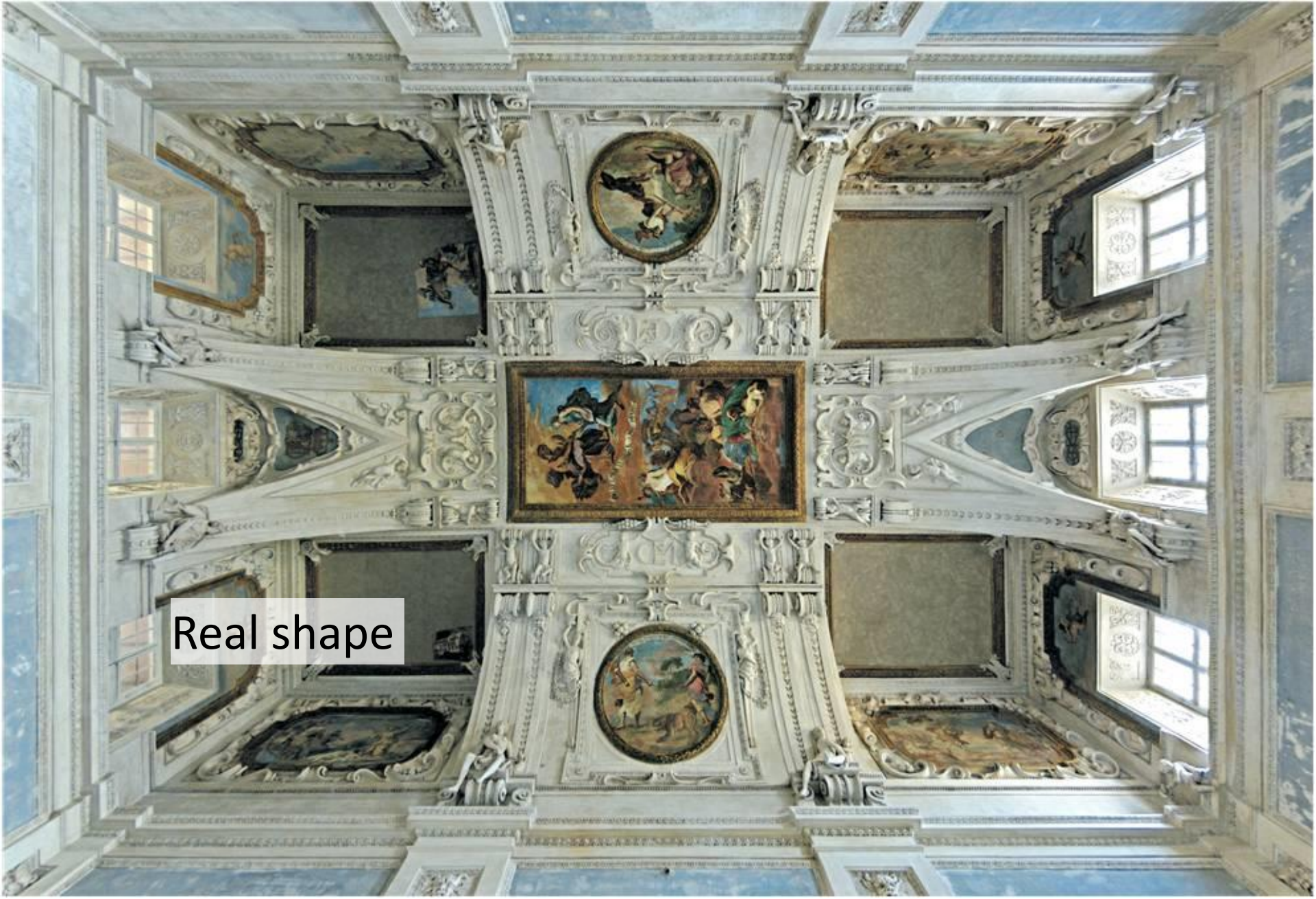
2015 10 - 13 June, Porto - Portugal
INTERNATIONAL MEETING
AMS / american mathematical society
/ european mathematical society
spm / sociedade portuguesa de matemática

MODELLING VAULTS:
FROM LASER SCANNING TECHNIQUES TO ORIGAMI

Ugo Comollo_ Politecnico di Torino, Dipartimento di Architettura e Design, DAD
Caterina Cumino_ Politecnico di Torino, Dipartimento di Scienze Matematiche, DISMA
Matteo Semplice_ Università degli Studi di Torino, Dipartimento di Matematica, DMAT
Maria Luisa Spreafico_ Politecnico di Torino, Dipartimento di Scienze Matematiche, DISMA
Ursula Zich_ Politecnico di Torino, Dipartimento di Architettura e Design, DAD

MODELLING VAULTS:
FROM LASER SCANNING TECHNIQUES TO ORIGAMI

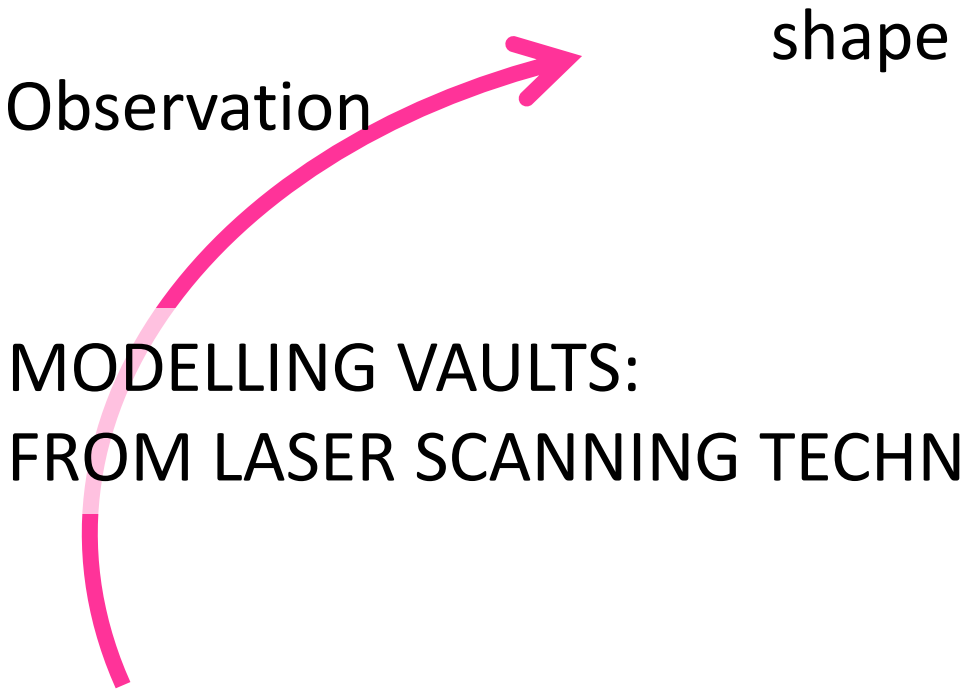
Real shape



Salone di Diana, Reggia di Venaria

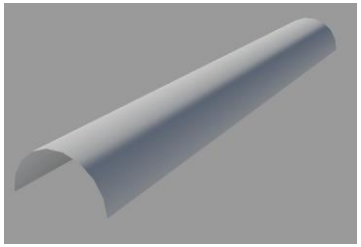
Chiesa di Sant'Uberto, Reggia di Venaria

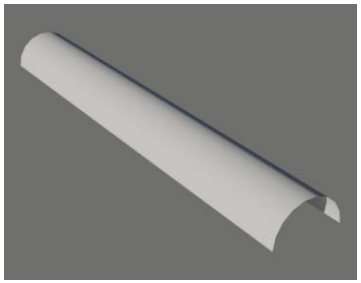
Galleria dei Ritratti, Reggia di Venaria

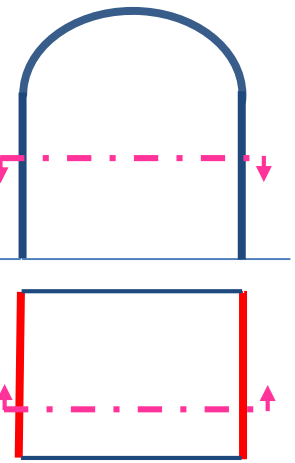


MODELLING VAULTS:
FROM LASER SCANNING TECHNIQUES TO ORIGAMI

Real shape





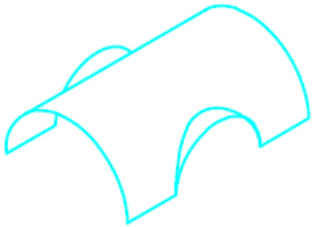


Barrel vault,
polycentric section

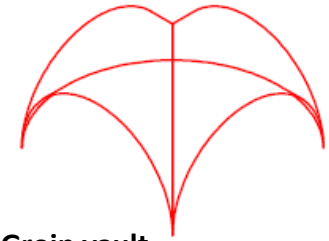
Galleria Grande,
Reggia di Venaria

shape

TRANSLATION

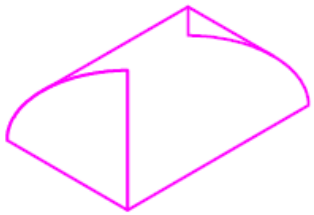


Barrel vault with lunettes



Groin vault

Cloister vault



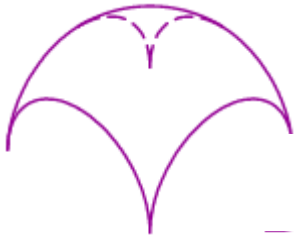
Barrel vault with cloister heads

Galleria Grande, Reggia di Venaria

TRANSLATION

Dome

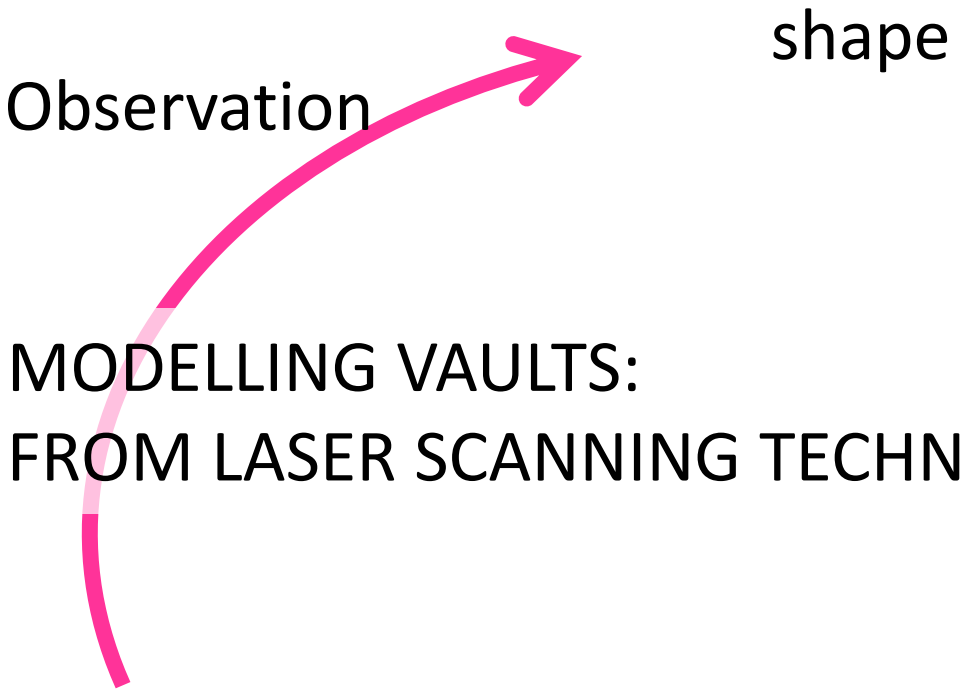
Basin



Sail vault

Pendentives

Galleria dei Ritratti, Reggia di Venaria



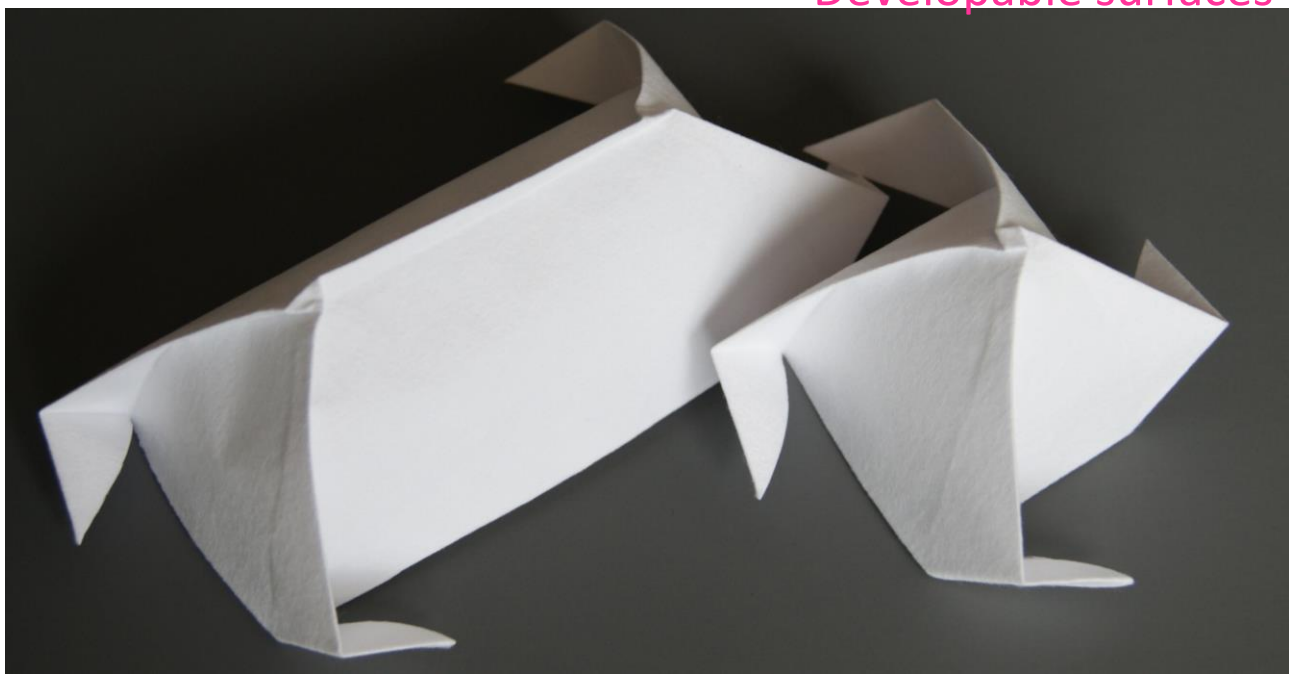
MODELLING VAULTS:
FROM LASER SCANNING TECHNIQUES TO ORIGAMI

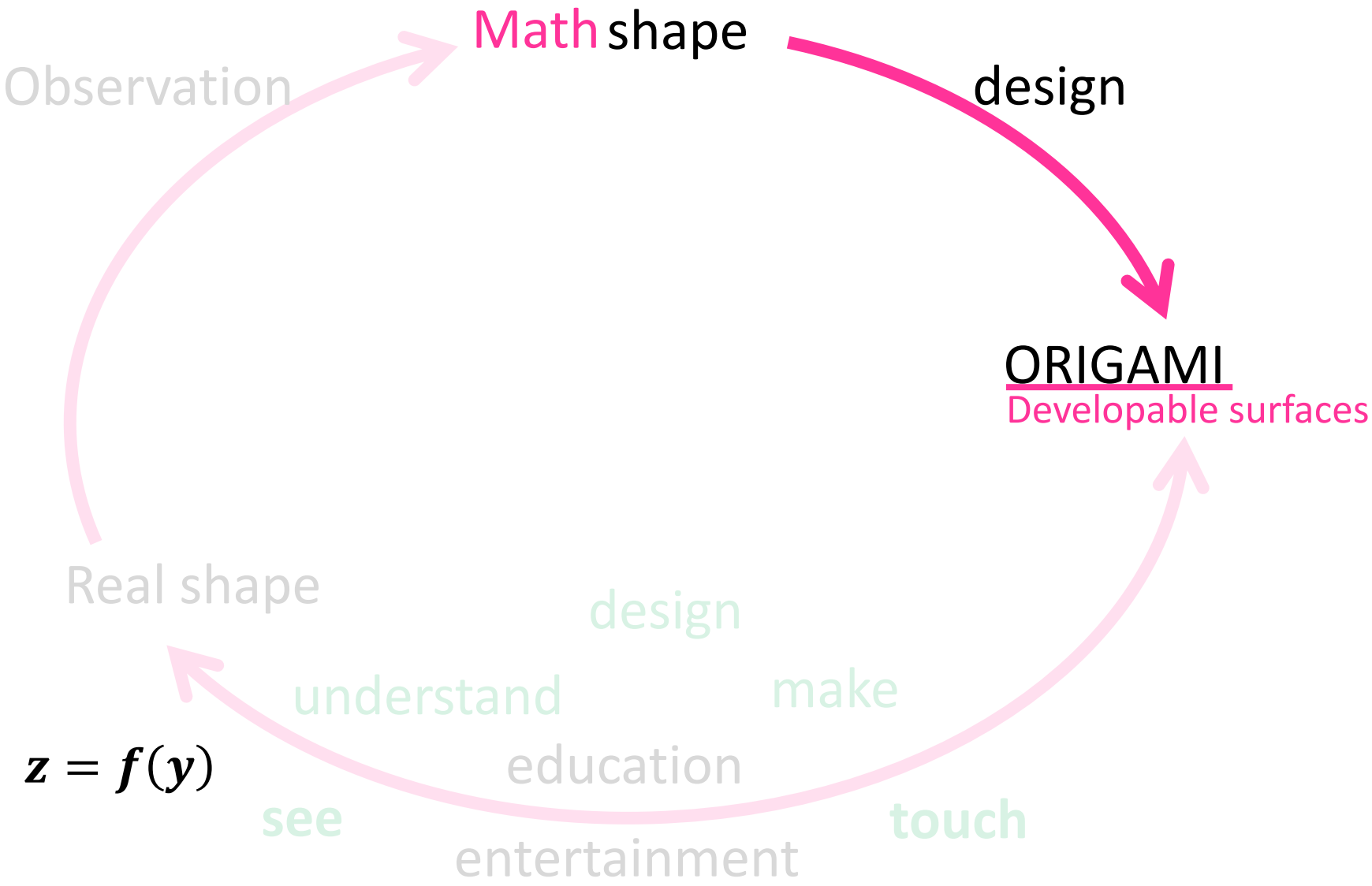
Real shape

Observation → Math shape → design → Real shape → Observation

MODELLING VAULTS: FROM LASER SCANNING TECHNIQUES TO ORIGAMI

Developable surfaces




Math shape
design
ORIGAMI
Developable surfaces
Real shape
Observation
design
understand
make
education
see
touch
entertainment
$z = f(y)$

Catenary

Math

Analytical description of surfaces

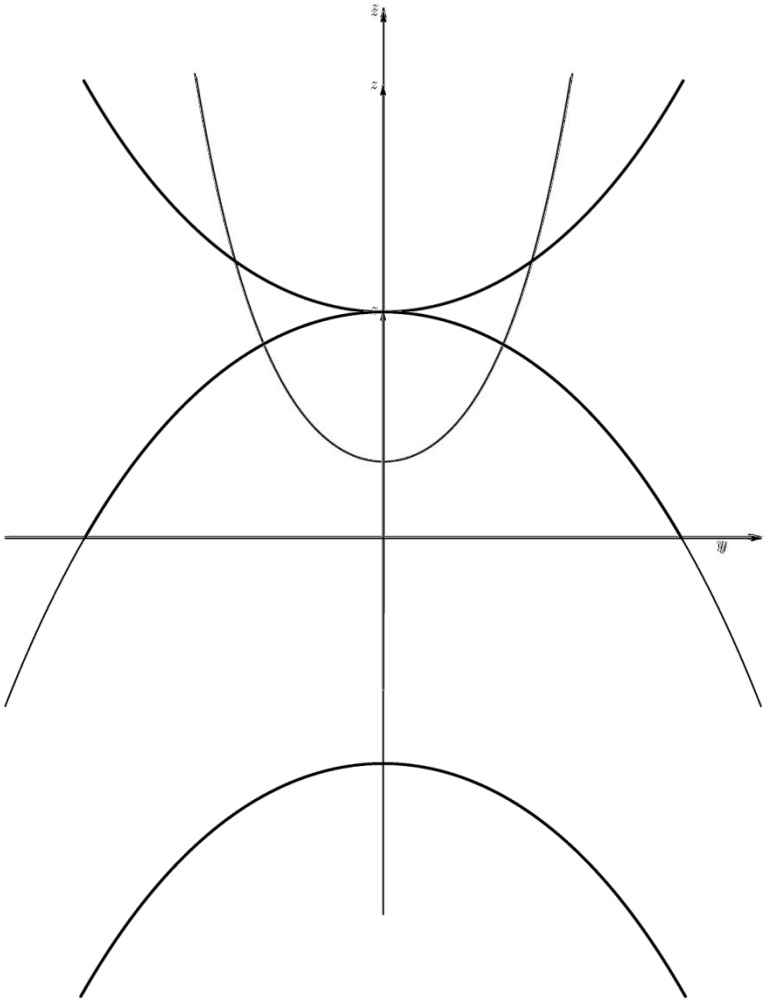
$$z = f(y)$$

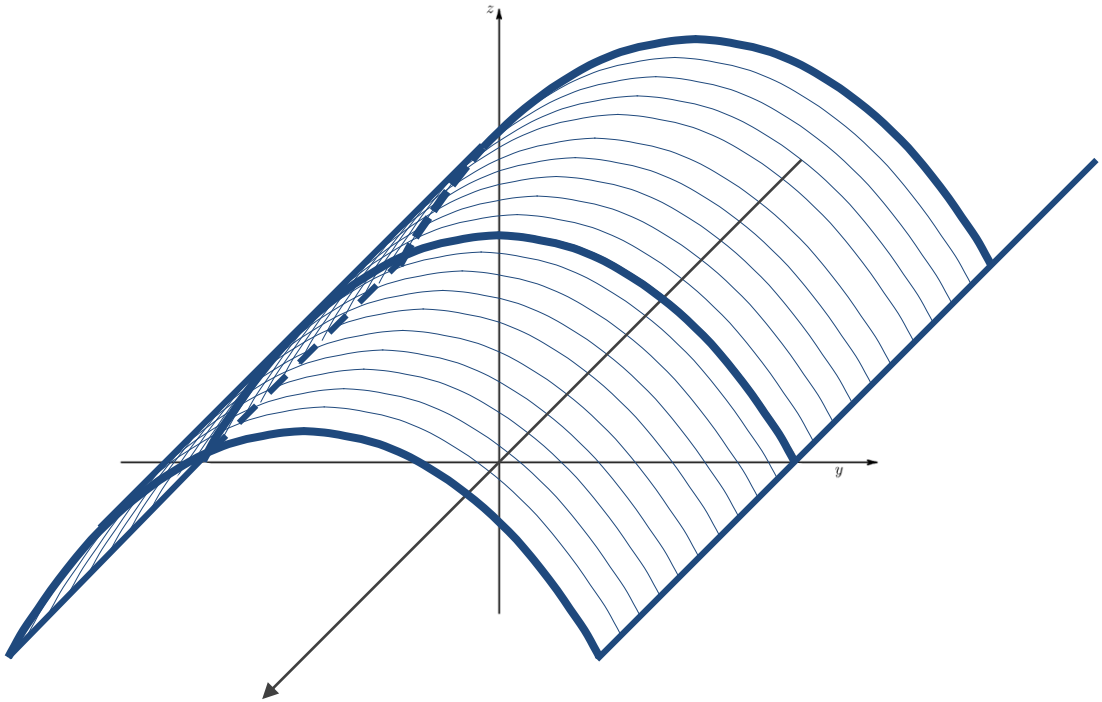
$$z = \cosh(y)$$

$$z = 3\cosh\left(\frac{y}{3}\right)$$

$$z = -3\cosh\left(\frac{y}{3}\right)$$

$$z = -3\cosh\left(\frac{y}{3}\right)+6$$



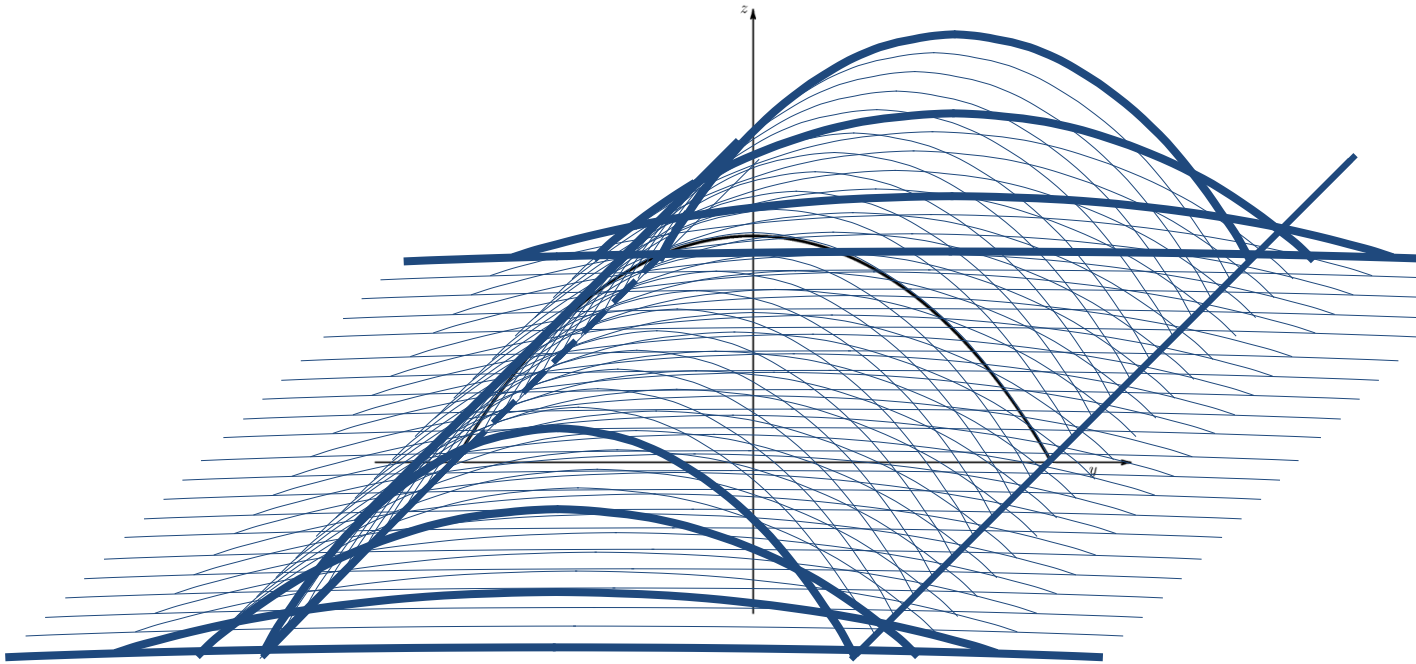


Math

Analytical description of surfaces

$$z = -3\cosh\left(\frac{y}{3}\right)+6$$

UNROLL THE CYLINDER
onto the xy plane !!!

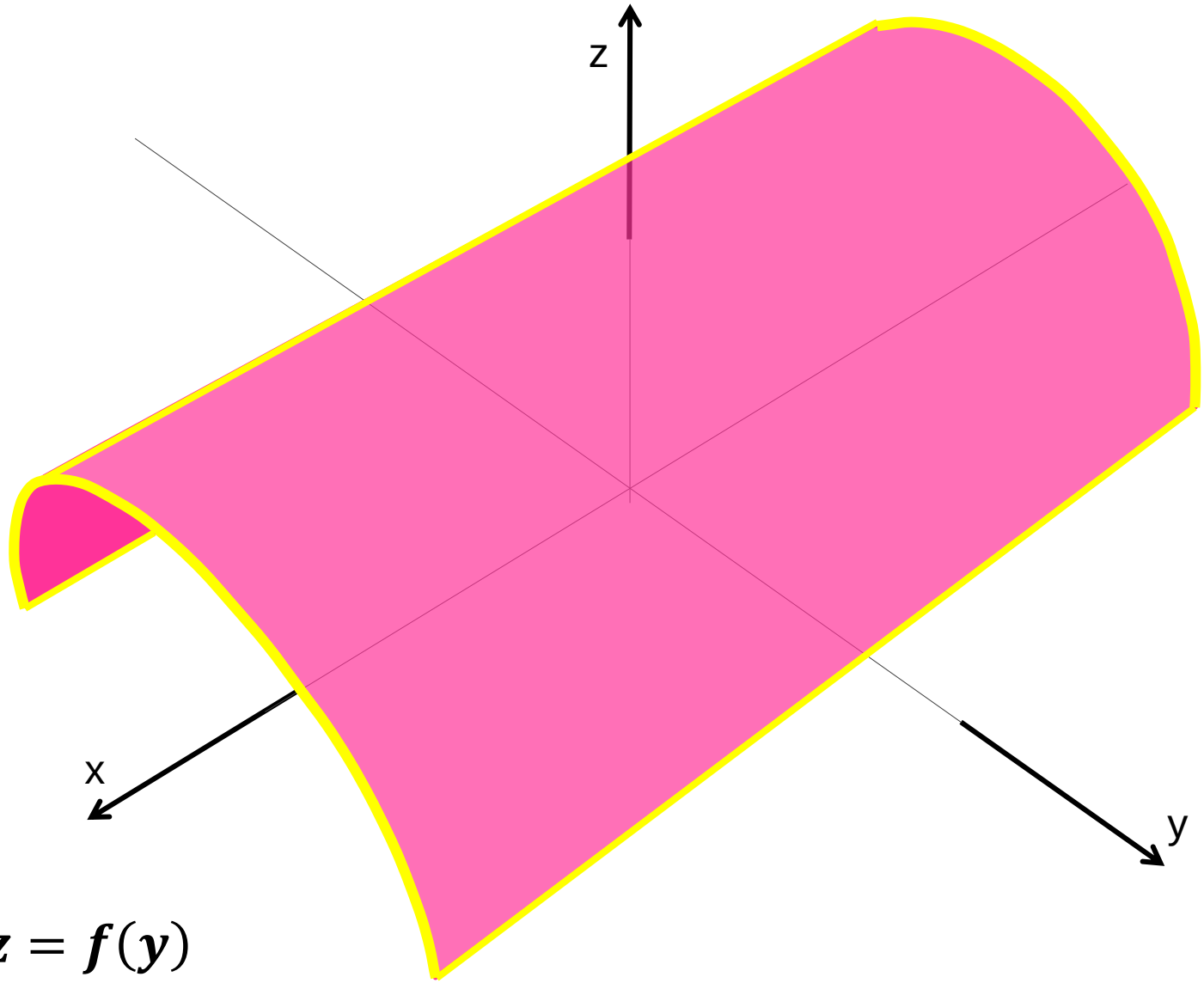


Math

Analytical description of surfaces

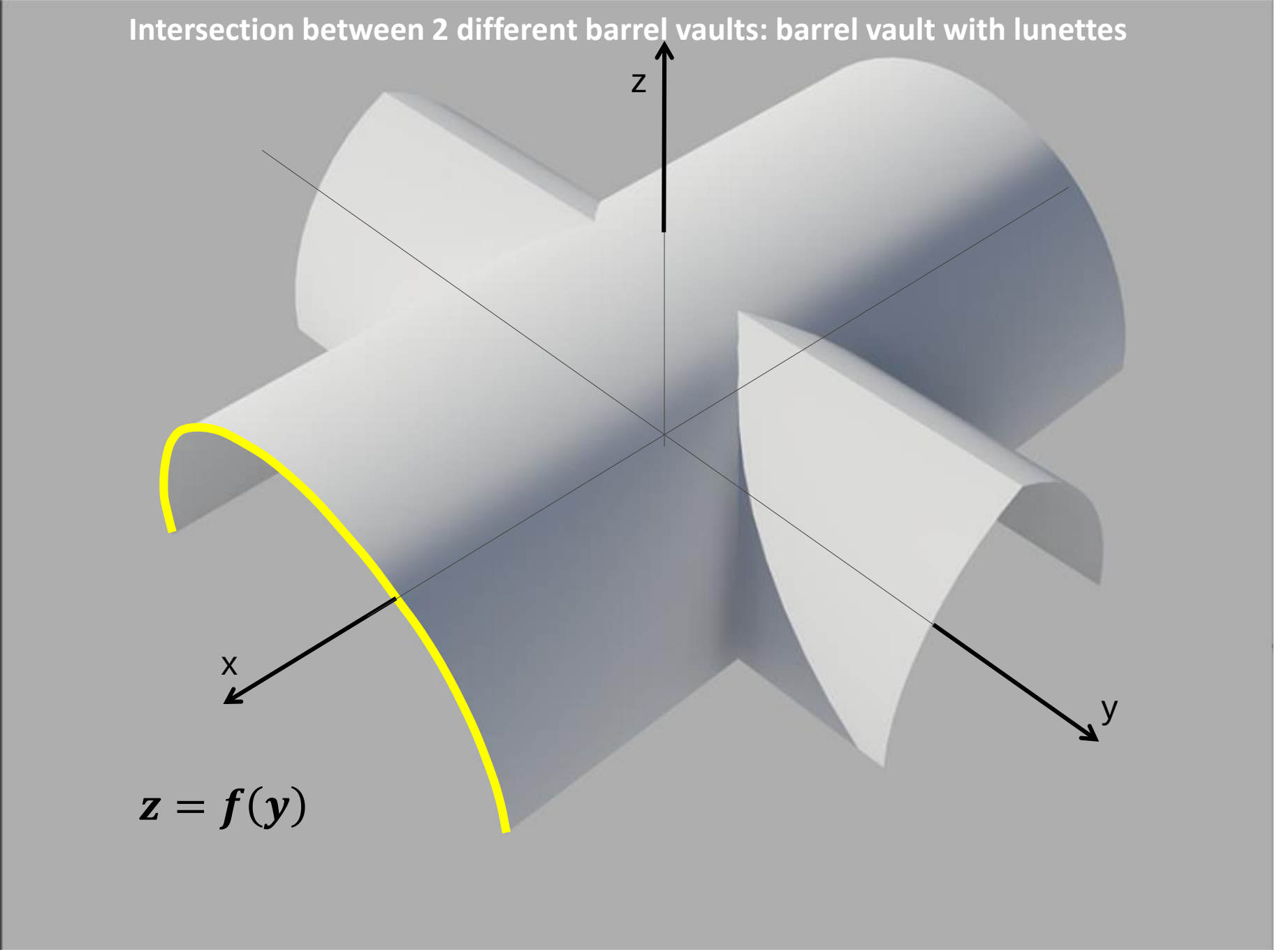
$$z = -3\cosh\left(\frac{y}{3}\right)+6$$

PATTERN **CREASE PATTERN**



$$z = f(y)$$

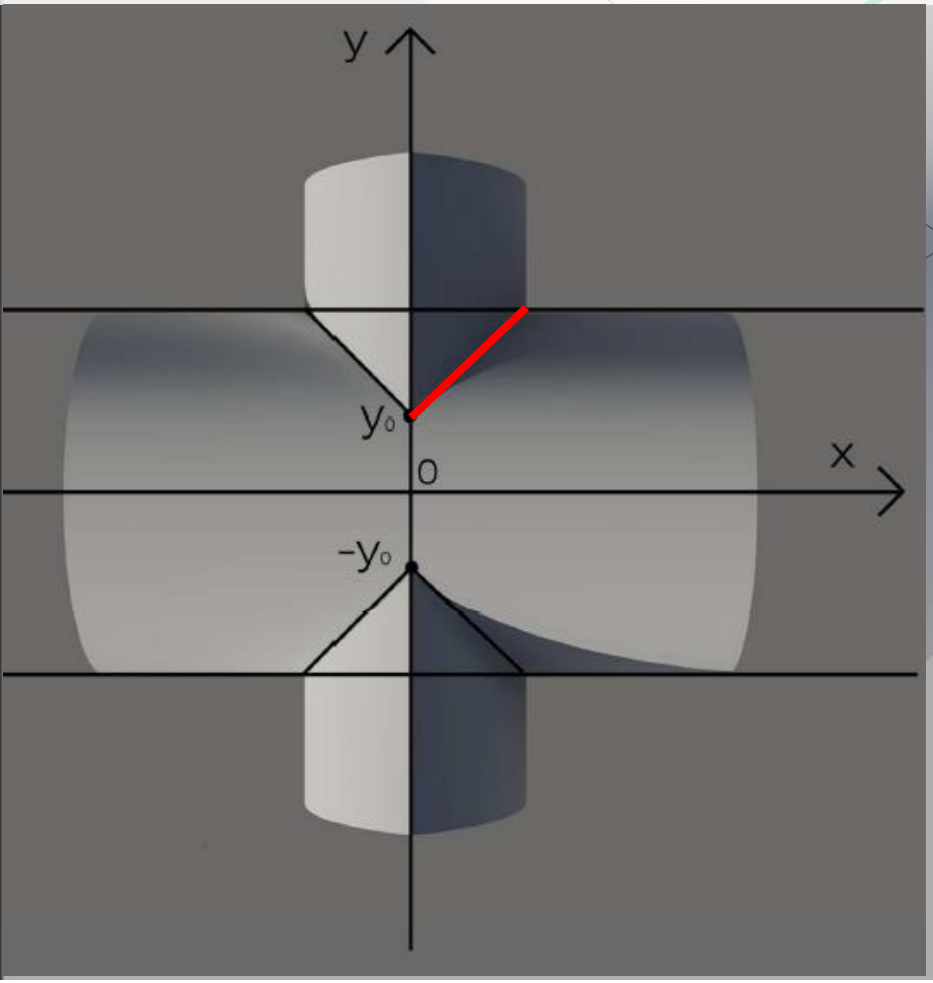
Intersection between 2 different barrel vaults: barrel vault with lunettes

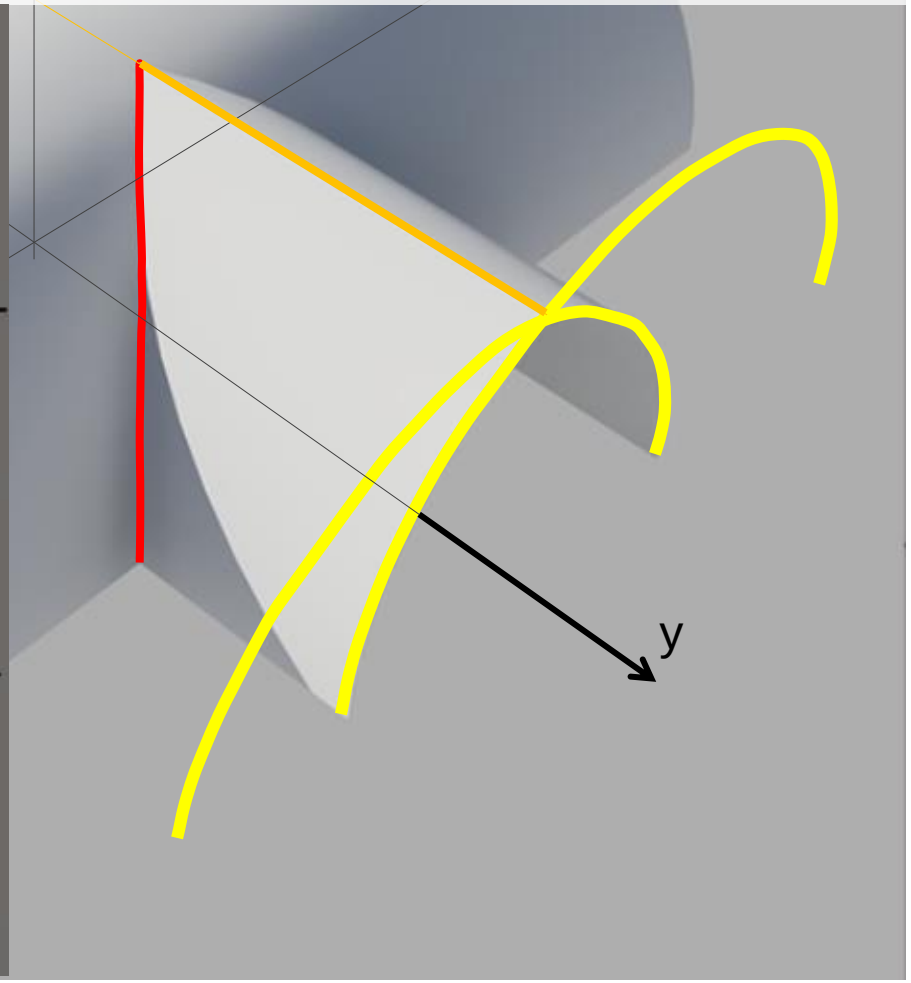


Intersection between 2 different barrel vaults: barrel vault with lunettes

planar intersection
&
equal curves on
the cylinders' developments

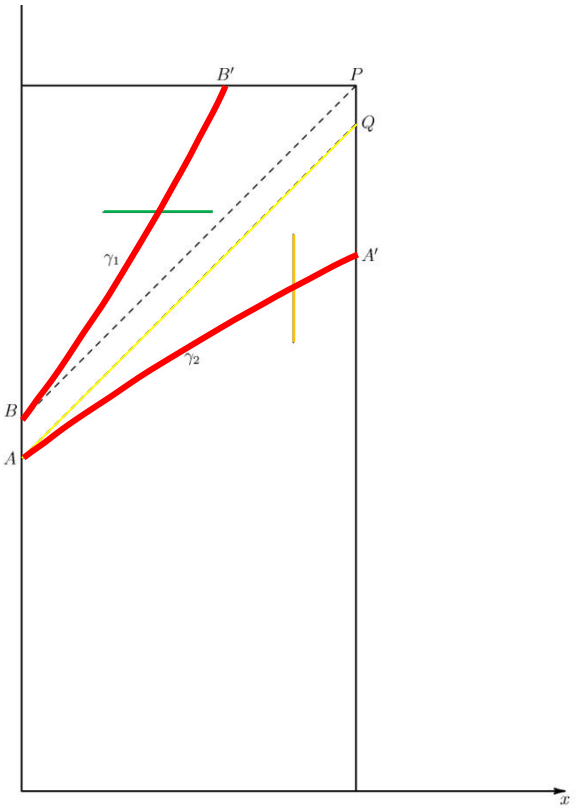
⇒ the second cylinder must be a portion of the first one

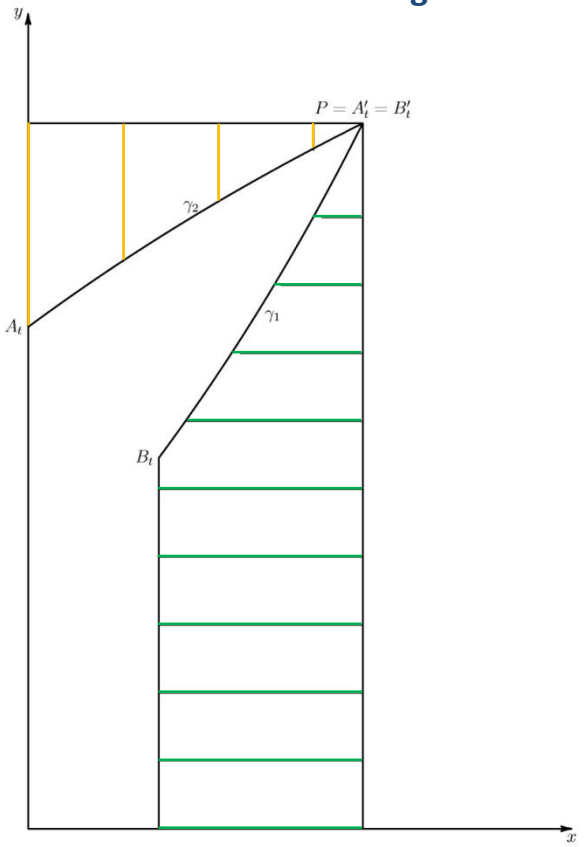




shift the curves to create the pattern...

Groin vault or generalization





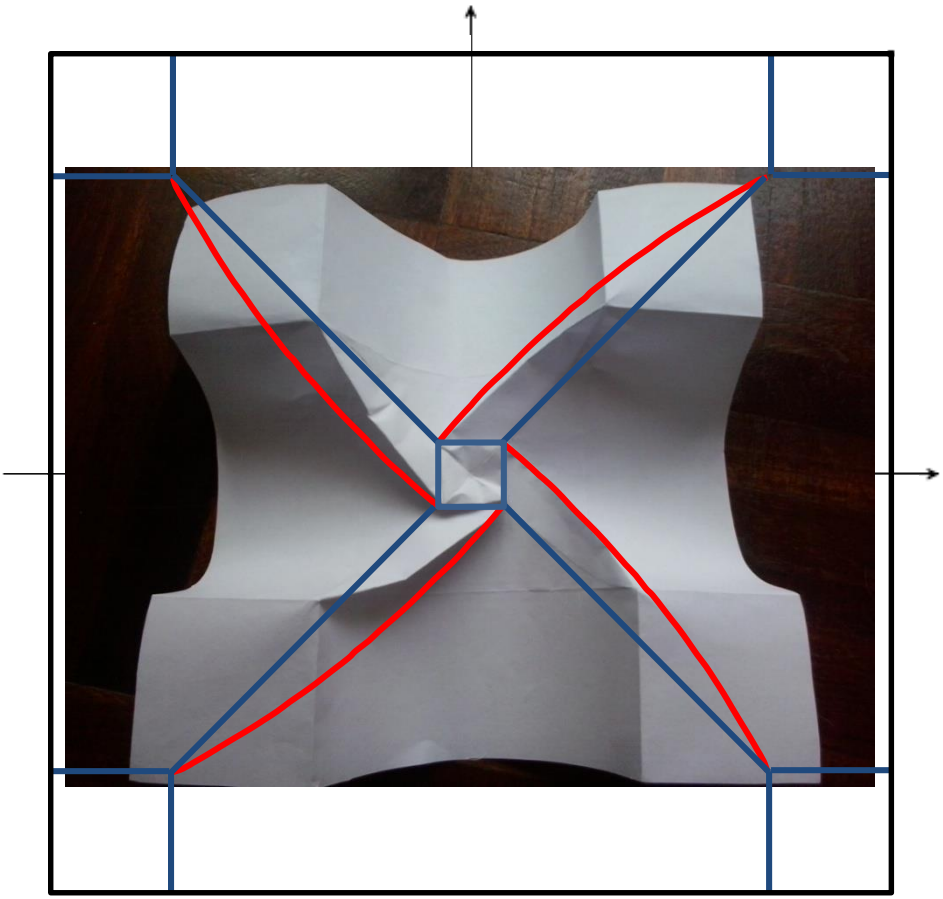
$$\boldsymbol{\gamma(v) = \left(v - y_0, \int_0^v \sqrt{1 + f'^2(t)dt}\,, 0 \right)}$$

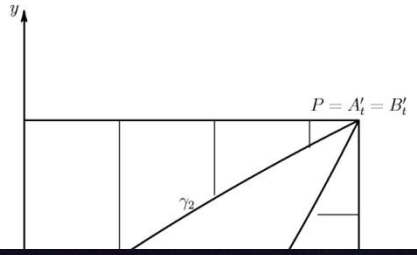
MOUNTAIN FOLD

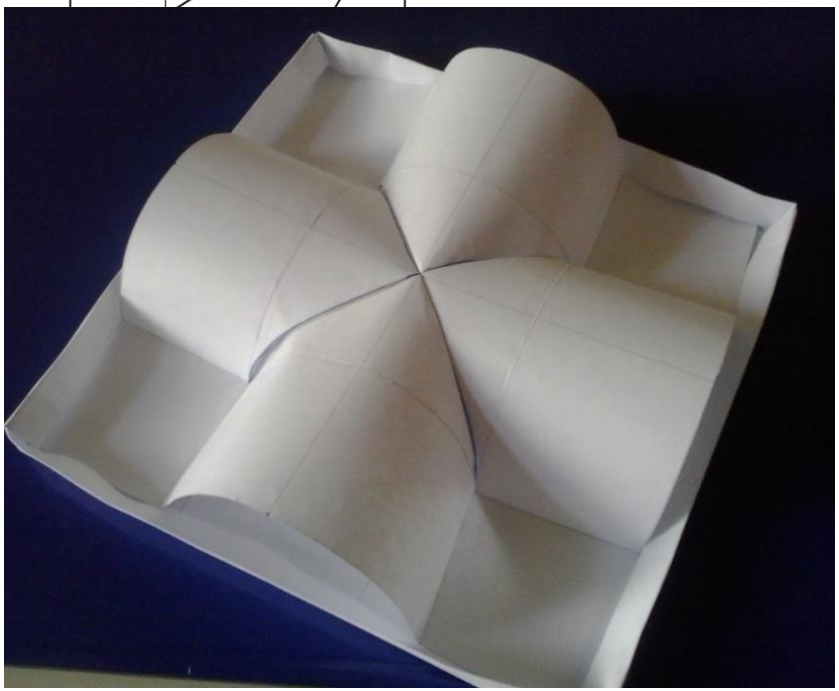
VALLEY FOLD

Groin vault

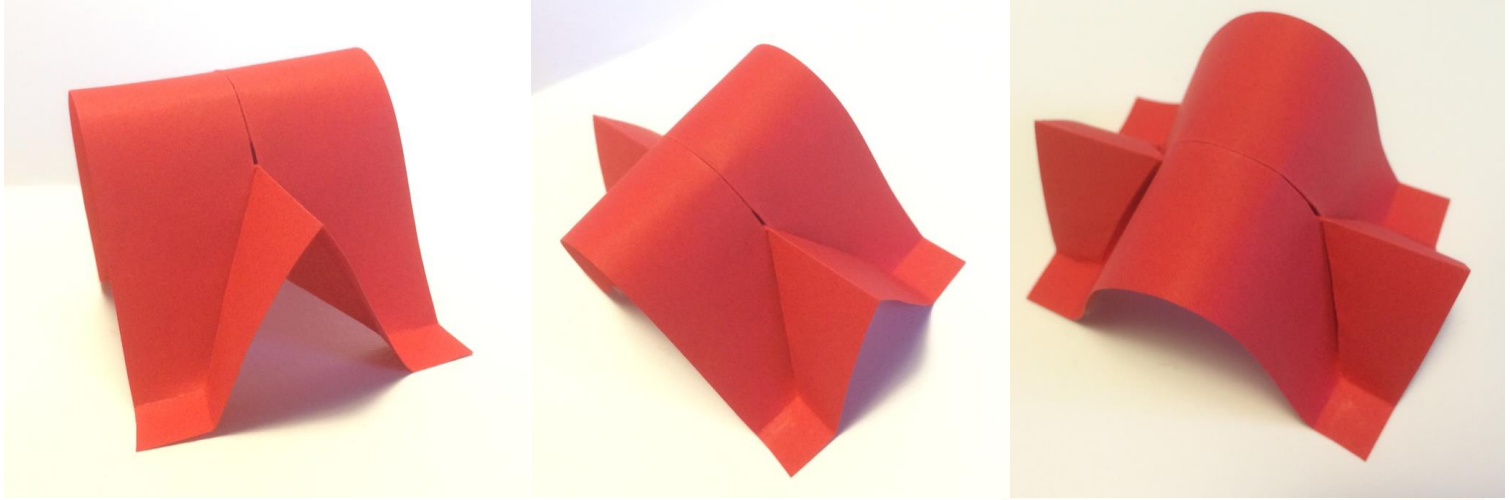
$$y_0 = 0$$

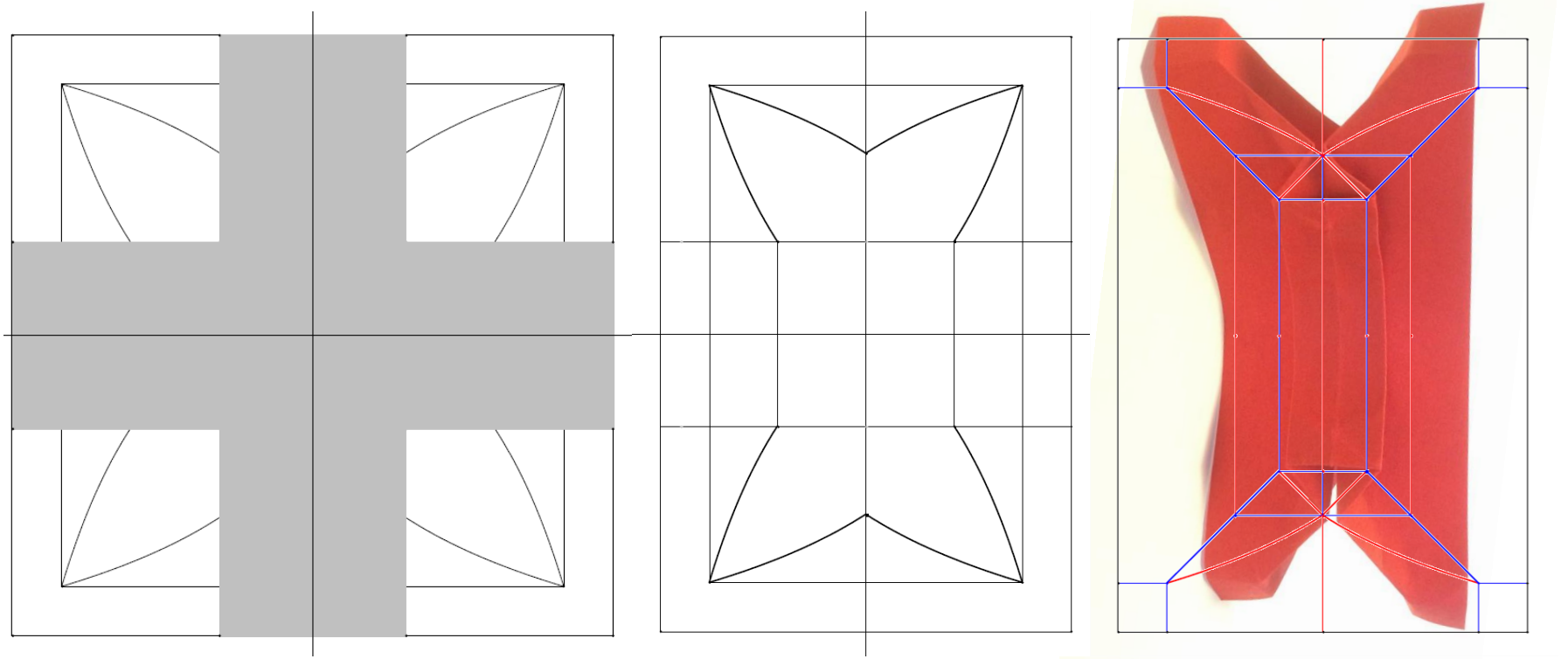






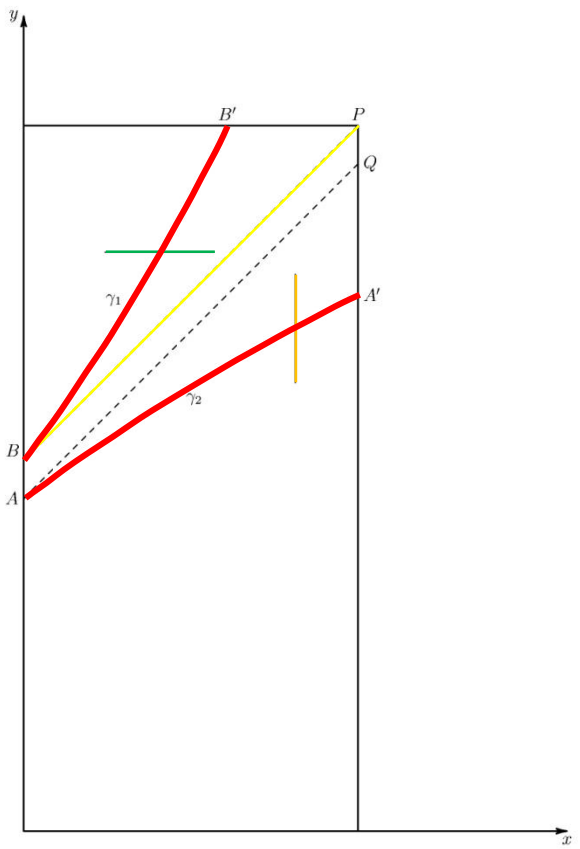
$y_0 > 0$

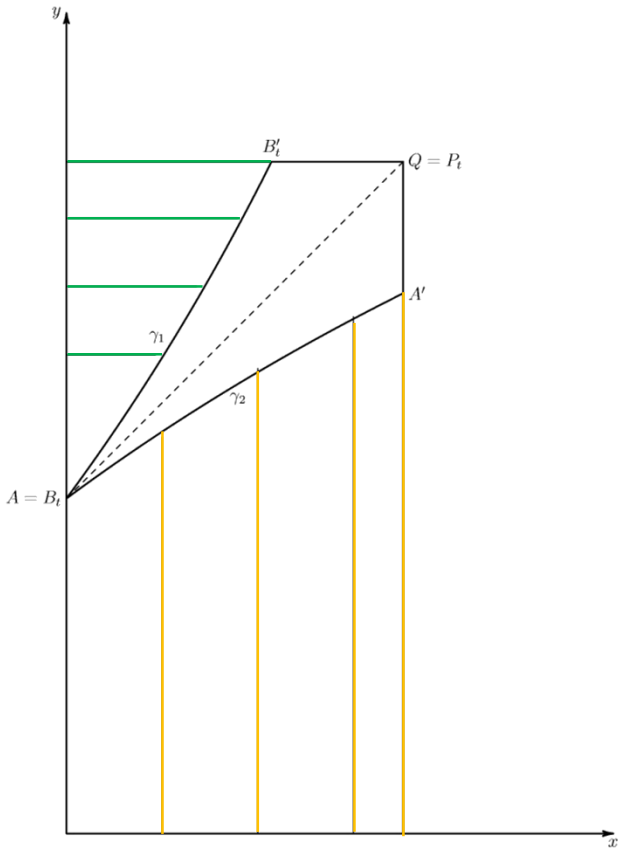




Cloister vault or generalization

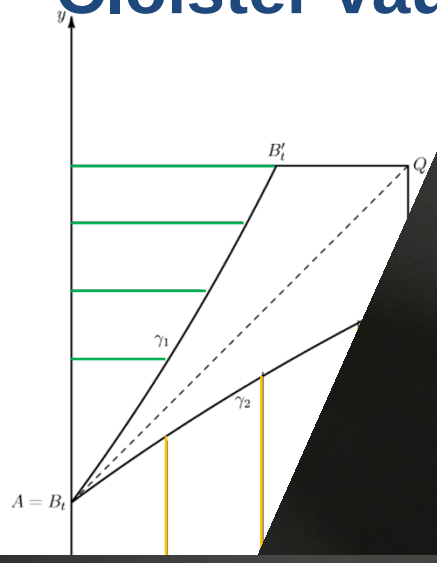
Shift the curves to create the pattern...

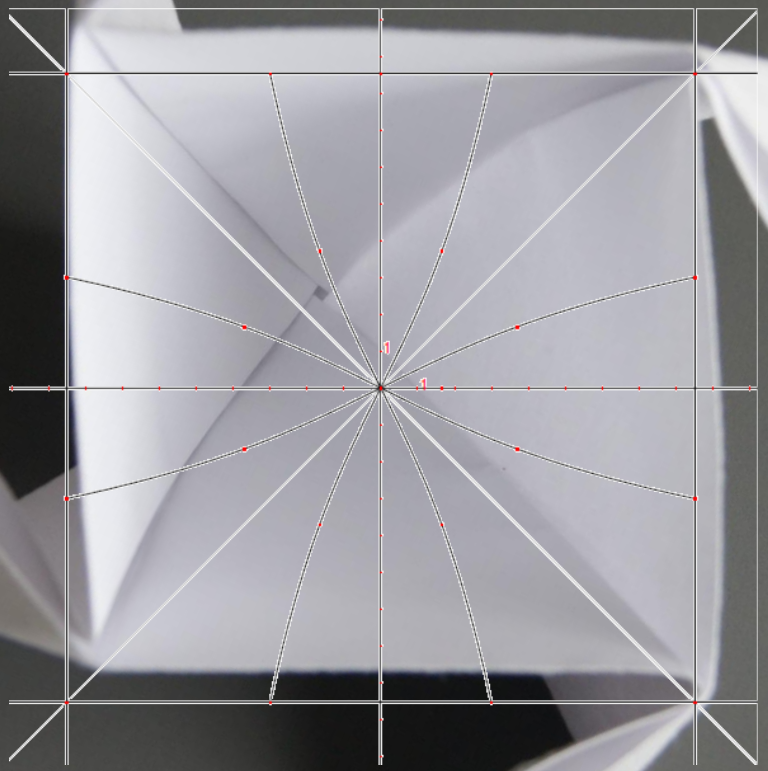


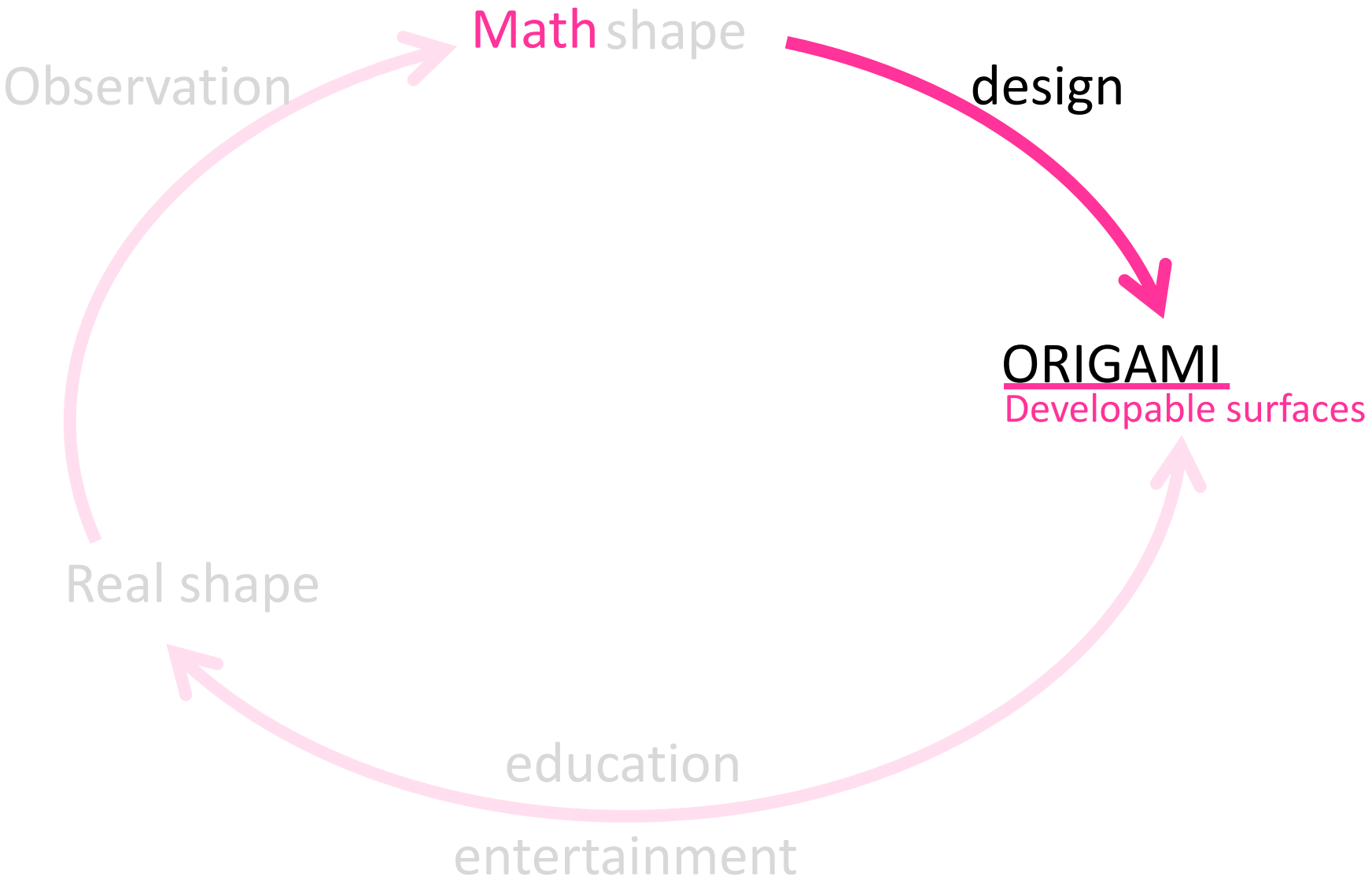


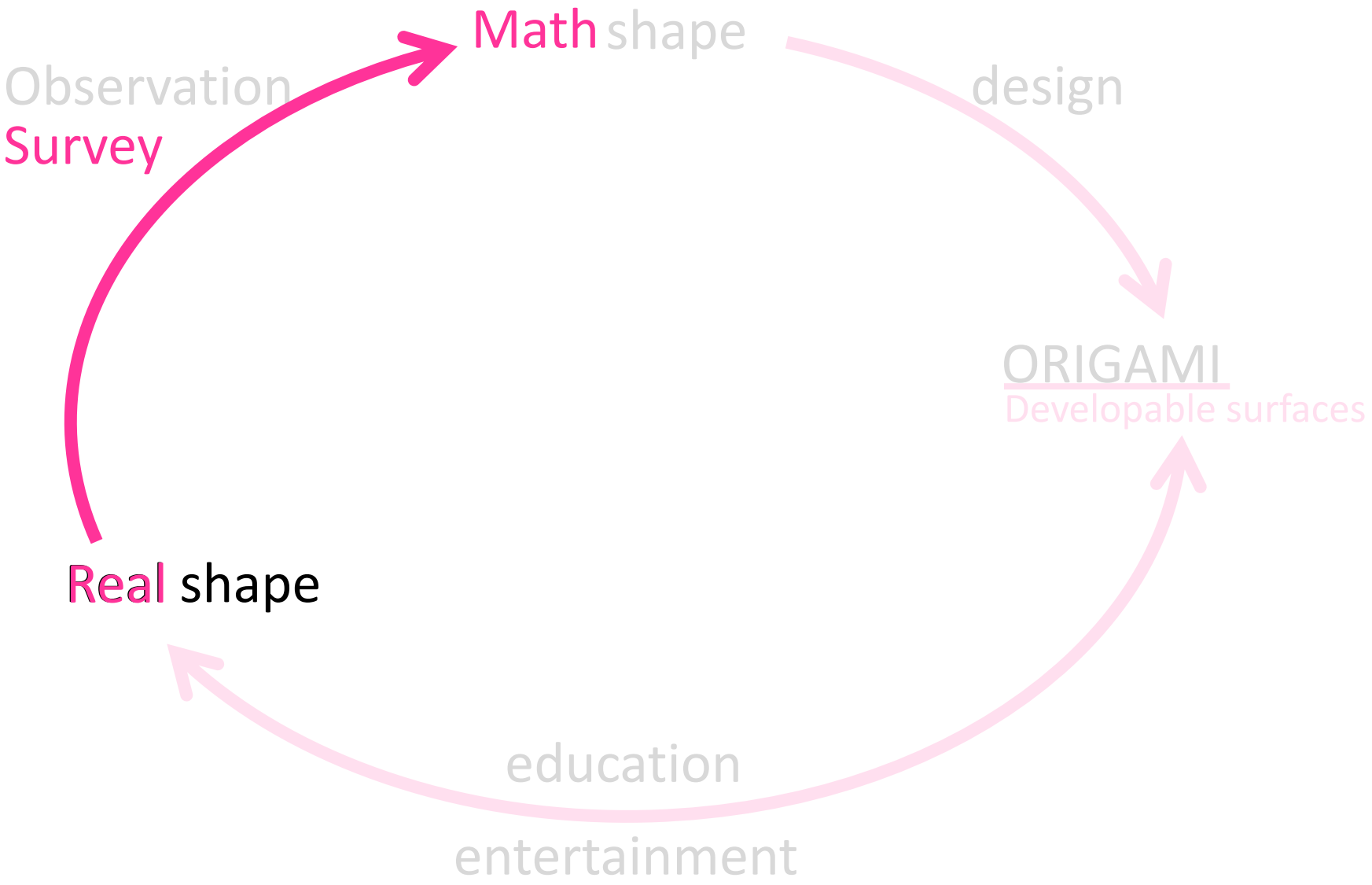
$$\boldsymbol{\gamma(v) = \left(v - y_0, \int_0^v \sqrt{1 + f'^2(t)dt}, 0 \right)}$$

Cloister vault or gen.






Math shape
design
ORIGAMI
Developable surfaces
Real shape
Observation
education
entertainment


Math shape
design
ORIGAMI
Developable surfaces
education
entertainment
Real shape
Observation
Survey

Texturized 3D point cloud

Two different steps from each Laser scanning station :

1) 3D point cloud capture

2) texture by photos

Texturized 3D point cloud

Two different steps from each Laser scanning station:

1) 3D point cloud capture

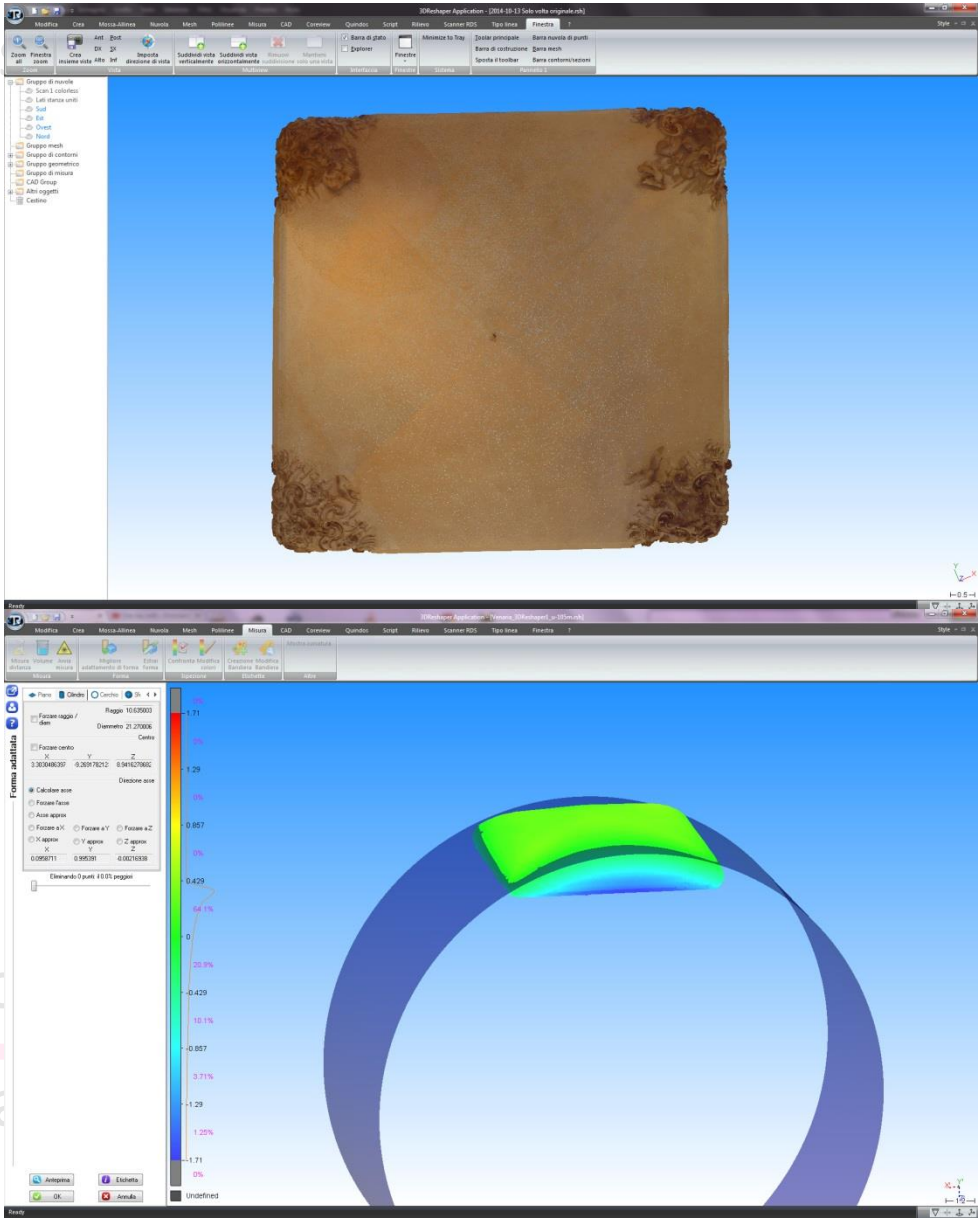
2) texture by photos

Observation

Survey

Math

Real shape



Texturized 3D point cloud

Two different steps from each Laser scanning station:

1) 3D point cloud capture

2) texture by photos

Math

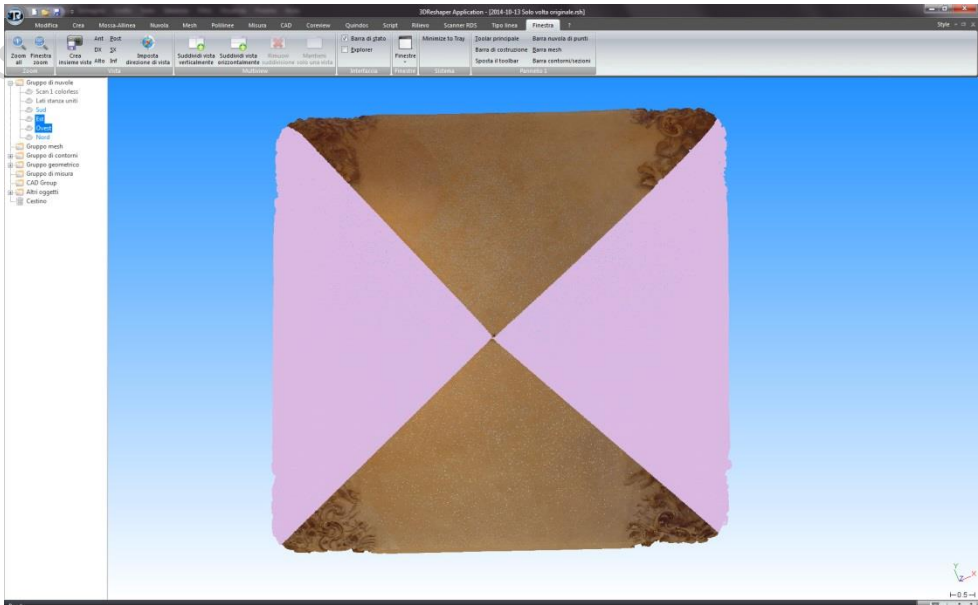
Observation

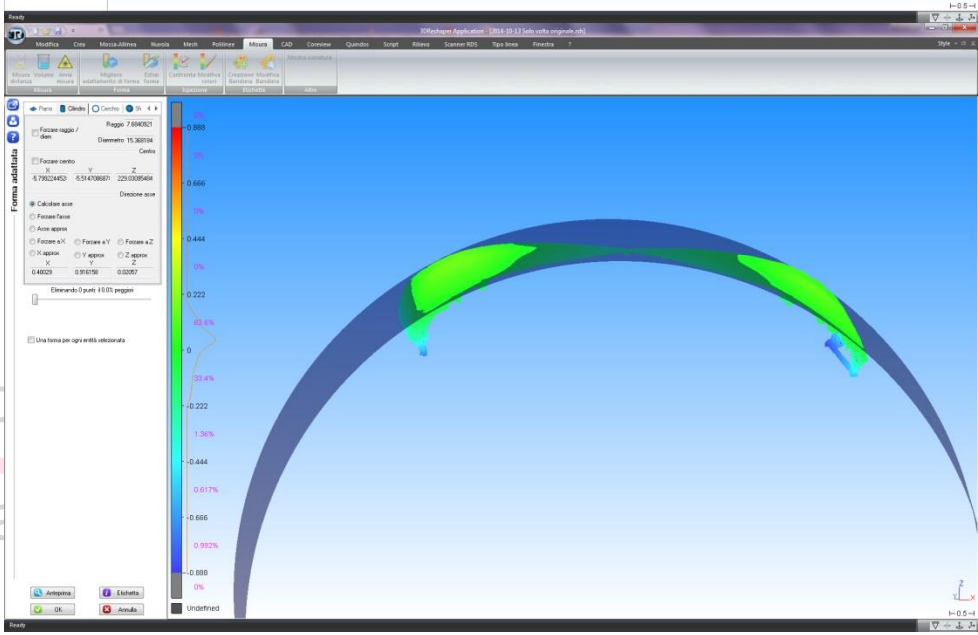
Survey

Real shape

educ

enterta





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Math shape

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Survey

Real shape

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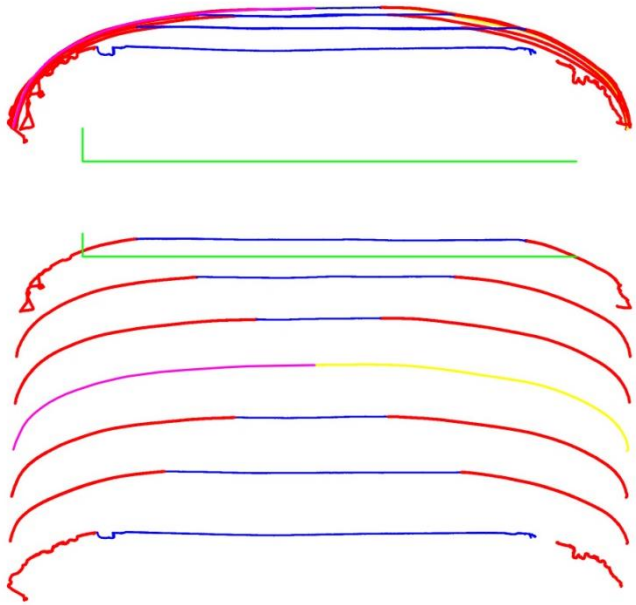
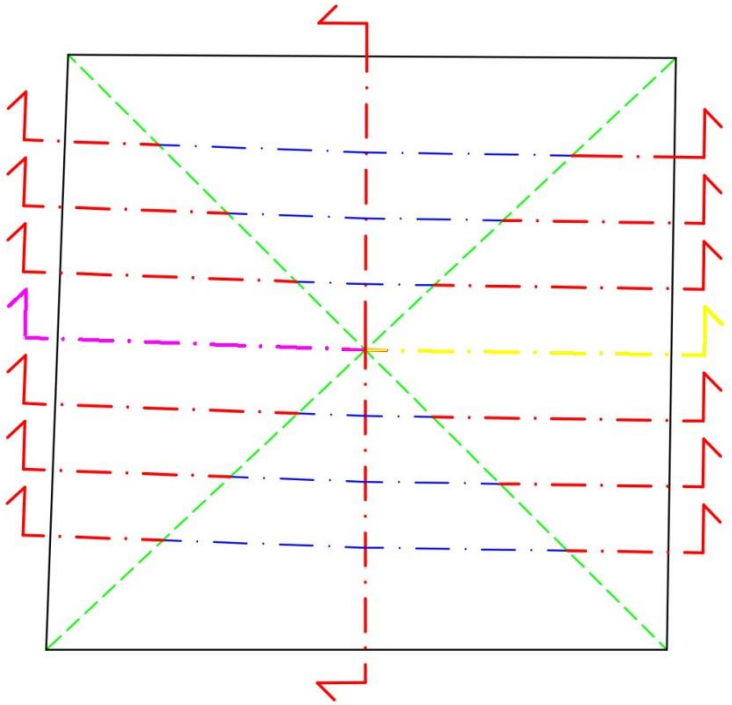




Our goal:

An algorithm to produce equations of a math shape from a cloud of points

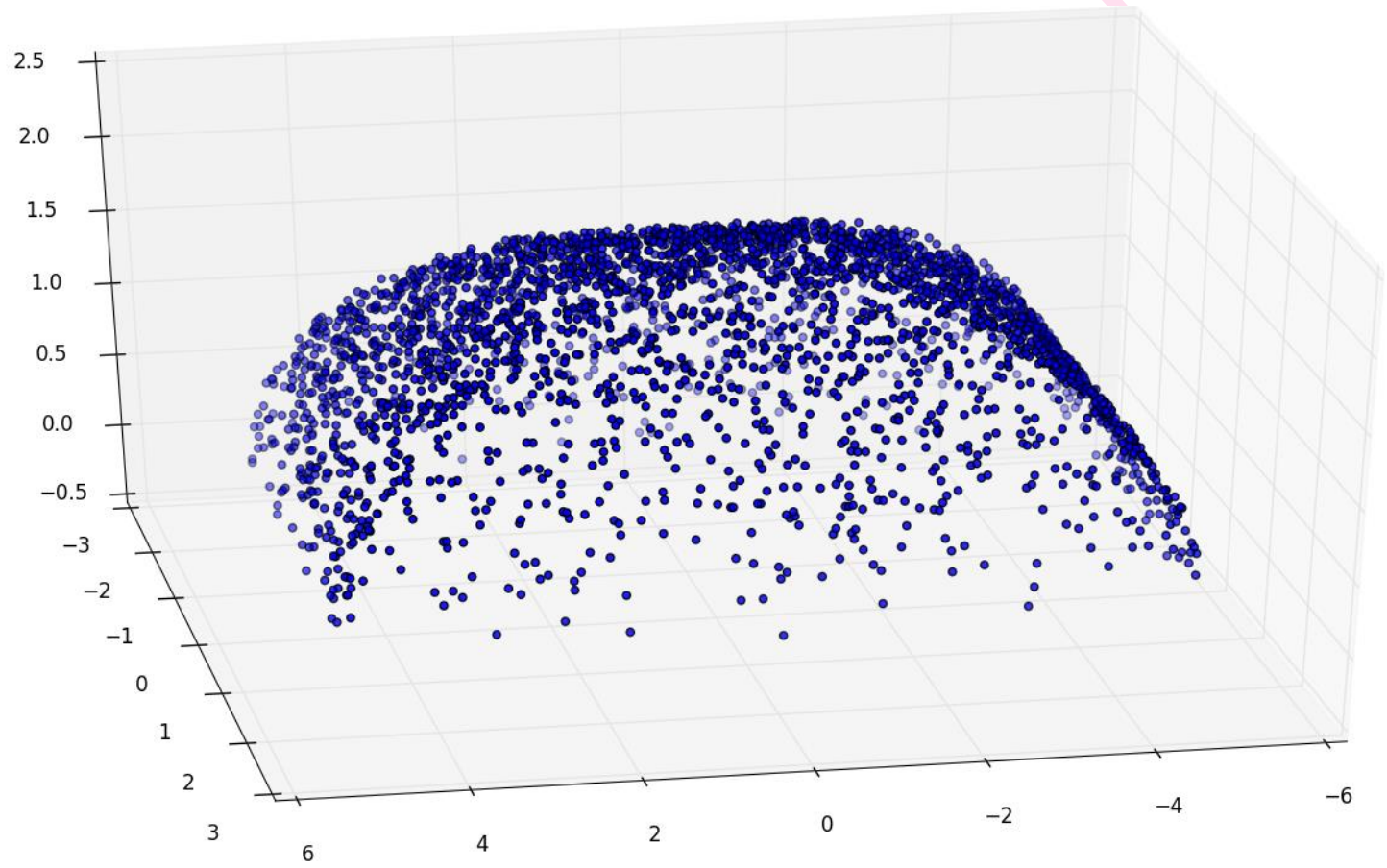
Math shape

Observation

Survey

design

Real



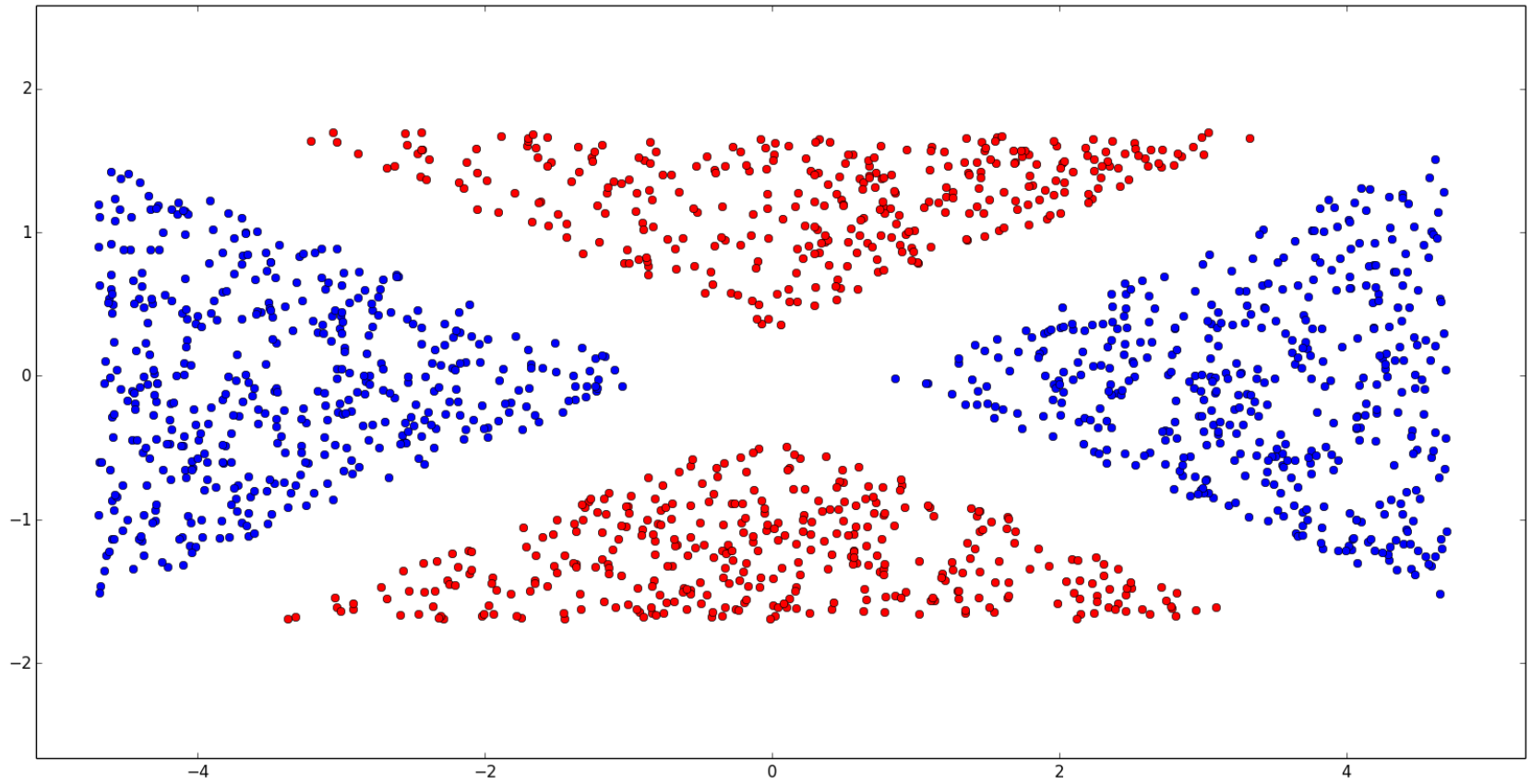
with **Davide Salvaggio**, Math Ing student

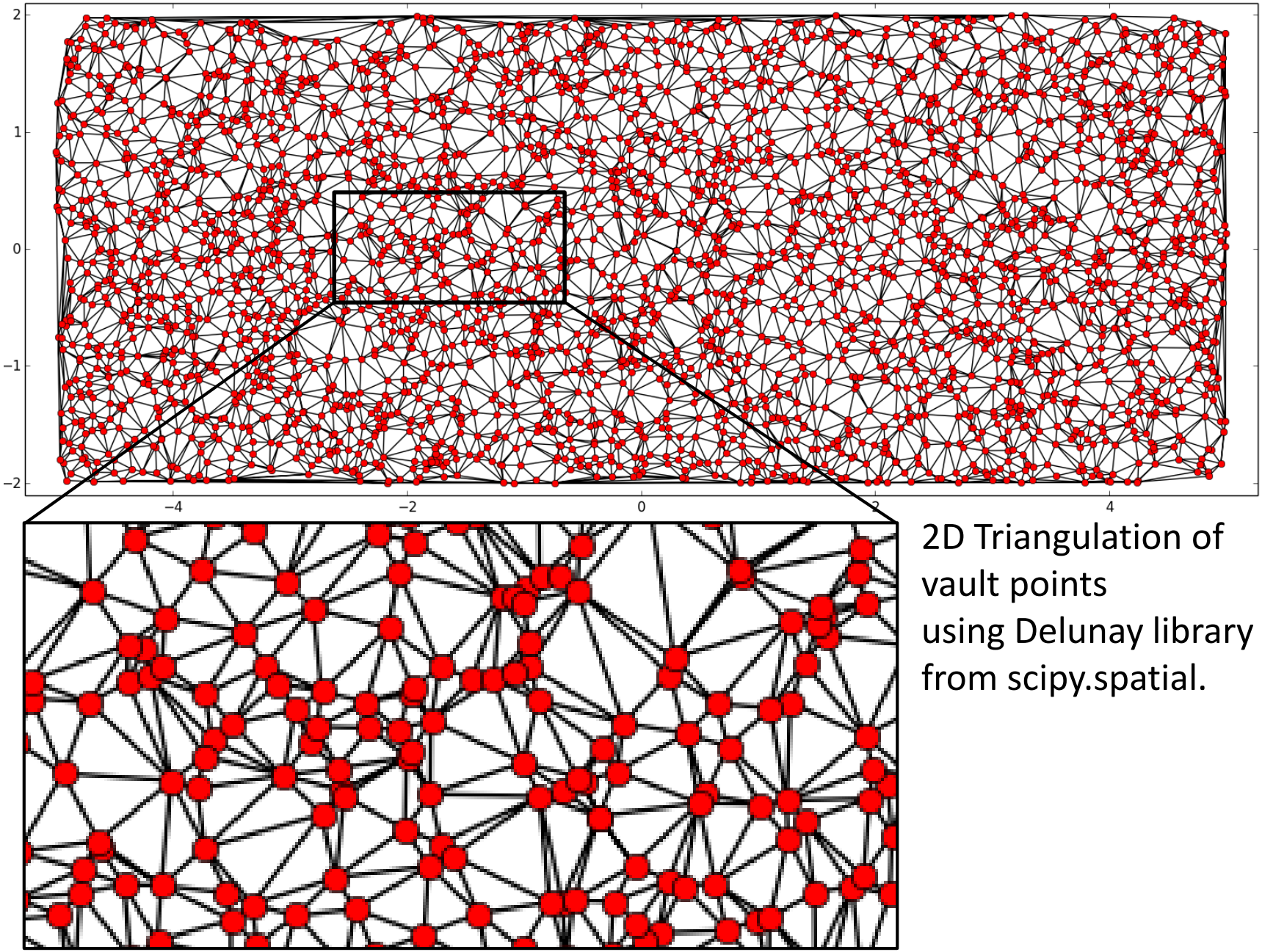
CLUSTERING

assign each point to the correct cylinder

Red zone: cylinder parallel to the x-axis

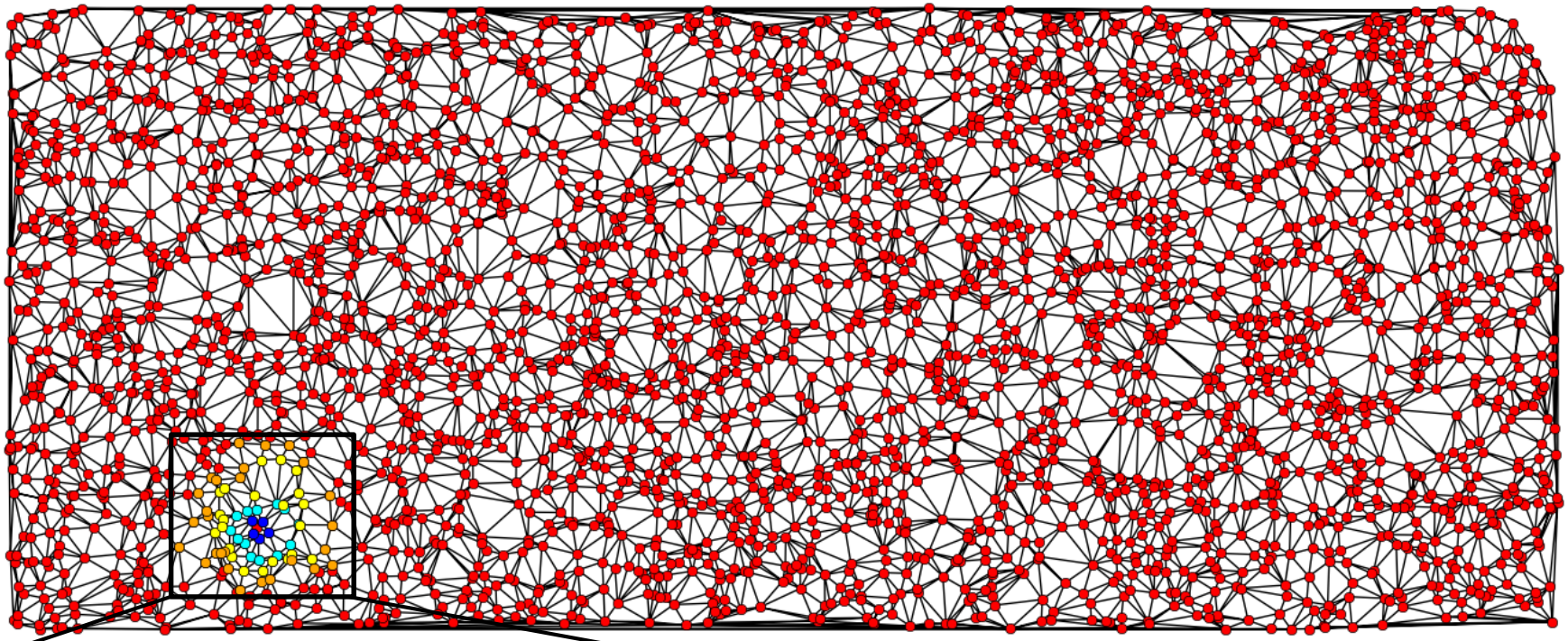
Blue zone: cylinder parallel to the y-axis

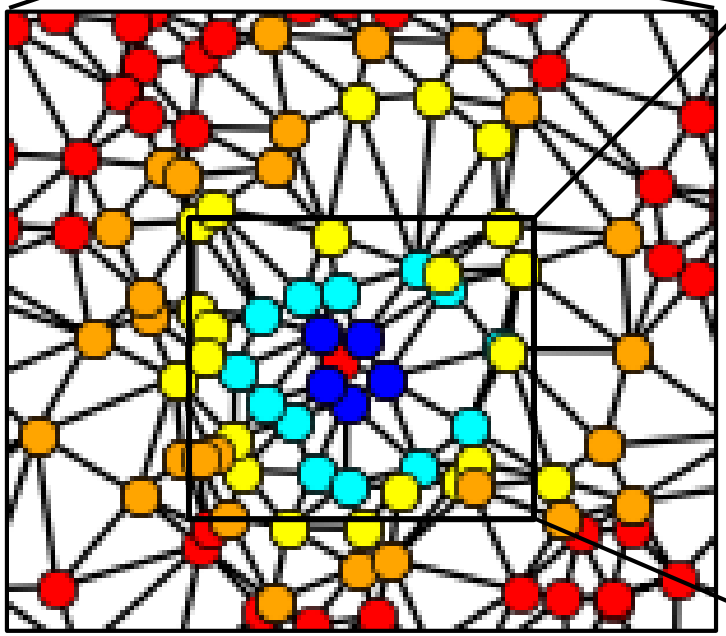


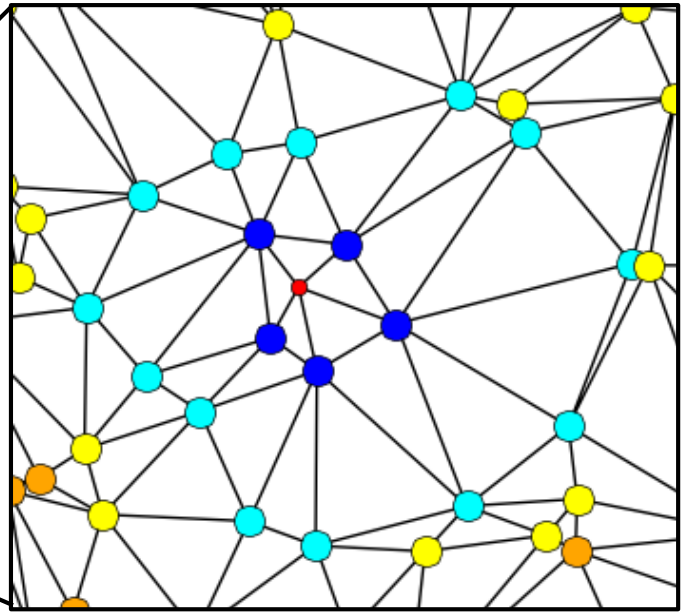


2D Triangulation of vault points using Delunay library from scipy.spatial.

with **Davide Salvaggio**, Math Ing student







The function “FindNeighbours”

1. Red: start point
2. Blue: nearest neighbours
3. Cyan: second level neighbours
4. Etc…

with **Davide Salvaggio**, Math Ing student

local osculating quadric:

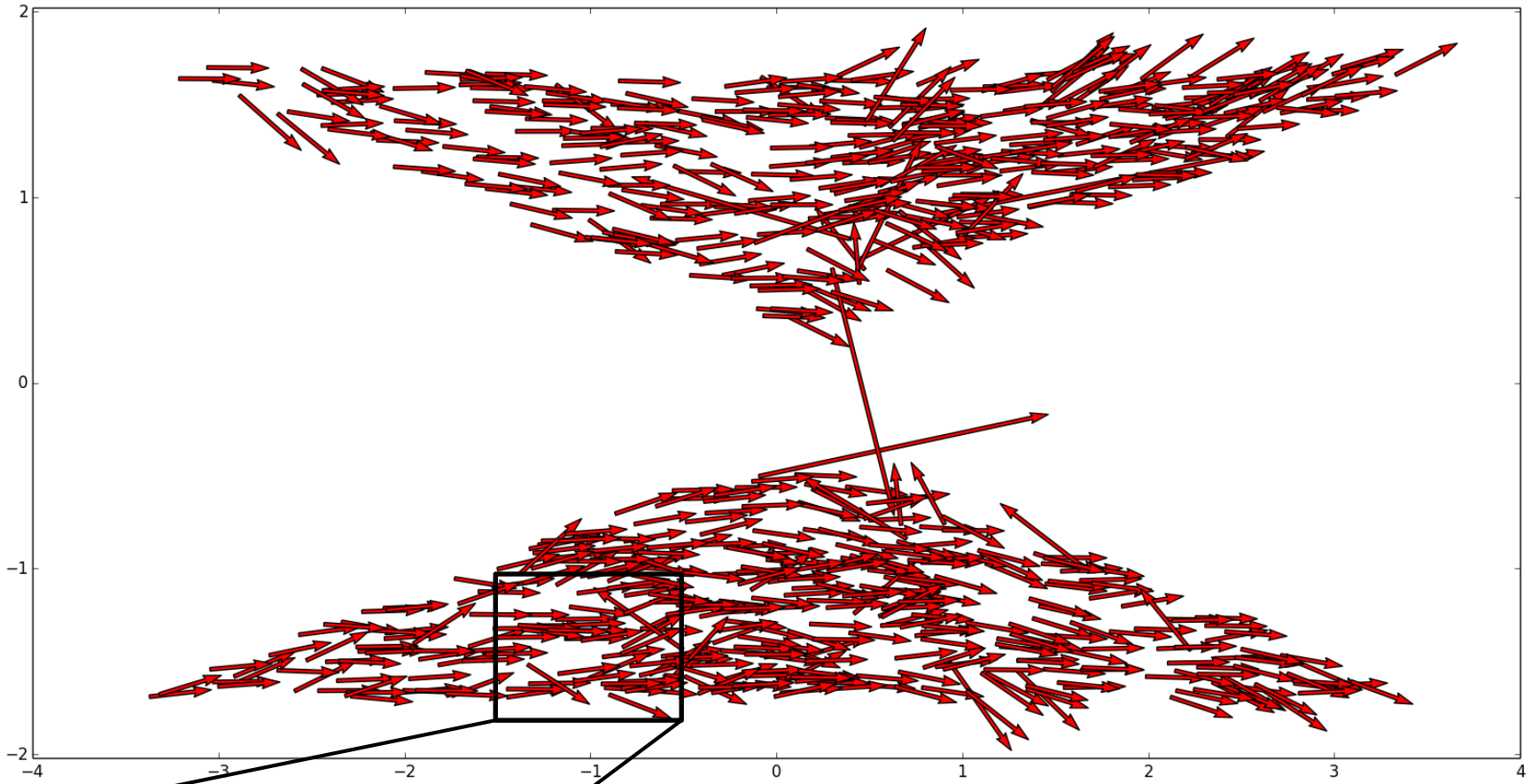
$$g(x, y) = a + bx + cy + dx^2 + 2exy + fy^2$$

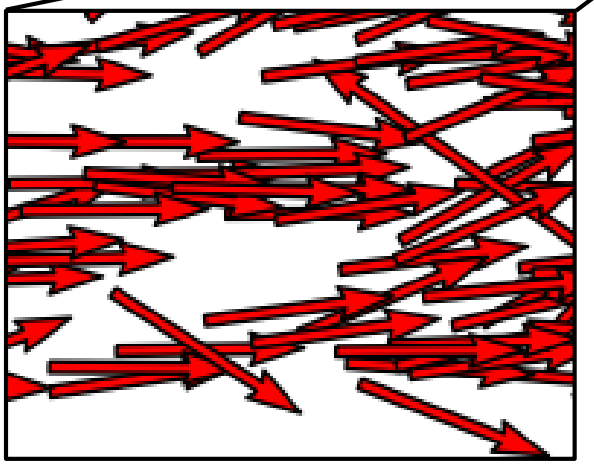
Hessian matrix

$$H = \begin{bmatrix} d & e \\ e & f \end{bmatrix}$$

Null eigenvalue ⟷ null curvature

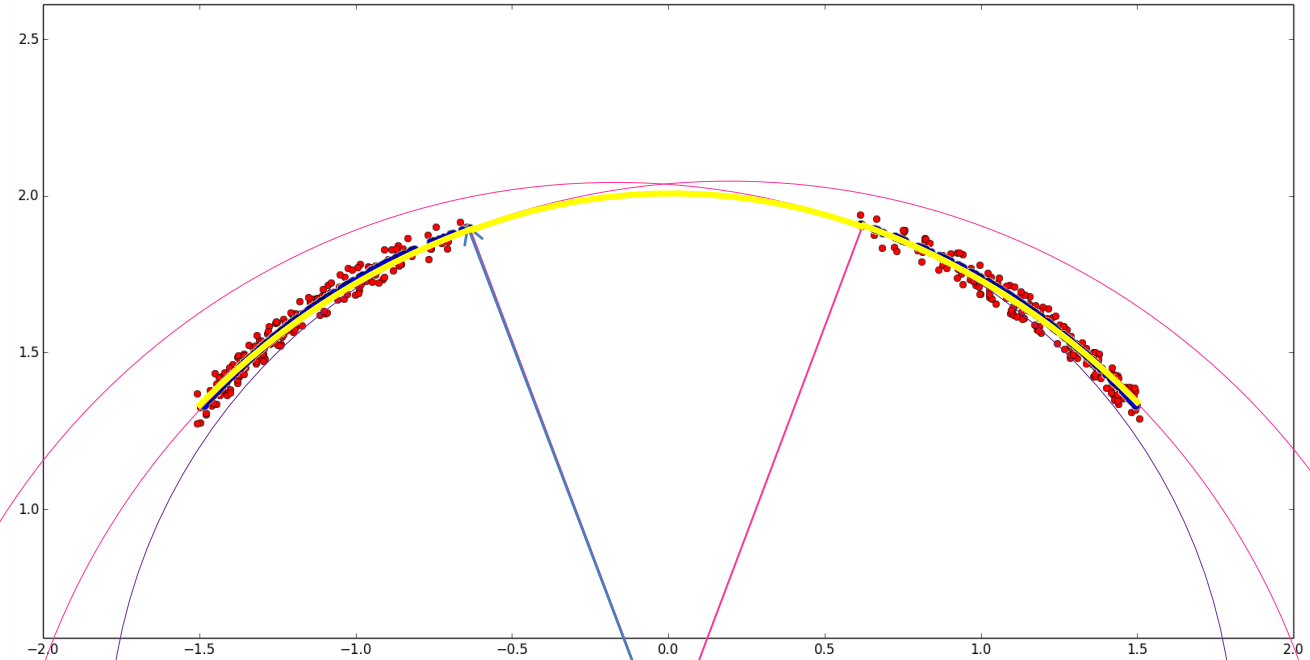
Eigenvector relative to the null eigenvalue ⟷ ruling direction





A statistical approach to find ruling direction

with **Davide Salvaggio**, Math Ing student



Projection of the red zone on a plane orthogonal to the median of all the vectors for that zone.

Generating curve used to create the syntetic vault

Interpolation of points with a polycentric curve

with **Giulia Guttilla**, Math student
with **Davide Salvaggio**, Math Ing student

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& Davide Salvaggio_ salvaggio.davide@gmail.com

UNITO_Università degli Studi di Torino

Matteo Semplice_ matteo.semplice@unito.it
& Giulia Guttilla_ giulia.guttilla@studenti.unito.it

We warmly thank the ***Consorzio La Venaria Reale*** for giving us the permission to use their pictures of the royal residence.