UNIVERSITÀ DEGLI STUDI DI BERGAMO

Kick-off: A review of basic concepts

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Aims of the lesson (Section 1)

- The time value of money;
- The future value of a cash flow today;
- Compounding;
- The present value of a cash flow in the future;
- Discounting.

All time worldwide box-office

Movie	Year	Box office revenue (USA dollars not adjusted for inflation)
1. Avatar	2009	$ 2,787,965,087
2. Titanic	1997	$ 2,186,772,032
3. Star Wars: The Force Awakens	2015	$ 2,040,854,468
9. Frozen	2013	$ 1,276,480,335
13. Lord of the Rings: Return of the King	2003	$ 1,119,929,521
20. Jurassic Park	1993	$ 1,029,153,882
58. Star Wars	1977	$ 775,400,00
216. Gone with the wind	1939	$ 400,200,000

Time value of money

- Supposed I asked you to lend me €100 today and promised to pay back €100 a year from now;

- Is that a good deal?

- What would make it a good deal?

Why time value of money?

- A euro today is worth more than a euro tomorrow;

- Why does money have time value?

…FUTURE VALUE…

Future value

- Future value of a cash flow today is the value of the funds invested at your opportunity cost;
- We call it: "*r*", i.e. *interest rate*;
- Let assume *r*=5%;
- If you invest €100, today, at an interest rate of 5%, how much will you have at the end of the year (one year)?
- If you invest €100, today, at an interest rate of 5%, how much will you have at the end of three years?

Future value

- How do you find the future value?
- Well, in general the future value of cash flow is called an upper *interest rate r*;
- We call this **COMPOUNDING**.

…PRESENT VALUE?…

Present value

- The present value of a future cash flow is the amount of cash flow you would take today instead of the promised future cash flow;
- As an example: you will be paid €50,000 in two years. How much would you accept today if your opportunity cost is 5%?
- We call this **DISCOUNTING**.

Present value

- In all these present value and future value calculations, the *r*, that we use, represents the discounts rate, interest rate, opportunity cost of capital and, I and you, we use these all interchangeably.

Aims of the lesson (Section 2)

- The future value of a stream of cash flows;
- The present value of a stream of cash flows;
- Annuity;
- The present or future value of an annuity.

Future value of a stream of cash flows

- Suppose you would like to have €50,000 in two years to start your new business idea. Impressed by your performance at work, you employer has just given you an annual bonus of €42,000 today. If you can invest at 5% per year, will you have enough at the end of two years to start your new business?

Uneven stream of cash flows

- Suppose over the next four years, you will receive the following cash flows: year 1= €3,000; year 2= €2,000; year 3= €4,000; year 4= €1,000;

- How much will you have at the end of four years (t=4), if the opportunity cost of funds is 5%?

Even stream of cash flows

- What if the cash flows were all the same?

- Say €3,000 every year?

Annuities

- An annuity is a series of equal fixed payments for a specified number of periods;
- Examples of annuities: bond payments, car loan payments, mortgage payments when you have a fixed cash flows;
- **Annuity Compound Factor** (ACF) sums up the compounding factors for *n* payments at a constant interest rate *r*;
- In general: $ACF(r,n) = \left[\frac{(1+r)^n - 1}{r}\right]$

Sinking fund problem

- $ACF(r,n) = \left[\frac{(1+r)^n - 1}{r}\right]$;
- We can look for the ***C***, i.e., the fixed equal payment, to accumulate to a target value;

$$Future\ value = C * ACF(r,n);$$
$$Future\ value = 3{,}000 * ACF(r = 5\%, n = 4)$$

Example: retirement problem

- Suppose you want to make sure you have €1,000,000 when you retire in 35 years. What even annual payments would you have to make to get to your goal if you can earn 6% per year?

Present value of a stream of cash flows

- How about we find the present value of a stream of cash flows?
- **Annuity Discount Factor** (ADF) sums up the discounting factors for *n* payments at a constant interest rate *r*;
- In general: $ADF(r,n) = \left[\frac{1-\frac{1}{(1+r)^n}}{r}\right]$
- Suppose you will get €2,000 every year for four years. What is the present value of these cash flows if the opportunity cost of funds is 5%?

Example: loan problem

- The purchase price of the car you would like to buy is €37,150. You want to take out a loan (100% financing) with a maturity of 60 months. The first loan payment will come in one month’s time, and the interest rate is 4% per year, compounded monthly. What are the monthly car payments?
- The statement «*the interest rate is 4% per year, compounded monthly*» tells us what the interest rate is per compounding period is.

Aims of the lesson (Section 3)

- How do you find effective interest rate?
- How do you adjust for compounding periods?
- What is continuous compounding?

Annual Percentage Rate (APR) vs Effective Rates

- The interest rate, provided by the loan problem, was given as 4% per year, compounded monthly;
- This is not the effective interest rate;
- This is the stated interest rate (or sometimes called the Annual Percentage Rate (APR));
- How do we find the effective annual interest rate?
- Suppose the stated interest rate is 4% per year, compounded monthly: what is the effective annual rate?

Effective Interest Rate

- In general: $(1+r) = \left(1+\frac{x}{t}\right)^t$ where: $r =$ effective rate; $x =$ APR; $t =$time;
- Let is calculate other effective rates: effective 2-month rate; 3-month rate; 6-month rate; 18-month rate;
- What if *r*=6% per year, compounded semi-annually?
- What about other effective rates: effective 6-month rate; effective three-month rate; effective 2-year rate (or 24-month rate); effective 15-month rate?

Continuous Compounding

- What if instead of every month, interest rate is compounded every instant, say continuously?
- Starting from: $(1+r) = \left(1+\frac{x}{t}\right)^t$ where: $r =$ effective rate; $x =$ APR; $t =$time;
- Now, let set t very large and define $r_c =$ the effective rate continuously compounded;
- Therefore: $e^x = (1+r_c)$; where is defined as: $r_c = e^x - 1$

Aims of the lesson (Section 4)

- How do value perpetuities?
- Growing annuities and perpetuities?

Perpetuity

- Perpetuity is a series of equal payments of a fixed amount for an infinite number of periods;
- How much would you pay for the opportunity to get paid €1,000 per year forever, if the interest rate is 10% per year?
- In general, we have: $V_0 = \frac{C}{r}$;
- What if C will grow at a certain *growth* for ever?
- In general, we have: $V_0 = \frac{C}{r-g}$

Growing Annuity

- Let is think back to annuity;
- What if the fixed payments grow at a constant rate?
- As an example: suppose you estimated your salary will start at €90,000 and will grow at 5% per year for the next five years. What is the present value of your future salary if the interest rate is 8% per year?
- Some hints: the **Annuity Discount Factor**, which includes the growth rate, becomes: $ADF(r,n,g) = \left[\frac{1-\frac{(1+g)^n}{(1+r)^n}}{r-g}\right]$;
- The **Annuity Compound Factor**, which includes the growth rate, becomes: $ACF(r,n,g) = \left[\frac{(1+r)^n-(1+g)^n}{r-g}\right]$