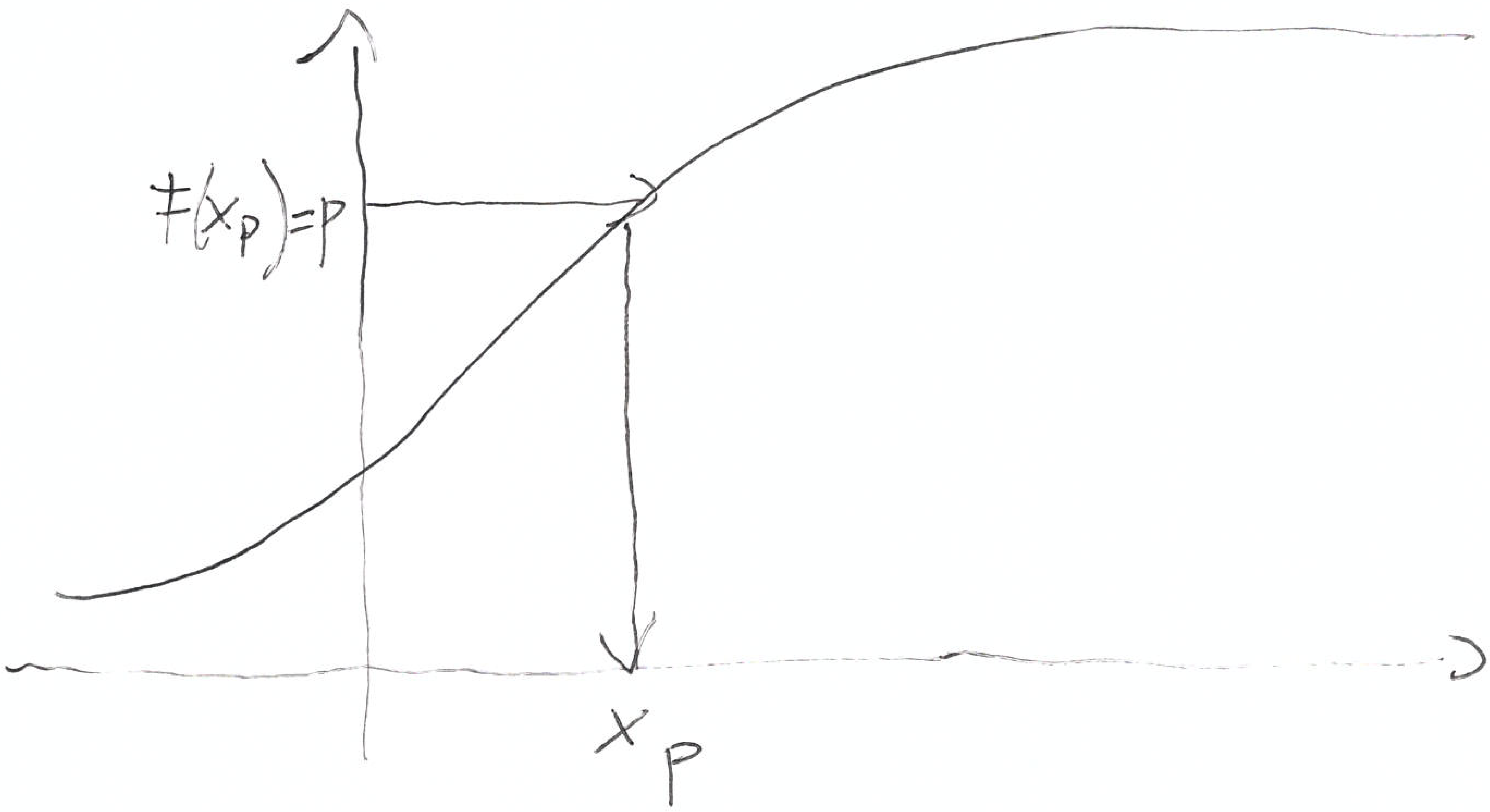
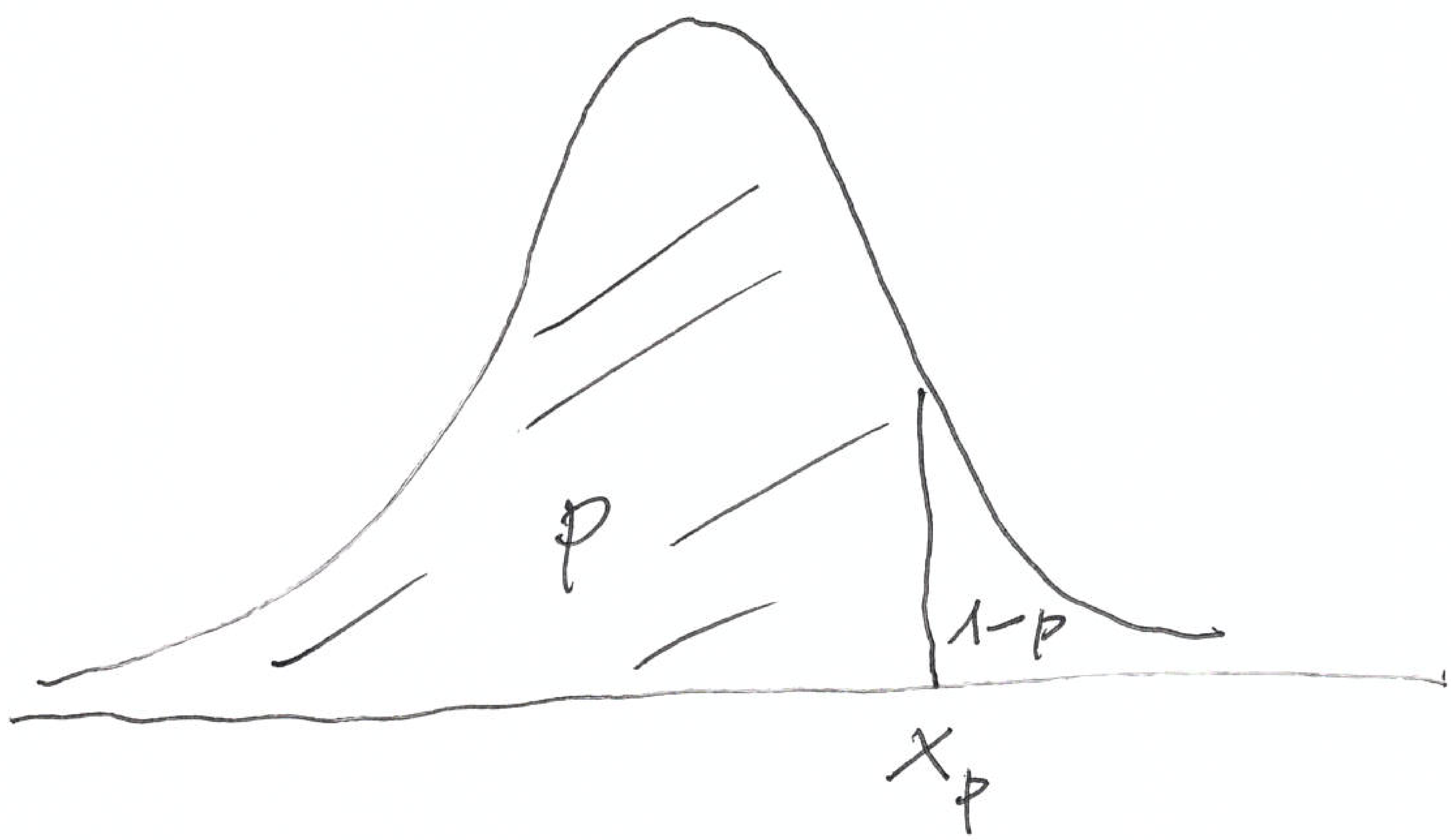
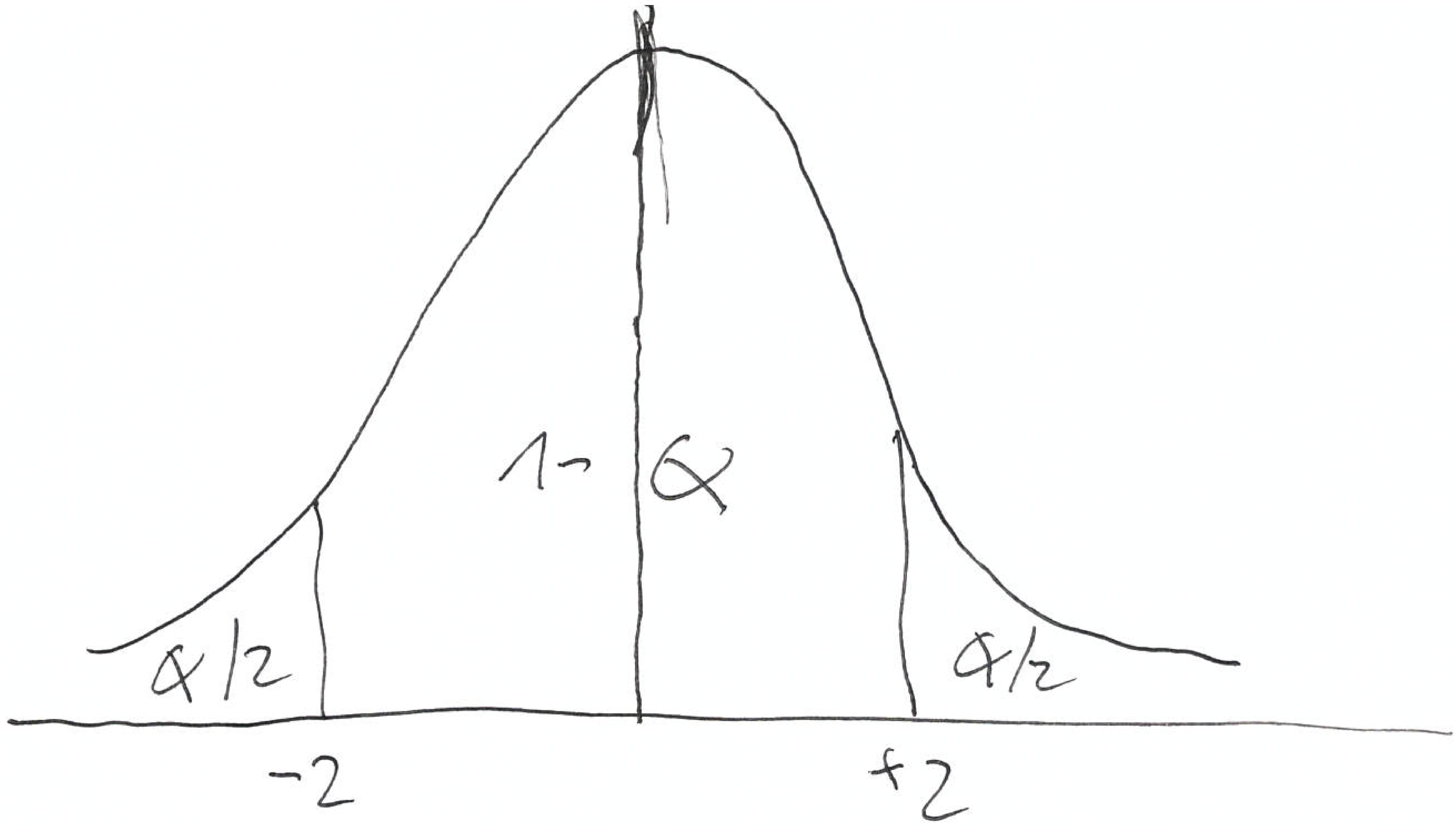
percentili



$$x_p \quad F(x_p) = p$$





$$z = z_{1-\alpha+\frac{\alpha}{2}} = z_{1-\alpha/2}$$

$$1-\alpha = 0.95$$

$$1-\alpha/2 = 0.975$$

$$z_{0.975} = 1.96$$

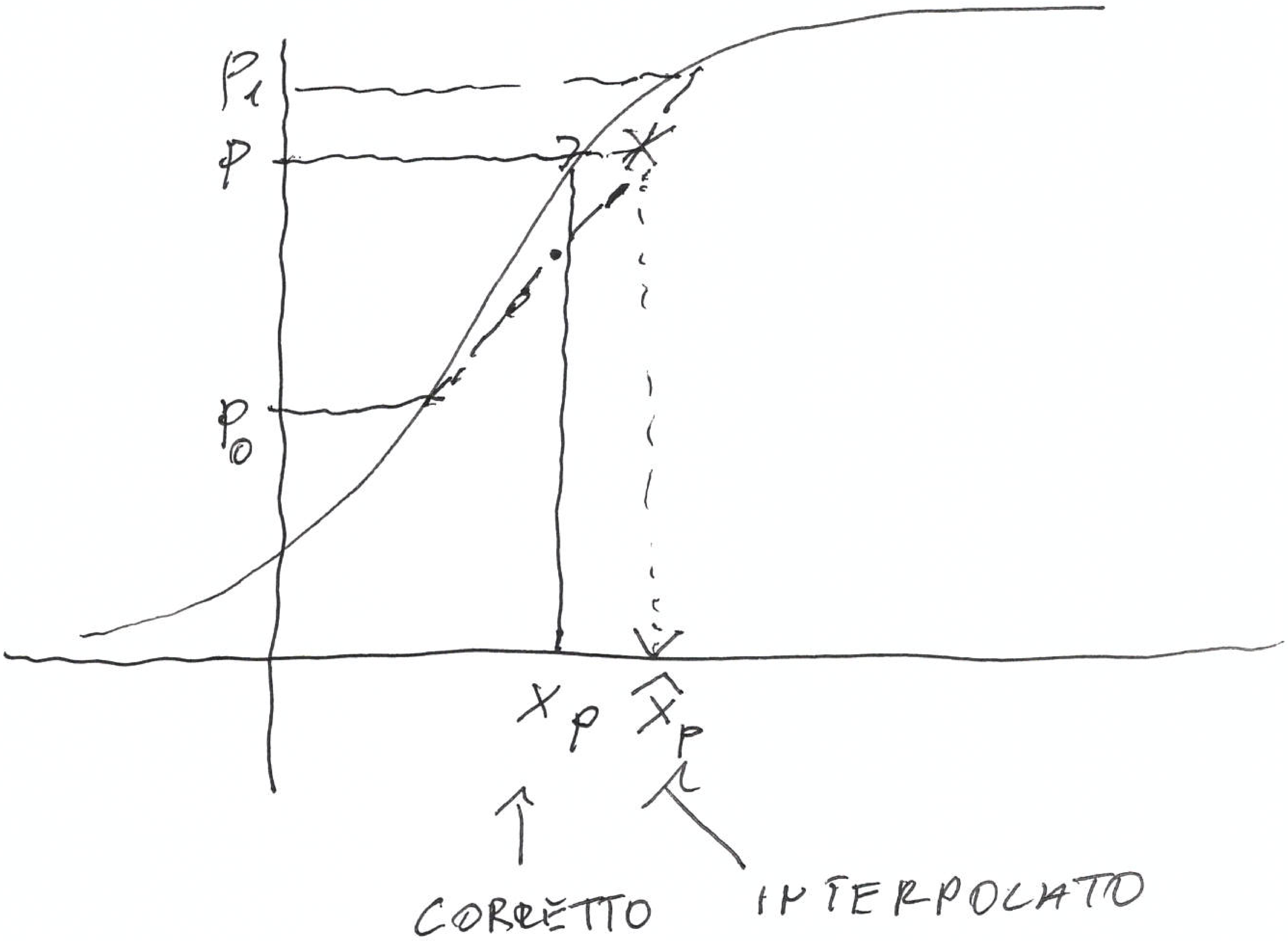
$$-z_{0.975} = z_{0.025} = -1.96$$

$$\mu = 5 \qquad \sigma^2 = 4$$

$$x_{0.975} = 5 + 2 * 1.96$$

$$x_{0.025} = 5 - 2 * 1.96$$

$$P(x_{0.025} \leq X \leq x_{0.975}) = 0.95$$



$$z_{0.975} = 1.96 \quad \text{(sulle tavole)}$$

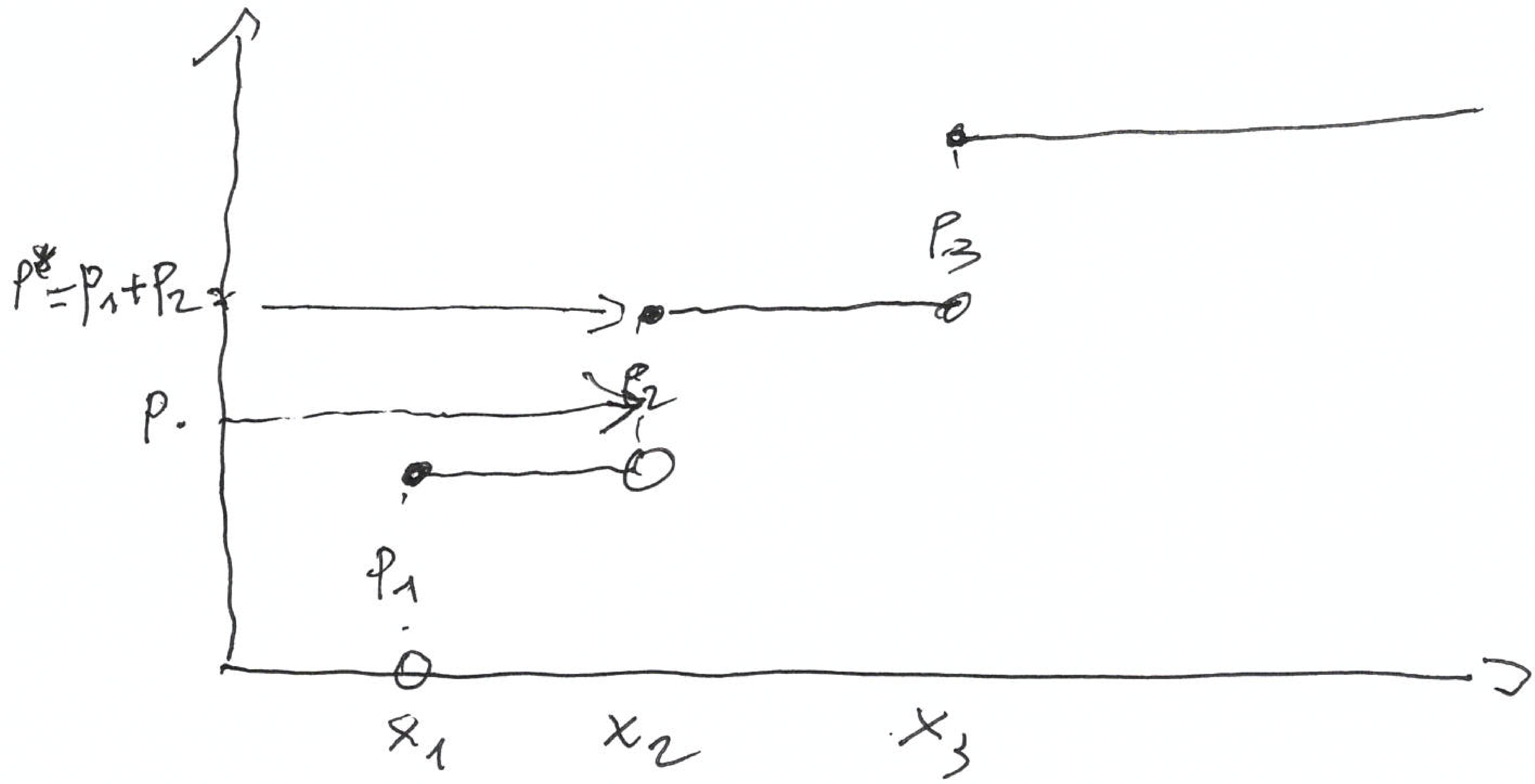
$$z_{0.95} = \frac{z_{0.9495} + z_{0.9505}}{2} =$$

$$= \frac{1.64 + 1.65}{2} = 1.645$$

con R $z_{0.95} = 1.6449$

qnorm(0.95, 0, 1)

percentili v.c. discrete



x_p non esiste

x_{p^*} infinite soluzioni

Definizione generale

$$x_p = \inf\{x : F(x) \geq p\}$$

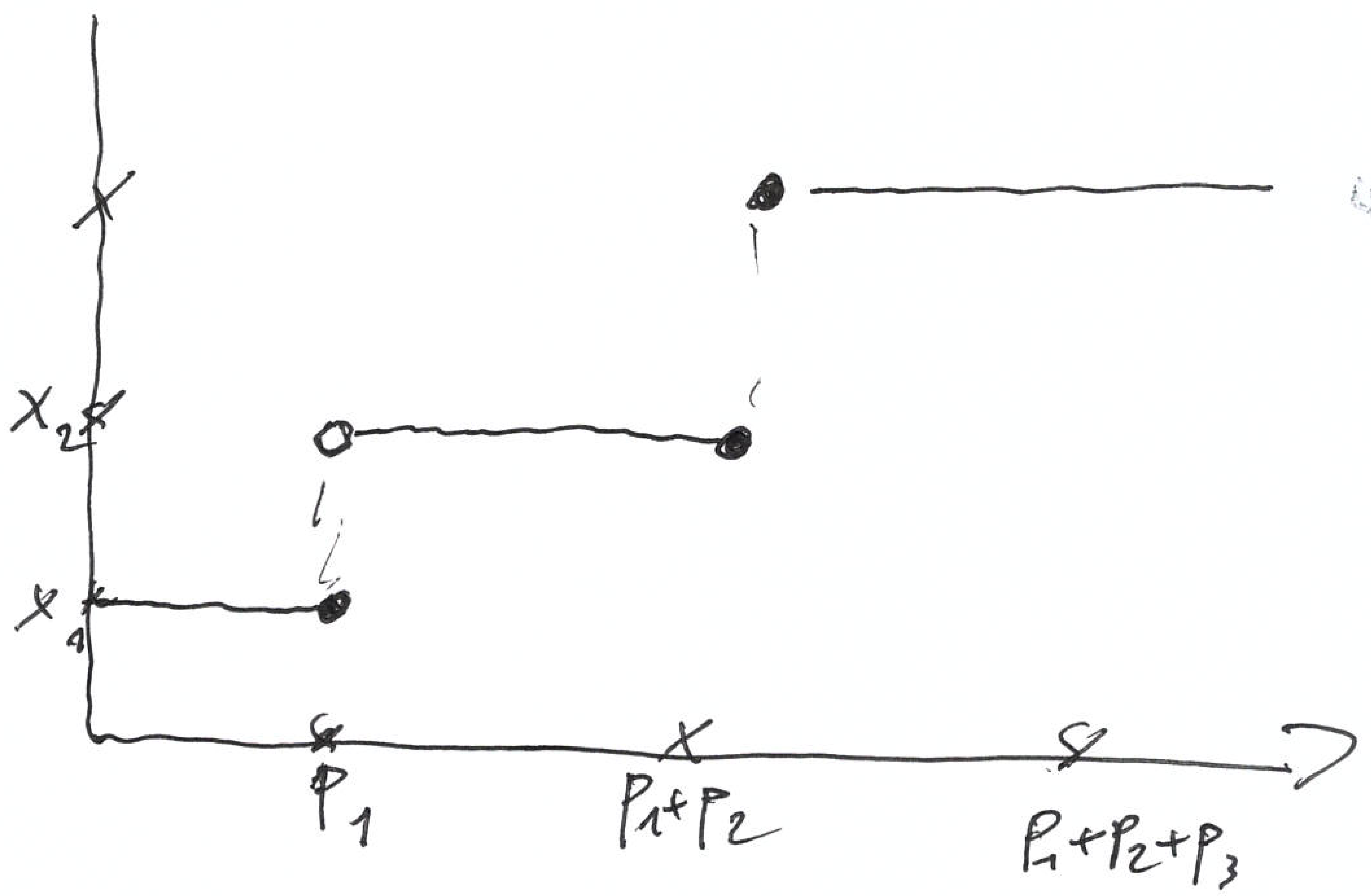
es: $x_p = x_{p^*} = x_2$

Funzione dei percentili

$$x_p = F^{-1}(p)$$

$$x_p = \inf\{x : F(x) \geq p\}$$

x	$F(x)$
x_1	p_1
x_2	p_2+p_1
x_3	$p_3+p_2+p_1$



Continua da simulare

Variabili casuali campionarie

$$\{X_1, X_2, \dots X_n\}$$

stessa $F(x)$

indipendenti

$$P(X_1 \le x_1, X_2 \le x_2, \dots, X_n \le x_n) =$$

$$= F(x_1)\,F(x_2) \dots F(x_n)$$

Somme Campionarie

$$S_n = X_1 + X_2 + \dots + X_n$$

X_i esponenziali $\Rightarrow$ S_n Gamma

λ $\qquad$ $\alpha = n, \lambda$

X_i normali $\Rightarrow$ S_n normale

μ, σ^2 $\qquad$ $n\mu, n\sigma^2$

X_i binomiali $\Rightarrow$ S_n binomiale

n_i, π $\qquad$ $\sum_{1}^{n} n_i, \pi$

$1 \quad \pi$ $\qquad$ $n \quad \pi$

Media Campionaria

$$M_n = \bar{X}_n = \frac{S_n}{n}$$

X_i normali $\Rightarrow M_n = \frac{S_n}{n}$ normale

μ, σ^2

$$S_n = n \cdot \mu + \sqrt{n}\, \sigma Z$$

$$M_n = \frac{S_n}{n} = \mu + \frac{\sigma}{\sqrt{n}} Z$$

$$E(M_n) = \mu \qquad Var(M_n) = \frac{\sigma^2}{n}$$

$$Z_n = \frac{M_n - \mu}{\sigma} \sqrt{n}$$ normale standard

Senza ipotesi
normalità

$$E(M_n) = E\left(\frac{1}{n}\sum_1^n X_i\right) = \frac{1}{n}\sum_1^n E(X_i) =$$

$$= \frac{1}{n} \times n \mu = \mu$$

$$Var(M_n) = Var\left(\frac{\sum_1^n X_i}{n}\right) = \frac{1}{n^2} Var\left(\sum_1^n X_i\right)$$

$$= \frac{1}{n^2}\sum_1^n Var(X_i) = \frac{1}{n^2} n \sigma^2 = \frac{\sigma^2}{n}$$

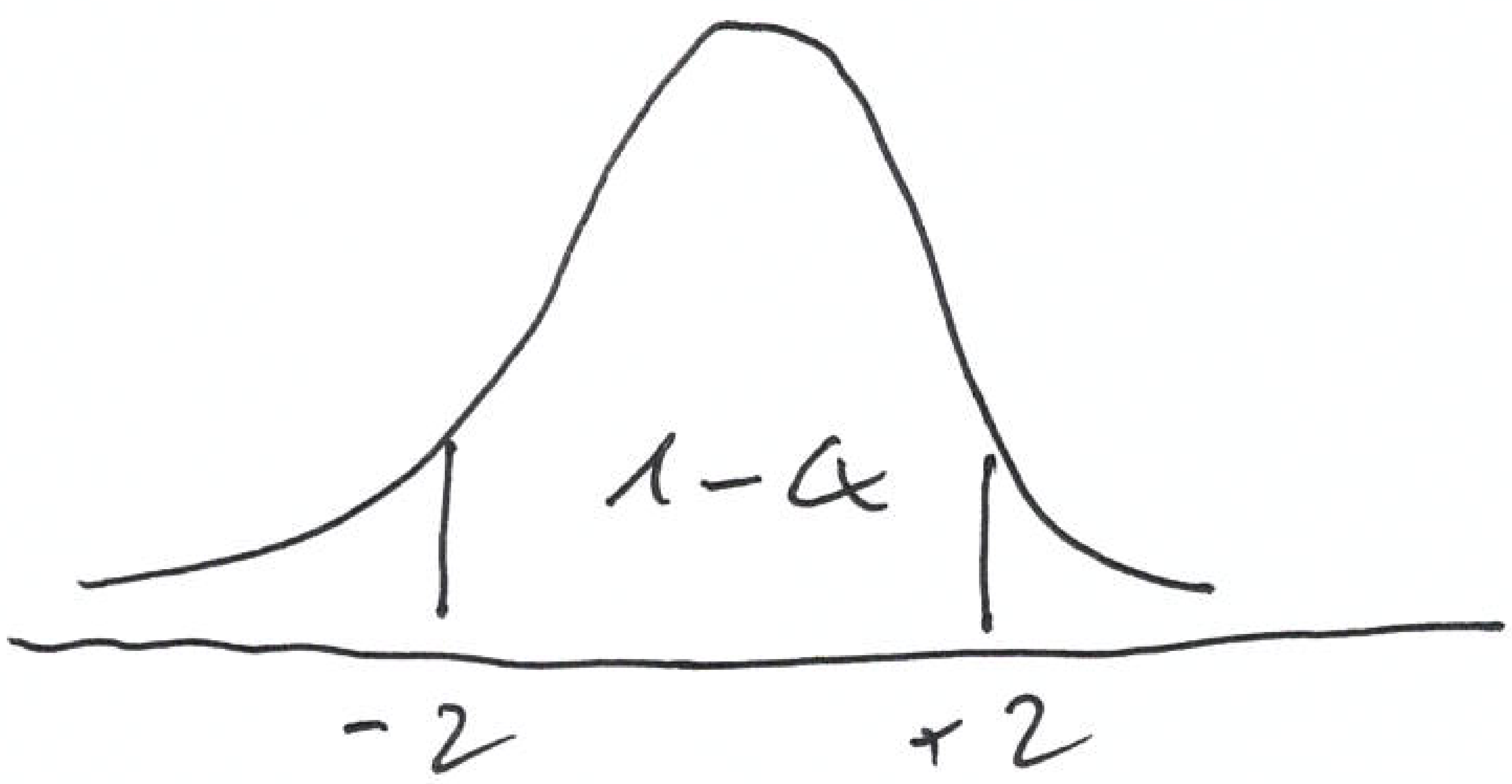
!!!

$$Z_n = \frac{M_n - \mu}{\sigma}\sqrt{n}$$

Teorema
normalità
asintotica

probleme

$$P\left(-2 \le \bar{Z}_n \le +2\right) \simeq 1-\alpha$$



$1-\alpha = 0.95$

$$-z_{0.975} \le \bar{Z}_n \le z_{0.975} \qquad (0.95)$$

$$-z_{0.975} \le \frac{\mu_n - \mu}{\sigma}\sqrt{n} \le z_{0.975} \qquad (0.95)$$

$$-z_{0.975} \times \frac{\sigma}{\sqrt{n}} \le \mu_n - \mu \le z_{0.975}\frac{\sigma}{\sqrt{n}} \qquad (0.95)$$

$$\boxed{\mu_n - z_{0.975}\frac{\sigma}{\sqrt{n}} \le \mu \le \mu_n + z_{0.975}\frac{\sigma}{\sqrt{n}}} \qquad (0.95)$$

Varianza Somma
in generale

$$Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(X,Y)$$

$$Cov(X,Y) = E(XY) - \mu_X \mu_Y$$