

$$\text{Cov}(X Y) = \frac{\sum_1^n x_i y_i}{n} - M_x M_y =$$

(1)

$$= \frac{\sum (x_i - M_x) y_i}{n} =$$

$$= \frac{\sum (x_i - M_x) (y_i - M_y)}{n}$$

$$\text{Var}(x) = \frac{\sum (x_i - M_x)^2}{n} = \frac{\sum (x_i - M_x) x_i}{n}$$

(2)

$$r = \frac{\text{Cov}(X Y)}{\sqrt{\text{Var}(x) \text{Var}(y)}}$$

(3)

$$\hat{y}_i = b_0 + b_1 x_i = M_y + b_1 (x_i - M_x)$$

(4)

$E(b_1)$        $Var(b_1)$

B

$$b_1 = \frac{\sum (x_i - M_x) y_i \cdot \frac{1}{n}}{Var(x)}$$

$$E(b_1) = \frac{\sum (x_i - M_x) [\beta_0 + x_i \beta_1] \cdot \frac{1}{n}}{Var(x)} =$$

$$= \frac{\beta_0 \sum (x_i - M_x) / n}{Var(x)} + \beta_1 \frac{\sum (x_i - M_x) x_i \cdot \frac{1}{n}}{Var(x)}$$

$$= \beta_1$$

$$Var(\beta_1) = \frac{1}{n^2} \frac{\sum (x_i - M_x)^2 \sigma^2}{(Var(x))^2} = \frac{\sigma^2}{n} Var(x)$$

$$E(b_0)$$

(1)

$$b_0 = M_y - b_1 M_x$$

$$E(b_0) = \frac{1}{n} \sum (\beta_0 + \beta_1 x_i) - \beta_1 M_x = (*)$$

$$= \beta_0 + \beta_1 M_x - \beta_1 M_x = 0$$

— . —

$$(*) \quad M_y = \frac{\sum y_i}{n} = \beta_0 + \beta_1 M_x + \frac{\sum \epsilon_i}{n}$$

$$E(M_y) = \beta_0 + \beta_1 M_x$$

~~Var (b0)~~

Var (b0)

C2

$$b_0 = \frac{\sum_{i=1}^n y_i}{n} - M_x \frac{\sum_{i=1}^n (x_i - M_x) y_i}{n} \cdot \frac{1}{\text{Var}(x)} =$$

$$= \frac{1}{n} \sum_{i=1}^n y_i \left[ 1 - \frac{M_x (x_i - M_x)}{\text{Var}(x)} \right] =$$

$$\text{Var}(b_0) = \frac{1}{n^2} \sigma^2 \sum_{i=1}^n \left[ 1 - \frac{M_x (x_i - M_x)}{\text{Var}(x)} \right]^2 =$$

$$= \frac{1}{n^2} \sigma^2 \sum_{i=1}^n \left[ 1 + \frac{M_x^2 (x_i - M_x)^2}{\text{Var}^2(x)} \right] \quad \begin{array}{l} \text{DOPPIO} \\ \text{PRODOTTO} \\ \text{NULLO} \end{array}$$

$$= \frac{1}{n} \sigma^2 \left[ 1 + \frac{M_x^2}{\text{Var}(x)} \right]$$

$$y^* = \beta_0 + \beta_1 x^* + e^* = \mu^* + e^*$$

(D)

$$\hat{y}^* = b_0 + b_1 x^*$$

$$E(\hat{y}^*) = \beta_0 + \beta_1 x^* = \mu^*$$

CORRETTA

Varianza

$$\hat{y}^* = \mu_y + b_1 (x^* - \mu_x)$$

$$= \mu_y + \frac{\sum (x_i - \mu_x) y_i}{n} (x^* - \mu_x)$$

$$= \frac{\sum y_i}{n} + (x^* - \mu_x) \frac{\sum (x_i - \mu_x) y_i}{n}$$

$$= \frac{1}{n} \sum y_i \left[ 1 + (x^* - \mu_x)(x_i - \mu_x) \right]$$

quindi (doppio prodotto nullo in  $[...]^2$ )

$$\text{Var}(y) = \frac{\sigma^2}{n} \left[ 1 + \frac{(x^* - \mu_x)^2}{\text{Var}(x)} \right]$$

Varianza residua

(±)

$$\sum_i (y_i - b_0 - b_1 x_i)^2 \cdot \frac{1}{n} =$$

$$\sum_i \left[ (y_i - \bar{y}) - b_1 (x_i - \bar{x}) \right]^2 \cdot \frac{1}{n} =$$

$$= \frac{\sum (y_i - \bar{y})^2}{n} + b_1^2 \frac{\sum (x_i - \bar{x})^2}{n} -$$

$$- 2 b_1 \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{n} =$$

$$= \text{Var}(y) - \frac{\text{Cov}(x, y)^2}{\text{Var}(x)} =$$

$$= \text{Var}(y) [1 - r^2]$$

RISULTATO UTILE X CONTI

$$s^2 = \frac{n}{n-2} \text{Var}(y) (1 - r^2)$$

$$M_{\hat{y}} = \frac{\sum \hat{y}_i}{n} = M_y + b_1 \frac{\sum (x_i - M_x)}{n} = M_y$$

$$\frac{\sum (y_i - M_{\hat{y}})^2}{n} = \frac{\sum (y_i - M_y)^2}{n}$$

$$= \frac{\sum (M_y + b_1(x_i - M_x) - M_y)^2}{n} =$$

$$= \frac{\text{Cov}(xy)^2}{\text{Var}(x)^2} \cdot \text{Var}(x) = \text{Var}(y) \cdot r^2$$

$$\frac{1}{n} \sum (y_i - \hat{y}_i)^2 = \text{Var}(y) (1 - r^2)$$

$$\frac{1}{n} \sum (\hat{y}_i - M_y)^2 = \text{Var}(y) r^2$$

$$\frac{1}{n} \sum (y_i - M_y) = r^2 \text{Var}(y)$$

Corneille 220

di  $\sigma^2$

(61)

$$E(\hat{\sigma}^2) = \sigma^2$$

WSD

$$\frac{\sum (y_i - M_y)^2}{n} = \frac{\sum (y_i - M_y)^2}{n} + \frac{\sum (y_i - \hat{y}_i)^2}{n}$$

$$y_i = \beta_0 + \beta_1 x_i + e_i$$

$$M_y = \beta_0 + \beta_1 M_x + \bar{e}$$

$$\bar{e} = \frac{\sum e_i}{n}$$

$$y_i - M_y = \beta_1 (x_i - M_x) + e_i - \bar{e}$$

$$\frac{\sum (y_i - M_y)^2}{n} = \beta_1^2 \text{Var}(x) + \frac{\sum (e_i - \bar{e})^2}{n} \cdot \sigma^2$$

$$+ 2 \beta_1 \sum_i (x_i - M_x)(e_i - \bar{e})$$

$$E \left[ \frac{\sum (y_i - M_y)^2}{n} \right] = \beta_1^2 \text{Var}(x) \cdot \frac{n-1}{n} \sigma^2 \begin{pmatrix} \text{USO} \\ \text{VALORE} \\ \text{ATTESO} \\ \sigma^2 \end{pmatrix}$$



$$\begin{aligned} \frac{\sum (y_i - M_y)}{n} &= \text{Var}(y) \cdot r^2 = \text{Var}(y) \cdot \frac{\text{Cov}^2(x, y)}{\text{Var}(x) \text{Var}(y)} = \text{Var}(x) b_1^2 \\ &= \text{Var}(x) b_1^2 \end{aligned}$$

$$E \left[ \frac{\sum (y_i - M_y)^2}{n} \right] = [\beta_1^2 + \text{Var}(b_1)] \text{Var}(x)$$

$$= \beta_1^2 \text{Var}(x) + \frac{\sigma^2}{n \text{Var}(x)} \text{Var}(x) =$$

$$= \beta_1^2 \text{Var}(x) + \frac{\sigma^2}{n}$$

PER DIFFERENZA

$$\begin{aligned} E \left( \frac{\sum (y_i - \hat{y}_i)^2}{n} \right) &= \beta_1^2 \text{Var}(x) + \frac{n-1}{n} \sigma^2 \\ &= \beta_1^2 \text{Var}(x) + \frac{\sigma^2}{n} \\ &= \frac{n-2}{n} \sigma^2 \end{aligned}$$

che implica  $E(s^2) = \sigma^2$