Multifactor Models

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Outline of talk

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The CAPM and the real world

- Portfolio theory and the CAPM have become accepted tools in the practitioner community;
- Many investment professionals think about the distinction between firm-specific and systematic risk and are comfortable with the use of *beta* to measure systematic risk;
- Still, the nuances of the CAPM are not nearly as well established in the community: compensation of portfolio managers is not based on appropriate risk-adjusted performance. What can we make of this?
- The CAPM was first published by Sharpe in the **Journal of Finance** in 1964;
- Early tests by **Black, Jensen & Scholes (1972)** and **Fama & MacBeth (1973)** were only partially supportive of the CAPM: average returns were higher for higher-beta portfolios, but the reward for beta risk was less than predicted by the simple version of the CAPM;

The CAPM and the real world

- In 1977, Roll shook the practitioner world. In the paper entitled *"A Critique of Capital Asset Pricing Tests"*, Roll argued that since the true market portfolio can never be observed, the CAPM is *necessarily* unstable;
- The publicity given the now classic *"Roll' critique"* resulted in popular articles such as *"Is Beta Dead?"*;
- Although Roll is correct on theoretical grounds, some tests suggest that the error introduced by using a broad market index as proxy for the true, unobserved market portfolio is perhaps not the greatest problem involved in testing the CAPM;

The CAPM and the real world

- **Fama & French (1992)** published a study that dealt the CAPM an even harsher blow. They claimed that in contradiction to the CAPM, certain characteristics of the firm, such as size and the ratio of market to book value, were far more useful in predicting future returns than beta;
- Fama & French and several others have published many follow-up studies of this topic;
- It seems clear from these studies that beta does not tell the whole story of risk;
- In particular, there seem to be risk factors that affect security returns beyond beta' one-dimensional measurement of market sensitivity;

The CAPM and the real world

- As an example, liquidity, a different kind of risk factor, was ignored for a long time;
- Although first analyzed by **Amihud & Mendelson (1986)**, it is yet to be accurately measured and incorporated in portfolio management;
- Measuring liquidity and the premium commensurate with illiquidity is part of a larger field in financial economics, namely market structure. Trading mechanisms on stock exchanges can affect the liquidity of assets traded on these exchanges and thus significantly affect their market value;
- Despite all these issues, **beta is not dead!**

The CAPM and the real world

- **Jagannathan & Wang (1996)** show that, when a more inclusive proxy for the market portfolio than the S&P 500 (specifically, an index that includes human capital) is used and when allowing for the fact that beta changes over time, the performance of beta in explaining security returns is considerably enhanced;
- The CAPM is not a perfect model: it will be greatly refined;
- However, the logic of the model is compelling and more sophisticated models of security pricing all rely on the key distinction between systematic and diversifiable risk;
- Therefore, the CAPM provides a useful framework for thinking rigorously about the relationship between security risk and return.

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Multifactor Models and the CAPM

- The index model allows us to decompose stock variance into systematic risk and firm-specific risk that can be diversified in large portfolios;
- In the index model, the returns on the market portfolio summarized the aggregate impact of macro factors;
- In reality, however, systematic risk is not due to one source but instead derives from uncertainty in many economywide factors such as business-cycle risk, interest or inflation rate risk, energy price risk, and so on;
- It stands to reason that a more explicit representation of systematic risk, allowing stocks to exhibit different sensitivities to its various facets, would constitute a useful refinement of the single-factor model;
- We can expect that models that allow for several systematic factors - **multifactor models** - can provide better descriptions of security returns;

Multifactor Models and the CAPM - Two-factor model

- Let is start with a **two-factor model**. Suppose the two most important macroeconomic sources of risk are: (i) the state of the business cycle reflected in returns on a broad market index such as the S&P 500; (ii) unanticipated changes in interest rates captured by returns on a Treasury-bond portfolio;
- The return on any stock will respond both to sources of macro risk and to its own firm-specific influences;
- Therefore, we can expand the single-index model $R_i = \alpha_i + \beta_i R_M + \varepsilon_i$, describing the excess rate of return on stock i in some time period t as follows:

$$R_{i,t} = \alpha_i + \beta_{i,M} R_{M,t} + \beta_{i,TB} R_{TB,t} + \varepsilon_{i,t} \tag{1}$$

Multifactor Models and the CAPM - Two-factor model

- where, $\beta_{i,TB}$ is the sensitivity of the stock' excess return to that of the T-bond portfolio; $R_{TB,t}$ is the excess return of the T-bond portfolio at time t;
- **Question:** how will the security market line of the CAPM generalize once we recognize the presence of multiple sources of systematic risk?;
- A multifactor index model gives rise to a multifactor security market line in which the risk premium is determined by the exposure to each systematic risk factor and by a risk premium associated with each of those factors (**Merton, 1973**);

Multifactor Models and the CAPM - Two-factor model

- In a two-factor economy of $R_{i,t} = \alpha_i + \beta_{i,M} R_{M,t} + \beta_{i,TB} R_{TB,t} + \varepsilon_{i,t}$, the expected rate of return on a security would be the sum of three terms:
  1. The risk-free rate of returns;
  2. The sensitivity to the market index (i.e., the market beta, $\beta_{i,M}$) times the risk premium of the index, $[E(r_M) - r_f]$;
  3. The sensitivity to interest rate risk (i.e., the T-bond beta, $\beta_{i,TB}$) times the risk premium of the T-bond portfolio, $[E(r_{TB}) - r_f]$;
- This assertion is expressed mathematically as a two-factor security line for security i:

$$E(r_i) = r_f + \beta_{i,M}[E(r_M) - r_f] + \beta_{i,TB}[E(r_{TB}) - r_f] \qquad (2)$$

Multifactor Models and the CAPM - Two-factor model

- Equation 2 is an expansion of the simple security market line. Once we generalize the single-index model to multiple risk sources, each with its own risk premium, the insights are similar;
- **Example**: Suppose Kos Financial Services has a hypothetical market beta of 1.2 and a T-bond beta of 0.7. Suppose the market index risk premium is 6%, while that of the T-bond portfolio (comprising a basket of European government bonds) is 3%, and the risk-free rate is 4%. What would be the equilibrium expected rate of return on Kos Financial Services?
- Suppose the risk premiums were $E(r_M - r_f) = 4\%$ and $E(r_{TB} - r_f) = 2\%$, what would be the new equilibrium expected rate of return on Kos Financial Services?

Multifactor Models and the CAPM - Two-factor model

- The multifactor model clearly gives us a richer way to think about risk exposures and compensation for those exposures than the single-index model or the CAPM;
- However, **what are the relevant additional systematic factors?**
- Three methodologies have been deployed to identify systematic factors in security returns: **(i) theory**; **(ii) regression analysis**; **(iii) other statistical tools**;

Multifactor Models and the CAPM - Two-factor model

- **The theory-based approach** specifies potential extra-market risk factors on the basis of their potential impact on lifetime consumption and bequests;
- These variables fall into two groups:
  1. Prices of items that make up a substantial part of the lifetime consumption basket of many consumers (e.g., health care or housing);
  2. Variables that affect future investment opportunities (e.g., interest rates or prices of inputs to major manufacturing and service industries).
- Investors are expected to respond to these sources of risk to their future consumption and investment opportunities by exhibiting excess demand for securities that can hedge those risks. This demand will drive up prices and drive down expected rates of return;

Multifactor Models and the CAPM - Two-factor model

- Thus, correlation with these sources of risk may induce its own risk premium;
- Variables that are important enough to affect security prices through a risk premium in such models are called **priced risk factors**;
- The theory therefore predicts a multi-index model in which portfolios that track each priced risk factor augment the market index in a multifactor version of the Security Market Line (SML);
- Some factors might help to explain returns but still might not carry a risk premium;
- As an example, securities of firms in the same industry may be highly correlated. If we were to run a regression of the returns on one such security on the returns of the market index and a portfolio of the other securities in the industry, we would expect to find a significant coefficient on the industry portfolio;

Multifactor Models and the CAPM - Two-factor model

- However, if this industry is a small part of the broad market, the industry risk can be diversified away;
- Thus, although an industry coefficient measures sensitivity to the industry factor, it does not necessarily represent exposure to systematic risk and will not result in a risk premium;
- We say that such factors are not priced, i.e., **they do not carry a risk premium**;
- The empirical content of a model of this type depends on the actual aggregate demand for these portfolios. So far, these models have not produced a clearly superior multi-index equation, suggesting that investors are not willing to pay significant premiums to hedge against these extra-market risk factors.

Multifactor Models and the CAPM - Two-factor model

- **The regression-based approach** seeks economic variables, or portfolios tracking those variables, that can significantly improve the explanatory power of the single-index equation;
- So far, one of these, the Fama-French factor model, has been most successful;
- **The statistics-based approach** deploys principle components and factor analysis procedures to identify systematic factors from only the return history of a security universe;
- This approach identifies a set of portfolios that explain returns well within a given sample. However, in practice, the composition of these portfolios appears to change quickly over time and tends to perform poorly when applied to out-of-sample data;
- As a consequence, this approach has largely been abandoned.

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The Fama-French Three-Factor Model

- **Fama & French (1996, JoF)** propose a three-factor model that has become a standard tool for empirical studies of asset returns;
- They add to the market-index portfolios formed on the basis of firm **size** and **book-to-market ratio** to explain average returns;
- These additional factors are motivated by the observations that average returns on stocks of small firms and on stocks of firms with a high ratio of book value of equity to market value of equity have historically been higher than predicted by the security market line of the CAPM;
- This observation suggests that size or the book-to-market (B/M) ratio may be proxies for exposures to sources of systematic risk not captured by the CAPM beta, and thus result in return premiums;

The Fama-French Three-Factor Model

- They point out that firms with high ratios of book-to-market value are more likely to be in financial distress and that small stocks may be more sensitive to changes in business conditions. Thus, these variables may capture sensitivity to macroeconomic risk factors;
- While the high book-to-market group includes many firms in financial distress, which depresses market value relative to book value, for the most part this group includes relatively mature firms. The latter derive a larger share of their market value form assets already in place, rather than growth opportunities. This group is called **value stocks**;
- In contrast, low-B/M companies are viewed as **growth firms** whose market values derive from anticipated future cash flows, rather than from assets already in place. Considerable evidence suggests that value stocks trade at lower prices than growth stocks (or, equivalently, have offered a higher average rate of return). The differential is knows as the **value premium**.

The Fama-French Three-Factor Model

- While a value premium may be appropriate compensation for risk for a firm whose high B/M ratio reflects potential financial distress, it would seem paradoxical for firms whose high B/M ratio reflects maturity and thus more predictable future cash flows. Therefore, other things equal, the required rate for growth stocks is lower than that of more mature value firms;
- **This is a puzzle**: one explanation is that mature firms with large amounts of installed capital deal with higher adjustment costs in adapting to shocks in the product markets in which they operate.

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First Assignment - Background

- A key goal of finance is to understand the sources of risk that cause different securities to offer different expected returns;
- The oldest model of this type is the CAPM, but in recent times researchers and practitioners have uncovered several findings that are inconsistent with the simple CAPM model;
- The current task builds on US research and proposes and tests a 4 factor model for UK asset returns based on research by Eugene Fama, Ken French and Mark Carhart.

First Assignment - Data

- The data file contains the following columns:
  1. Dates for each observation (monthly);
  2. Returns on a size factor portfolio (SMB): a portfolio that is long small cap stocks and short large cap stocks;
  3. Returns on a value portfolio (HML): a portfolio that is long stocks with high book-to-market value and short stocks with low book-to-market value;
  4. Returns on a momentum factor portfolio (UMD): a portfolio that is long recent winners and short recent losers (in cumulative return terms). For more details, see Carhart (**1997, JoF**) *"On Persistence in Mutual Fund Performance"*;
  5. Returns on the UK stock market (RM);
  6. The UK risk-free return (RF);
  7. The difference between the UK stock market return and the UK risk free return (RMRF);
  8. The returns on 10 portfolios of UK stocks (S1-S10): stocks have been assigned to portfolios based on their market cap. The smallest stocks are in S1 and the largest stocks are in S10.

First Assignment - Data

- The four factor portfolios are supposed to represent sources of risk (SMB, HML, UMD and RMRF);
- UMD the monthly premium on winners minus losers from Fama & French (1993) and Carhart (1997);
- The ten size based portfolios (S1-S10) represent the basic assets whose pricing we want to understand.

First task: basic statistics

- The first part of your task:
  1. Read the data: excel file "First_Assignment_dataset";
  2. From S1-S10 and the risk-free rate data, create excess returns for the ten portfolios. Call these excess returns XS1-XS10;
  3. Inspect the returns for each portfolio and comment on any exceptional features;
  4. Compute summary statistics for the 10 excess return series. Comment on the variation in statistics across the portfolios and the implications of this variation.

Second task: time-series regressions

- The second part of your task is to work out how sensitive each excess return is to all of the risk factors. For each return XS1 to XS10 in turn:
  1. Run a regression of the excess return on a constant term, RMRF, SMB, HML, and UMD;
  2. Interpret the regression coefficients, their t-ratios or p-values;
  3. Store the slope coefficients form these regressions in an Excel file;
  4. Comment on the variations in risk exposures across factors and how they might be interpreted;
  5. Also interpret the R^2 from these regressions.

Third task: cross-sectional regression

- The final part of our job. Do risk exposures explain the way in which mean returns vary across portfolios?
  1. Run a series of cross section regressions. The y-variable is the set of 10 mean returns on the ten size-based portfolios and the x-variables are a constant and two sets of risk exposures form your time series regressions; those on RMRF and one other. In turn, the other risk factor will be SMB, HML and UMD. Thus, in each regression there are 10 observations and two explanatory variables plus a constant term;
  2. Interpret your results;
  3. Write up your results into a *"research report"* of at least 1,500 and at most 2,000 words. **Deadline: Sunday, 27th November 2022**;
  4. Write up your results into a set of at most 10 slides and be prepared to present your findings to the class. **Conference presentation: Thursday, 1st December, 2022**;
  5. Students, who are not going to do the assignment, have to prepare (entirely) for the exam the following paper: **Fama, E., & French, K. (1996). Multifactor Explanations of Asset Pricing Anomalies.** *Journal of Finance*, **51(1), 55-84.**

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