RISULTATI UTILI

A

$$Cov(x\,y) = \frac{\sum_{1}^{n} x_i y_i}{n} - \mu_x \mu_y =$$ (1)

$$= \frac{\sum (x_i - \mu_x) y_i}{n} =$$

$$= \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Var(x) = \frac{\sum (x_i - \mu_x)^2}{n} = \frac{\sum (x_i - \mu_x) x_i}{n}$$ (2)

$$r = \frac{Cov(x\,y)}{\sqrt{Var(x)\,Var(y)}}$$ (3)

$$\hat{y}_i = b_0 + b_1 x_i = \mu_y + b_1 (x_i - \mu_x)$$ (4)

$E(b_1)$ $\qquad$ $Var(b_1)$ $\qquad$ $\underline{\beta}$

$$b_1 = \frac{\sum (X_i - M_x) y_i \cdot \frac{1}{n}}{Var(x)}$$

$$E(b_1) = \frac{\sum (X_i - M_x)\left[\beta_0 + X_i \beta_1\right] \cdot \frac{1}{n}}{Var(x)} =$$

$$= \frac{\beta_0 \cdot \sum (X_i - M_x)/n}{Var(x)} + \beta_1 \frac{\sum (X_i - M_x) x_i \cdot \frac{1}{n}}{Var(X)}$$

$$= \beta_1$$

$$Var(\beta_1) = \frac{1}{n^2} \frac{\sum (X_i - M_x)^2 \sigma^2}{Var(X)^2} = \frac{\sigma^2}{n} / Var(X)$$

$$E(b_0)$$ (1

$$b_0 = M_y - b_1 M_x$$

$$E(b_0) = \frac{1}{n} \sum (\beta_0 + \beta_1 x_i) - \beta_1 M_x = \quad (*)$$

$$= \beta_0 + \beta_1 M_x - \beta_1 M_x = \beta_0$$

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$$(*) \quad M_y = \frac{\sum y_i}{n} = \beta_0 + \beta_1 M_x + \frac{\sum \varepsilon_i}{n}$$

$$E(M_y) = \beta_0 + \beta_1 M_x$$

$\cancel{E(b_0)}$ $\quad Var(b_0)$ C-2

$$b_0 = \frac{\sum_1^n y_i}{n} - M_x \frac{\sum_1^n (x_i - M_x) y_i}{n} \cdot \frac{1}{Var(x)} =$$

$$= \frac{1}{n} \sum_1^n y_i \left[1 - \frac{M_x (x_i - M_x)}{Var(x)} \right] =$$

$$Var(b_0) = \frac{1}{n^2} \sigma^2 \sum_1^n \left[1 - \frac{M_x (x_i - M_x)}{Var(x)} \right]^2 =$$

$$= \frac{1}{n^2} \sigma^2 \sum_1^n \left[1 + \frac{M_x^2 (x_i - M_x)^2}{Var^2(x)} \right] =$$ DOPPIO PROD NULLO

$$= \frac{1}{n} \sigma^2 \left[1 + \frac{M_x^2}{Var(x)} \right]$$

$$y^* = \beta_0 + \beta_1 x^* + e^* = \mu^* + e^*$$

D

$$\hat{y}^* = b_0 + b_1 x^* \qquad \mu^* = \beta_0 + \beta_1 x^*$$

CORRETTEZZA

$$E(\hat{y}^*) = \beta_0 + \beta_1 x^* = \mu^*$$

Varianza

$$\hat{y}^* = \mu_y + b_1 (x^* - \mu_x)$$

$$= \mu_y + \frac{\sum (x_i - \mu_x) y_i}{n \, Var(x)} (x^* - \mu_x)$$

$$= \frac{\sum y_i}{n} + (x^* - \mu_x) \frac{\sum (x_i - \mu_x) y_i}{n \, Var(x)}$$

$$= \frac{1}{n} \sum y_i \left[1 + \frac{(x^* - \mu_x)(x_i - \mu_x)}{Var(x)} \right]$$

quindi (doppio prodotto nullo in $[\dots]^2$)

$$Var(y) = \frac{\sigma^2}{n} \left[1 + \frac{(x^* - \mu_x)^2}{Var(x)} \right]$$

Varianza residua (E)

$$\sum_i \left(y_i - b_0 - b_1 x_i\right)^2 \cdot \frac{1}{n} =$$

$$\sum_i \left[\left(y_i - \mu_y\right) - b_1\left(x_i - \mu_x\right)\right]^2 \cdot \frac{1}{n} =$$

$$= \frac{\sum\left(y_i - \mu_y\right)^2}{n} + b_1^2 \frac{\sum\left(x_i - \mu_x\right)^2}{n} -$$

$$- 2 b_1 \frac{\sum\left(y_i - \mu_y\right)\left(x_i - \mu_x\right)}{n} =$$

$$= Var(y) - \frac{Cov^2(x, y)}{Var(x)} =$$

$$= Var(y)\left[1 - r^2\right]$$

RISULTATO UTILE X CONTI

$$\hat{s}^2 = \frac{n}{n-2} Var(y)\left(1 - r^2\right)$$

VARIANZA SPIEGATA

T 1

$$M_{\hat{y}} = \frac{\sum \hat{y}_i}{n} = M_y + b_1 \frac{\sum (x_i - M_x)}{n} = M_y$$

$$\frac{\sum (\hat{y}_i - M_{\hat{y}})^2}{n} = \frac{\sum_1^n (\hat{y}_i - M_y)^2}{n} =$$

$$= \frac{\sum \left(M_y + b_1(x_i - M_x) - M_y\right)^2}{n} =$$

$$= \frac{Cov(xy)^2}{Var(x)^2} \cdot Var(x) = Var(y) \cdot r^2$$

σ_y^2

$$\boxed{\begin{aligned} &\frac{1}{n} \sum (y_i - \hat{y}_i)^2 = \cancel{Var}(y)(1 - r^2) \\ &\frac{1}{n} \sum (\hat{y}_i - M_y)^2 = \cancel{Var}(y)\, r^2 \\ &\frac{1}{n} \sum (y_i - M_y) = \sigma_y^2 \; Var(y) \end{aligned}}$$

correttezza di $\hat{s}^2$ (61)

$$E(\hat{s}^2) = \sigma^2$$

USO

$$\frac{\sum (y_i - M_y)^2}{n} = \frac{\sum (y_i - M_y)^2}{n} + \frac{\sum (y_i - \hat{y}_i)^2}{n}$$

$$y_i = \beta_0 + \beta_1 X_i + e_i$$

$$M_y = \beta_0 + \beta_1 M_x + \bar{e} \qquad \bar{e} = \frac{\sum e_i}{n}$$

$$y_i - M_y = \beta_1 (X_i - M_x) + e_i - \bar{e}$$

$$\frac{\sum (y_i - M_y)^2}{n} = \beta_1^2 Var(X) + \frac{\sum (e_i - \bar{e})^2}{n \sigma^2} \cdot \sigma^2 + 2 \beta_1 \sum_{i}^{n} (X_i - M_x)(e_i - \bar{e})$$

$$E\left[\frac{\sum (y_i - M_y)^2}{n}\right] = \beta_1^2 Var(X) + \frac{n-1}{n} \sigma^2 \quad \left(\begin{array}{c}\text{USO} \\ \text{VALORE} \\ \text{ATTESO} \\ \chi^2\end{array}\right)$$

$$\frac{\sum(y_i - M_{\hat{y}})}{n} = Va(y) \cdot r^2 =$$

$$= Var(y) \cdot \frac{Cov^2(x,y)}{Va(x)\,Va(y)} = \sigma 2$$

$$= Var(x)\, b_1^2$$

$$E\left[\frac{\sum(y_i - M_y)^2}{n}\right] = \left[\beta_1^2 + Va(b_1)\right] Va(x)$$

$$= \beta_1^2\, Va(x) + \frac{\sigma^2}{n\, Var(x)}\, Va(x) =$$

$$= \beta_1^2\, Var(x) + \frac{\sigma^2}{n}$$

PER DIFFERENZA

$$E\left(\frac{\sum(y_i - \hat{y}_i)^2}{n}\right) = \beta_1^2\, Va(x) + \frac{n-1}{n}\sigma^2$$

$$- \beta_1^2\, Va(x) - \frac{\sigma^2}{n}$$

$$= \frac{n-2}{n}\sigma^2$$

che implica $E(\hat{s}^2) = \sigma^2$