LET US ASSUME TO COMPOSE A PORTFOLIO (ONLY RISKY ASSETS) AS FOLLOWS:

STOCK X	WEIGHT = 60%	RETURNS = 9,80%	$\sigma^2 = 0.25\%$	$\sigma = 5.02\%$
STOCK Y	WEIGHT = 40%	RETURNS = 10.5%	$\sigma^2 = 0.08\%$	$\sigma = 2.75\%$
$\sum$ PORTFOLIO	100%			

COMPUTE THE EXPECTED RETURNS OF THE PORTFOLIO.

$$r_{PTF} = (9.8 \cdot 0.6) + (10.5 \cdot 0.4) = 10.08\%$$

COMPUTE THE RISK OF THE PORTFOLIO

$$\boxed{\sigma_{PTF} = (5.02\% \cdot 0.6) + (2.75 \cdot 0.4) = 4.11\%}$$

ABSOLUTELY NOT

LET ASSUME THAT : $\begin{cases} \sigma_{(x,y)} = -0.107\% \Rightarrow \text{COVARIANCE} \\ \rho_{x,y} = -77.6345\% \Rightarrow \dfrac{\sigma(x,y)}{\sigma_x \cdot \sigma_y} \end{cases}$

LET US DENOTE: THE WEIGHT OF $x = a$

THE WEIGHT OF $y = b$

$$\begin{array}{c|cc} & x & y \\ \hline x & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{bmatrix} a^2 \cdot \sigma_x^2 & \sigma(x,y) \cdot a \cdot b \\ \sigma(x,y) \cdot a \cdot b & b^2 \cdot \sigma_y^2 \end{bmatrix}$}} \\ y & & \end{array}$$

SYMMETRIC MATRIX

$$\sigma^2_{PTF} = \sqrt{a^2 \cdot \sigma_x^2 + b^2 \cdot \sigma_y^2 + 2 \cdot \sigma(x,y) \cdot a \cdot b}$$

INSTEAD OF $\sigma(x,y)$, WE CAN USE $\rho_{x,y} = \dfrac{\sigma(x,y)}{\sigma_x \cdot \sigma_y}$

$$\sigma^2_{PTF} = \sqrt{a^2 \cdot \sigma_x^2 + b^2 \cdot \sigma_y^2 + 2 \cdot a \cdot b \cdot \sigma_x \cdot \sigma_y \cdot \rho_{x,y}}$$

$$\sigma^2_{PTF} = 0.6^2 \cdot 0.25 + 0.4^2 \cdot 0.08 + 2 \cdot 0.6 \cdot 0.4 \, (-0.107\%) = 0.0513\%$$

$$\sigma_{PTF} = \sqrt{0.000513} = 2.26\%$$

$$\sigma^2_{PTF} = 0.6^2 \cdot 0.25 + 0.4^2 \cdot 0.08 + 2 \cdot 0.6 \cdot 0.4 \cdot 0.0502 \cdot 0.027 \cdot (-0.776345) = 0.0513\%$$

$$\sigma_{PTF} = \sqrt{0.000513} = 2.26\%$$

APPLICATION

STOCK	RETURNS	σ	WEIGHT
A	12%	5%	30%
B	4%	2%	50%
C	15%	6%	20%

PORTFOLIO COMPOSITION

$\sigma(A,B) = 0.0001$

$\sigma(A,C) = 0.0006$

$\sigma(B,C) = 0.00048$

COMPUTE THE RETURN AND THE RISK OF THE PORTFOLIO.

$$r_{PTF} = (12\% \cdot 0.3) + (4\% \cdot 0.5) + (15\% \cdot 0.2) = 8.6\%$$

$$\sigma_{PTF} = \sqrt{0.3^2 \cdot 0.05^2 + 0.5^2 \cdot 0.02^2 + 0.2^2 \cdot 0.06^2 + (2 \cdot 0.0001 \cdot 0.3 \cdot 0.5) + (2 \cdot 0.0006 \cdot 0.2 \cdot 0.3) + (2 \cdot 0.00048 \cdot 0.2 \cdot 0.5} =$$

$$\sigma_{PTF} = 2.58\%$$

$$
\begin{array}{c|ccc}
 & A & B & C \\
\hline
A & 0.3^2 \cdot 0.05^2 & & \\
B & 0.0001 \cdot 0.3 \cdot 0.5 & 0.5^2 \cdot 0.02^2 & \\
C & 0.0006 \cdot 0.2 \cdot 0.3 & 0.00018 \cdot 0.5 \cdot 0.2 & 0.2^2 \cdot 0.06^2
\end{array}
$$

$$\sigma_{PTF} = \sum_i [\,\cdot\cdot\cdot\cdot\cdot\,] = 2.58\%$$

VARIANCE - COVARIANCE MATRIX

$$\begin{array}{c|cc} & x & y \\ \hline x & \sigma_x^2 & \sigma(y,x) \\ y & \sigma(x,y) & \sigma_y^2 \end{array}$$

CORRELATION MATRIX

$$\begin{array}{c|cc} & x & y \\ \hline x & \rho_{x,x} = 1 & \rho_{x,y} \\ y & \rho_{y,x} & \rho_{y,y} = 1 \end{array}$$

APPLICATION

STOCK	WEIGHT	RETURNS	σ
1	50%	10%	20%
2	30%	15%	30%
3	20%	20%	40%

CORRELATION MATRIX

$$
\begin{array}{c c}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.5 & 1 & 0.1 \\ 0.3 & 0.1 & 1 \end{bmatrix}
\end{array}
$$

COMPUTE RETURN AND RISK OF THE PORTFOLIO.

$$r_{PTF} = \underset{1}{(10 \cdot 0.5)} + \underset{2}{(15 \cdot 0.3)} + \underset{3}{(20 \cdot 0.2)} = 13.5\%$$

$$
\begin{array}{c c}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.5^2 \cdot 0.2^2 & & \\ 0.3 \cdot 0.5 \cdot 0.5 \cdot 0.2 \cdot 0.3 & 0.3^2 \cdot 0.3^2 & \\ 0.2 \cdot 0.5 \cdot 0.3 \cdot 0.2 \cdot 0.4 & 0.2 \cdot 0.3 \cdot 0.1 \cdot 0.4 \cdot 0.2 & 0.2^2 \cdot 0.4^2 \end{bmatrix}
\end{array}
$$

$$\sigma^2_{PTF} = \left[0.01 + 0.0081 + 0.0064 + 2(0.0045 + 0.0024 + 0.00048)\right]$$

$$\sigma^2_{PTF} = 0.03926 = 3.926\%$$

$$\sigma_{PTF} = \sqrt{0.03926} = 19.81\%$$

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THE ROLE OF DIVERSIFICATION

CONSIDER A PORTFOLIO WITH ONLY RISKY ASSETS. WE CAN PROVIDE AS MANY COMBINATIONS AS WE WISH. REGARDING ρ, WE CAN OBTAIN THE FOLLOWING RELATIONSHIP BETWEEN RISK & RETURNS