SUPPOSE THE CURRENT ZERO-COUPON YIELD CURVE FOR RISK-FREE BOND IS AS FOLLOWS:

MATURITY (YEARS)	1	2	3	4	5
YTM	3.25%	3.50%	3.90%	4.25%	4.60%

*) THE PRICE PER 100€ FACE VALUE OF A THREE-YEAR, ZERO COUPON, RISK-FREE BOND IS: $\frac{100}{(1.039)^3} = 89.15$

*) THE PRICE PER 100€ FACE VALUE OF A FOUR-YEAR, ZERO COUPON, RISK-FREE BOND IS: $\frac{100}{(1.0425)^4} = 84.66$

CONSIDER A ZERO-COUPON BOND WITH 20 YEARS TO MATURITY. THE PRICE AT WHICH THIS BOND WILL TRADE IF THE YIELD TO MATURITY (YTM) IS 6% IS:

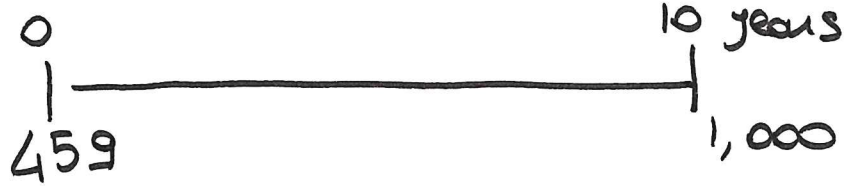
THE FACE VALUE IS 1,000 €

$$PV = \frac{1,000}{(1.06)^{20}} = 311.80 \text{ €}$$

*) CONSIDER A ZERO-COUPON BOND WITH A 1,000 € FACE VALUE AND 10 YEARS LEFT UNTIL MATURITY. IF THE YIELD TO MATURITY OF THIS BOND IS 10.4%, THEN THE PRICE OF THIS BOND IS CLOSEST TO:

$$PV = \frac{1,000}{(1.104)^{10}} = 371.80 \text{ €}$$

(*) CONSIDER A ZERO-COUPON BOND WITH A 1,000 € FACE VALUE AND 10 YEARS LEFT UNTIL MATURITY. IF THE BOND IS CURRENTLY TRADING FOR 459 €, THEN THE YIELD TO MATURITY ON THIS BOND IS:



$$459 = \frac{1000}{(1+YTM)^{10}}$$

$$YTM = \sqrt[10]{\frac{1,000}{459}} - 1 = 8.1\%$$

THE MILOS COMPANY HAS A BOND OUTSTANDING WITH A FACE VALUE OF 1,000 € THAT REACHES MATURITY IN 15 YEARS. THE BOND CERTIFICATE INDICATES THAT THE STATED COUPON RATE FOR THIS BOND IS 8%. AND THAT THE COUPON PAYMENTS ARE TO BE MADE SEMIANNUALLY.

*) HOW MUCH WILL EACH SEMIANNUAL COUPON PAYMENT BE?

COUPON = (COUPON RATE · FACE VALUE) / NUMBER OF COUPONS PER YEAR:

$= (0.08 \cdot 1{,}000\,€)/2 = 40\,€$

*) ASSUMING THE APPROPRIATE YTM ON THE MILOS BOND IS 7.5%, THEN THE PRICE AT WHICH THIS BOND TRADES WILL BE:

$$PV = \underbrace{\frac{40}{0.0375}}_{\frac{7.5\%}{2}} \cdot \left(1 - \frac{1}{(1.0375)^{30}}\right) + \frac{1{,}000}{(1.0375)^{30}} = 1{,}044{,}57$$

$30 \rightarrow 15.2$

(*) ASSUMING THE APPROPRIATE YTM ON THE MILOS BOND IS 7.5%, THEN THIS BOND WILL TRADE AT:

→ PAR

→ A DISCOUNT

→ NONE OF THE ABOVE

→ A PREMIUM

(*) ASSUMING THE APPROPRIATE YTM ON THE MILOS BOND IS 9 %, THEN THE PRICE AT WHICH THIS BOND TRADES WILL BE:

$$PV = \frac{40}{\underbrace{0.045}_{\frac{9\%}{2}}} \cdot \left(1 - \frac{1}{(1.045)^{30}}\right) + \frac{1{,}000}{(1.045)^{30}} = 918.55\ €$$

(*) ASSUMING THE APPROPRIATE YTM ON THE MILOS BOND IS 9%, THEN THIS BOND WILL TRADE AT:

→ PAR

→ A DISCOUNT

→ NONE OF THE ABOVE

→ A PREMIUM

(*) ASSUMING THAT THIS BOND TRADES FO 1,112 €, THE THE YIELD TO MATURITY FOR THIS BOND IS CLOSEST TO:

EXCEL FORMULA = RATE (30, 40, -1112, 1000)

BECAUSE THE BOND PAYS COUPONS SEMIANNUALLY, THIS YIELD IS FOR SIX-MONTH PERIOD.

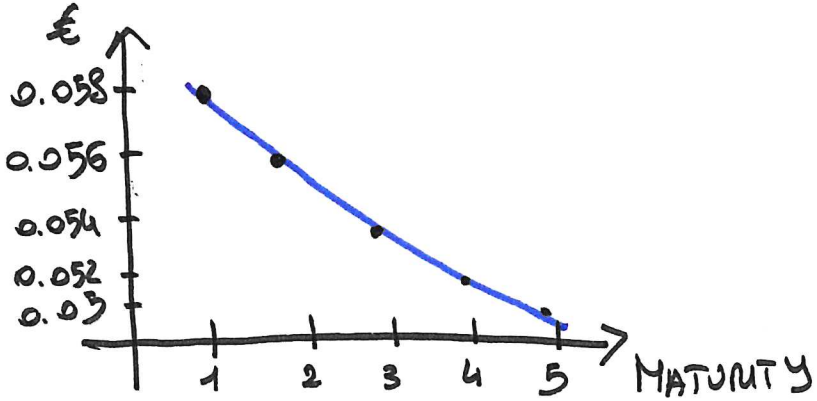
WE CONVERT IT TO AN ANNUAL YTM BY MULTIPLYING BY THE NUMBER OF COUPON PAYMENTS PER YEAR = 3.3987% · 2 = 6.7975%.

THE FOLLOWING TABLE SUMMARIZES PRICE OF VARIOUS DEFAULT-FREE ZERO-COUPON BONDS (EXPRESSED AS A PERCENTAGE OF FACE VALUE)

MATURITY (YEARS)	1	2	3	4	5
PRICE (per 100 FACE VALUE)	94.52	89.68	85.40	81.65	78.35
YTH	5.8%	5.6%	5.4%	5.2%	5.0%

(*) COMPUTE THE YIELD TO MATURITY FOR EACH OF THE FIVE ZERO-COUPON BONDS. (SEE THE TABLE) EACH YIELD TO MATURITY ABOVE IS CALCULATED USING THE FORMULA $YTH = \sqrt[n]{\frac{100}{PRICE}} - 1$ WHERE n = MATURITY, 1, 2, 3, 4, 5

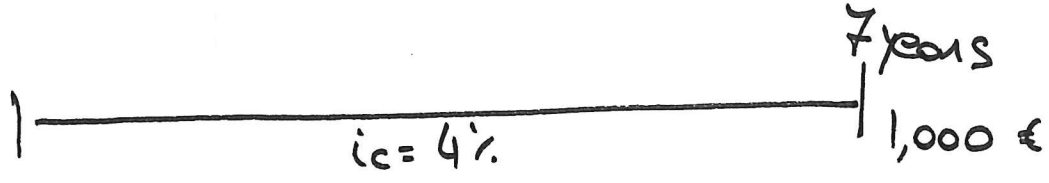
(*) PLOT THE ZERO-COUPON YIELD CURVE (FOR THE FIRST FIVE YEARS)



SUPPOSE THAT PROCTER & GAMBLE ISSUED A BOND THAT HAS 7 YEARS REMAINING UNTIL MATURITY, A € 1,000 FACE VALUE, AND A 4% COUPON RATE WITH ANNUAL COUPON PAYMENTS.

IF THE CURRENT MARKET INTEREST RATE IS 3%, (i) WHAT IS BOND'S PREMIUM OR DISCOUNT? (ii) WHAT IF THE CURRENT MARKET RATE IS 6%?

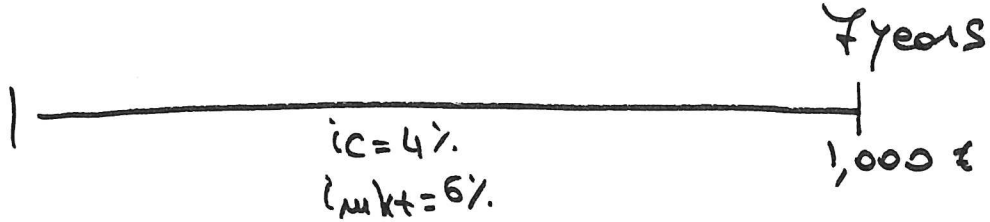
(i)



$$PV = \frac{40}{0.03} \cdot \left(1 - \frac{1}{(1.03)^7}\right) + \frac{1,000}{(1.03)^7} = 1,062,3$$

→ THE PREMIUM IS $1,062,3 - 1,000 =$ € 62,30

(ii)



$$PV = \frac{40}{0.06} \cdot \left(1 - \frac{1}{(1.06)^7}\right) + \frac{1,000}{(1.06)^7} = 888,35$$

→ THE DISCOUNT IS $1,000 - 888,35 = 111,65$

WHAT WE CAN ARGUE IS THE FOLLOWING:

(i) HOLDING ALL OTHER THINGS CONSTANT, A BOND'S YIELD TO MATURITY WILL NOT CHANGE OVER TIME;

(ii) HOLDING ALL OTHER THINGS CONSTANT, THE PRICE OF DISCOUNT OR PREMIUM BOND WILL MOVE TOWARD PAR VALUE OVER TIME;

(iii) IF A BOND'S YIELD TO MATURITY HAS NOT CHANGED, THEN THE IRR OF AN INVESTMENT IN THE BOND EQUALS ITS YIELD TO MATURITY EVEN IF YOU SELL THE BOND EARLY

LET'S SEE

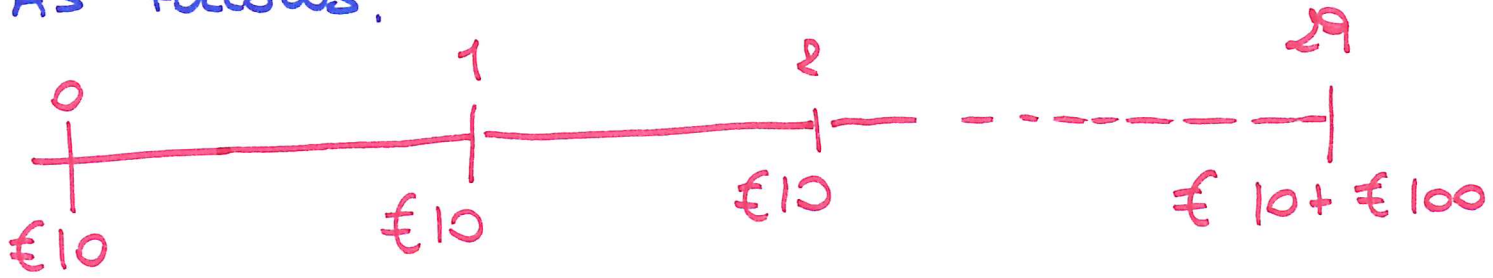
CONSIDER A 30-YEAR BOND WITH A 10% COUPON RATE (ANNUAL PAYMENTS) AND A €100 FACE VALUE.

i) WHAT IS THE INITIAL PRICE OF THIS BOND IF IT HAS A 5% YTM?

ii) IF THE YTM IS UNCHANGED, WHAT WILL THE PRICE BE IMMEDIATELY BEFORE AND AFTER THE FIRST COUPON IS PAID?

(i) $$PV = \frac{10}{0.05}\cdot\left(1-\frac{1}{(1.05)^{30}}\right)+\frac{100}{(1.05)^{30}} = €176.86$$

(ii) NOW, CONSIDER THE CASH FLOWS OF THIS BOND IN ONE YEAR, IMMEDIATELY BEFORE THE FIRST COUPON IS PAID. THE BOND NOW HAS 29 YEARS UNTIL IT MATURES, AND THE TIMELINE IS AS FOLLOWS:



AGAIN, WE COMPUTE THE PRICE BY DISCOUNTING THE CASH FLOWS BY THE YIELD TO MATURITY.

NOTE THAT THERE IS A CASH FLOW OF €10 AT THE ZERO, THE COUPON THAT IS ABOUT TO BE PAID. IN THIS CASE, WE CAN TREAT THE FIRST COUPON SEPARATELY. AND VALUE THE REMAINING CASH FLOWS AS FOLLOWS:

$$PV\left(\begin{matrix}\text{JUST BEFORE}\\ \text{FIRST COUPON}\end{matrix}\right) = 10 + \frac{10}{0.05}\cdot\left(1-\frac{1}{(1.05)^{29}}\right) + \frac{100}{(1.05)^{29}} = 185.71$$

NOTE THAT THE BOND PRICE IS HIGHER THAN IT WAS INITIALLY. IT WILL MAKE THE SAME TOTAL NUMBER OF COUPON PAYMENTS, BUT AN INVESTOR DOES NOT NEED TO WAIT AS LONG TO RECEIVE THE FIRST ONE.

WE COULD ALSO COMPUTE THE PRICE BY NOTING THAT BECAUSE THE YIELD TO MATURITY REMAINS AT 5% FOR THE BOND, INVESTORS IN THE BOND SHOULD EARN A RETURN OF 5% OVER THE YEAR

$$176.86\,(1.05) = 185.71$$

WHAT HAPPENS TO THE PRICE OF THE BOND JUST AFTER THE FIRST COUPON IS PAID?

WELL, THE TIMELINE IS THE SAME AS THAT GIVEN EARLIER, EXCEPT THE NEW OWNER OF THE BOND WILL NOT RECEIVE THE COUPON AT $t=0$. THUS, JUST AFTER THE COUPON IS PAID, THE PRICE OF THE BOND (GIVEN THE SAME YTM) WILL BE:

$$PV\left(\begin{matrix}\text{JUST AFTER}\\ \text{FIRST COUPON}\end{matrix}\right) = \frac{10}{0.05}\cdot\left(1-\frac{1}{(1.05)^{29}}\right)+\frac{100}{(1.05)^{29}} = €175.71$$

THE PRICE OF THE BOND WILL DROP BY THE AMOUNT OF THE COUPON (€10) IMMEDIATELY AFTER THE COUPON IS PAID, REFLECTING THE FACT THAT THE OWNER WILL NO LONGER RECEIVE THE COUPON. IN THIS CASE, THE PRICE IS LOWER THAN THE INITIAL PRICE OF THE BOND.

BECAUSE THERE ARE ARE FEWER COUPON PAYMENTS REMAINING, THE PREMIUM INVESTORS WILL PAY FOR THE BOND DECLINES. STILL, AN INVESTOR WHO BUYS THE BOND INITIALLY, RECEIVES THE FIRST COUPON, AND THEN SELLS IT EARNS A 5% RETURN IF THE BOND'S YIELD DOES NOT CHANGE:

$$\frac{(10+175.71)}{176.86} = 1.05$$

SUPPOSE THAT APPLE ISSUED A BOND THAT HAS TEN YEARS REMAINING UNTIL MATURITY, A €1,000 FACE VALUE, AND A 3% COUPON RATE WITH ANNUAL COUPON PAYMENTS.

i) IF THE CURRENT MARKET INTEREST RATE IS 5%, WHAT IS THE CURRENT PRICE OF THE BOND?

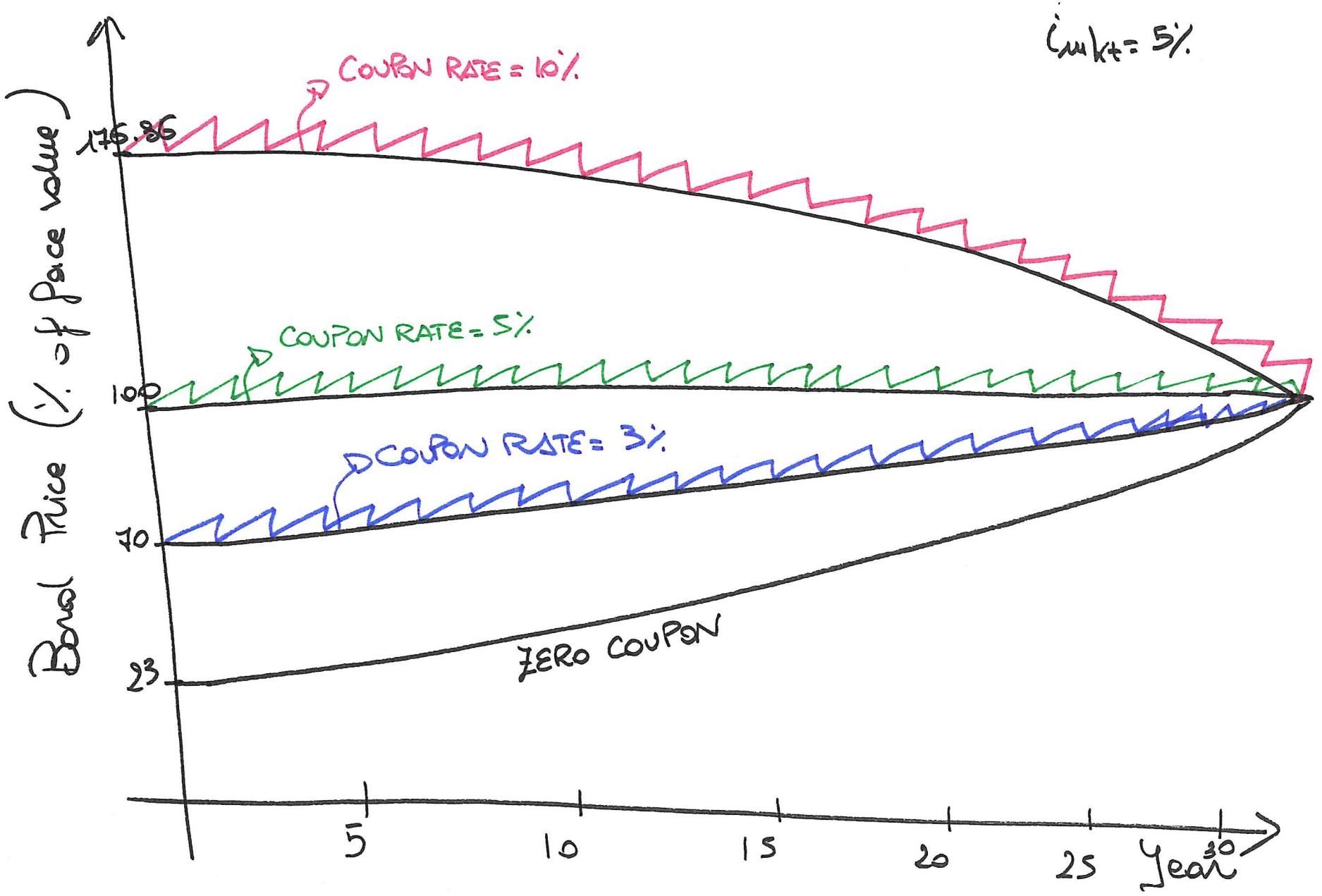
ii) WHAT WILL THE PRICE BE IN 4 YEARS, ASSUMING THE CURRENT MARKET RATE REMAINS UNCHANGED?

$1{,}000 \cdot 3\%$

$$(i) = PV = \frac{30}{0.05} \cdot \left(1 - \frac{1}{(1.05)^{10}}\right) + \frac{1{,}000}{(1.05)^{10}} = €845.57$$

$$(ii) = PV = \frac{30}{0.05} \cdot \left(1 - \frac{1}{(1.05)^{6}}\right) + \frac{1{,}000}{(1.05)^{6}} = €898.48$$

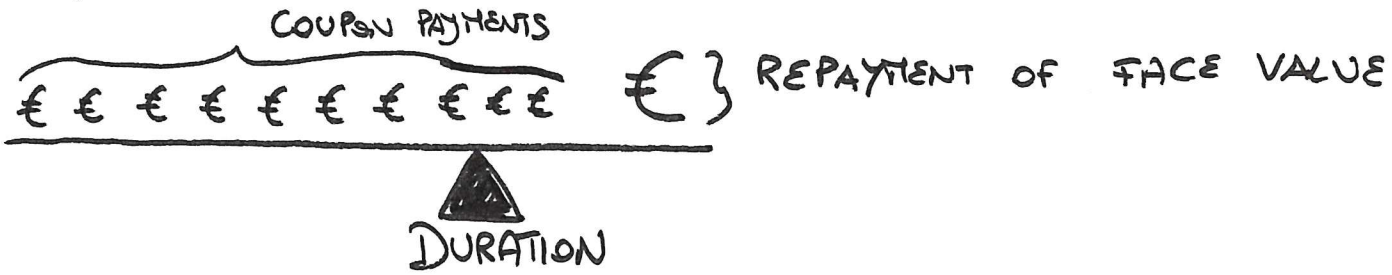
THE EFFECT OF TIME ON BOND PRICES



INTEREST RATE CHANGES AND BOND PRICES

- THERE IS AN INVERSE RELATIONSHIP BETWEEN INTEREST RATE AND BOND PRICES
  - (*) AS INTEREST RATES AND BOND YIELDS RISE, BOND PRICES FALL
  - (*) AS INTEREST RATES AND BOND YIELDS FALL, BOND PRICES RISE

- THE SENSITNITY OF A BOND'S PRICE TO CHANGES IN INTEREST RATES IS MEASURED BY THE BOND'S **DURATION**: IS A WEIGHTED AVERAGE OF THE PRESENT VALUE OF A BOND'S CASH FLOWS, WHICH INCLUDE A SERIES OF REGULAR COUPON PAYMENTS FOLLOWED BY A MUCH LARGER PAYMENT AT THE END WHEN THE BOND MATURES AND THE FACE VALUE IS REPAID.



- DURATION < MATURITY.
- DURATION IS ALSO AFFECTED BY THE SIZE OF REGULAR COUPON PAYMENTS AND THE BOND'S FACE VALUE
- FOR A ZERO COUPON BOND, [MATURITY = DURATION] SINCE THERE ARE NO REGULAR COUPON PAYMENTS AND ALL CASH FLOWS OCCUR AT MATURITY.
- ZERO COUPON BOND TEND TO PROVIDE THE MOST PRICE MOVEMENT FOR A GIVEN CHANGE IN INTEREST RATES, WHICH CAN MAKE ZERO COUPON BONDS ATRACTIVE TO INVESTORS EXPECTING A DECLINE IN RATES.
- BONDS WITH HIGH DURATIONS ⇒ HIGHLY SENSITIVE TO INTEREST RATE CHANGES
- BONDS WITH LOW DURATIONS ⇒ LESS SENSITIVE TO INTEREST RATE CHANGES