

1 ES1 2 CAMPIORI

$$S_x^2 = \frac{\sum_{i=1}^n (X_i - M_x)^2}{n-1}$$

$$S_y^2 = \frac{\sum_{i=1}^m (Y_i - M_y)^2}{m-1}$$

$$E(S_x^2) = \sigma^2$$

$$E(S_y^2) = \sigma^2$$

$$S^2 = p_1 S_x^2 + (1-p_1) S_y^2 \quad 0 \leq p_1 \leq 1$$

$$E(S^2) = p_1 \sigma^2 + (1-p_1) \sigma^2 = \sigma^2$$

$$\text{Var}(S^2) = \text{Var}(P_1 S_x^2) + \text{Var}((1-P_1) S_y^2)$$

$$= \text{Var}\left(\frac{P_1 \sigma^2}{n-1} \frac{S_x^2(n-1)}{\sigma^2}\right) + \text{Var}\left(\frac{(1-P_1) \sigma^2}{m-1} \frac{S_y^2(m-1)}{\sigma^2}\right) =$$

$$= \frac{P_1^2 \sigma^4}{(n-1)^2} 2(n-1) + \frac{(1-P_1)^2 \sigma^4}{(m-1)^2} 2(m-1) =$$

$$= 2\sigma^4 \left( \frac{1}{n-1} P_1^2 + \frac{1}{(m-1)} (1-P_1)^2 \right)$$

$$\frac{\delta \text{Var}(S^2)}{\delta P_1} = 2\sigma^4 \left[ 2P_1 \frac{1}{n-1} + \frac{2(1-P_1)(-1)}{m-1} \right]$$

$$= 4\sigma^2 \left[ \frac{P_1}{n-1} + \frac{P_1}{m-1} - \frac{1}{m-1} \right]$$

Condizione x minima

$$\frac{P_1}{n-1} + \frac{P_1}{m-1} = \frac{1}{m-1}$$

$$P_1 \left( \frac{n+m-2}{(n-1)(m-1)} \right) = \frac{1}{m-1}$$

$$P_1 = \frac{n-1}{n+m-2} \quad (1-P_1) = \frac{m-1}{n+m-2}$$

$$S^2 = \frac{n-1}{n+m-2} S_x^2 + \frac{m-1}{n+m-2} S_y^2$$

$$= \frac{\sum_{i=1}^n (x_i - \mu_x)^2 + \sum_{j=1}^m (y_j - \mu_y)^2}{n+m-2}$$

$$\frac{(n+m-2) S^2}{\sigma^2}$$

$\chi^2$  con  $(n+m-2)$  g.l.e

$$\frac{\mu_x - \mu_y - (\mu_x - \mu_y)}{\left[ \sigma^2 \left( \frac{1}{n} + \frac{1}{m} \right) \right]^{1/2}}$$

Normal Standard

~~Ass~~

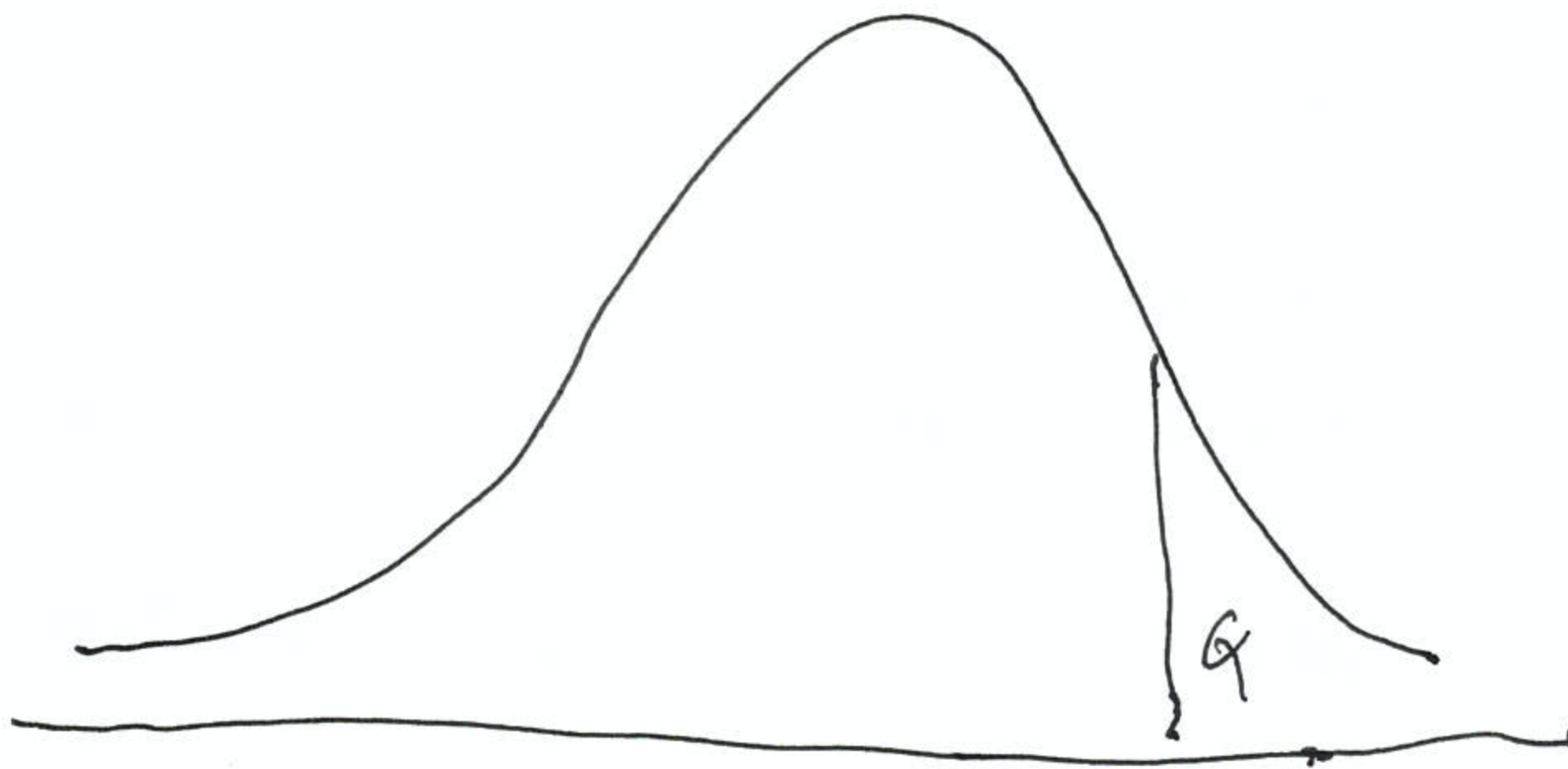
$$\frac{\mu_x - \mu_y}{\sigma} \cdot \frac{1}{\left( \frac{1}{n} + \frac{1}{m} \right)^{1/2}}$$

t Student  
 $n+m-2$   
g.l.e

$$\sqrt{\frac{S^2}{\sigma^2}}$$

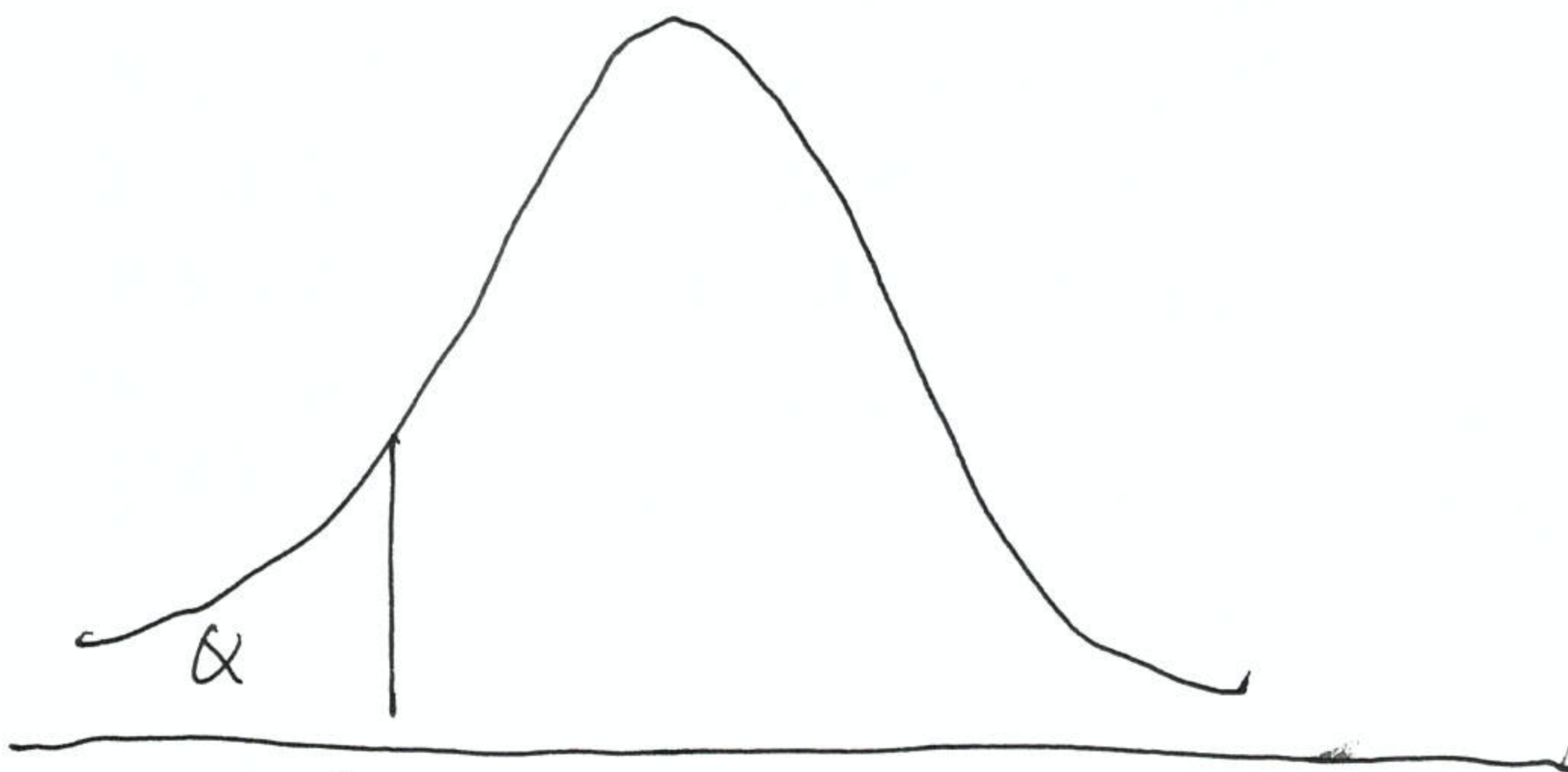
$$H_0: \mu_x - \mu_y = 0$$

$$T = \frac{\mu_x - \mu_y}{\sqrt{s^2 \left( \frac{1}{n} + \frac{1}{m} \right)}}$$



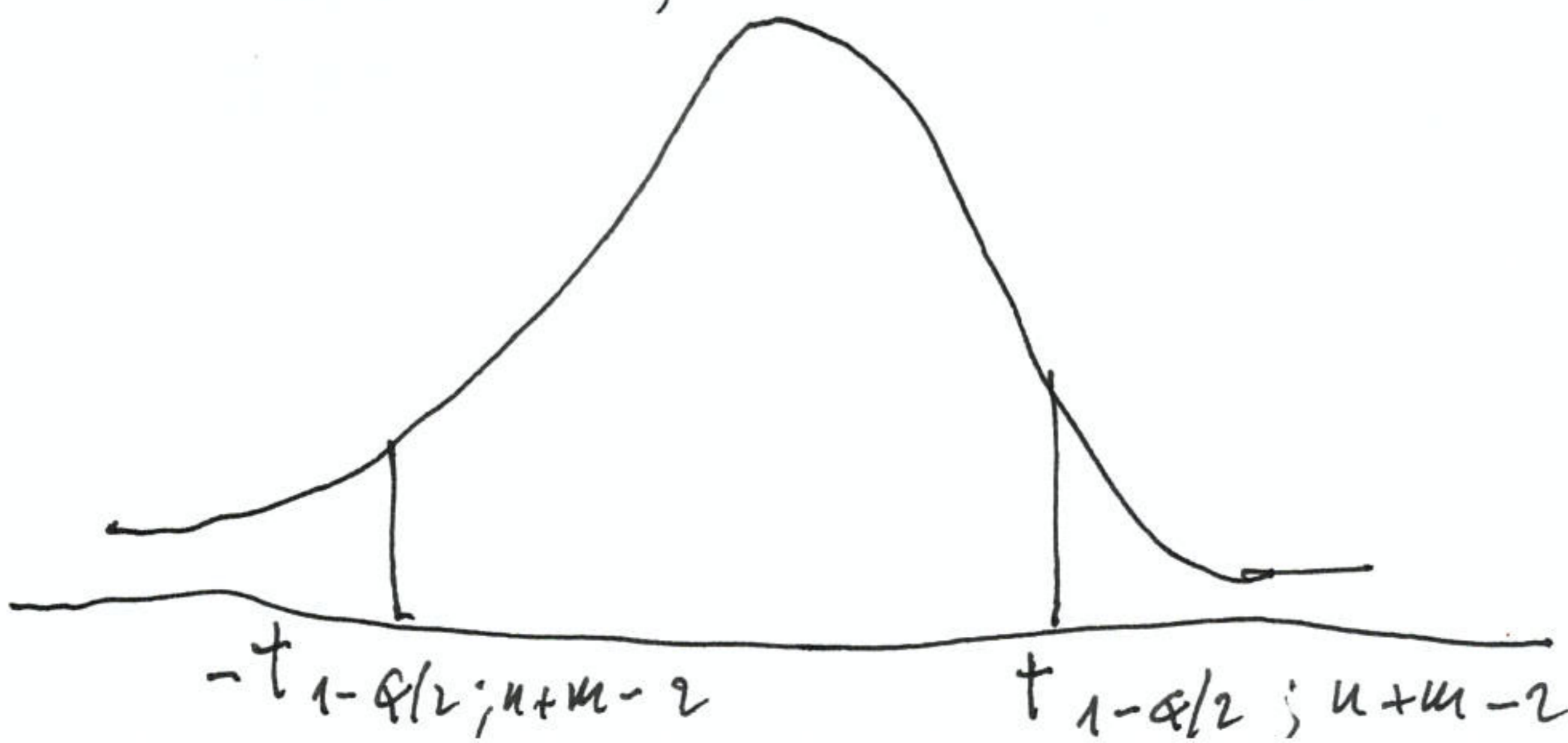
DX

$$t_{1-\alpha; n+m-2}$$



SX

$$-t_{1-\alpha/2; n+m-2}$$



BI

$$-t_{1-\alpha/2; n+m-2}$$

$$t_{1-\alpha/2; n+m-2}$$