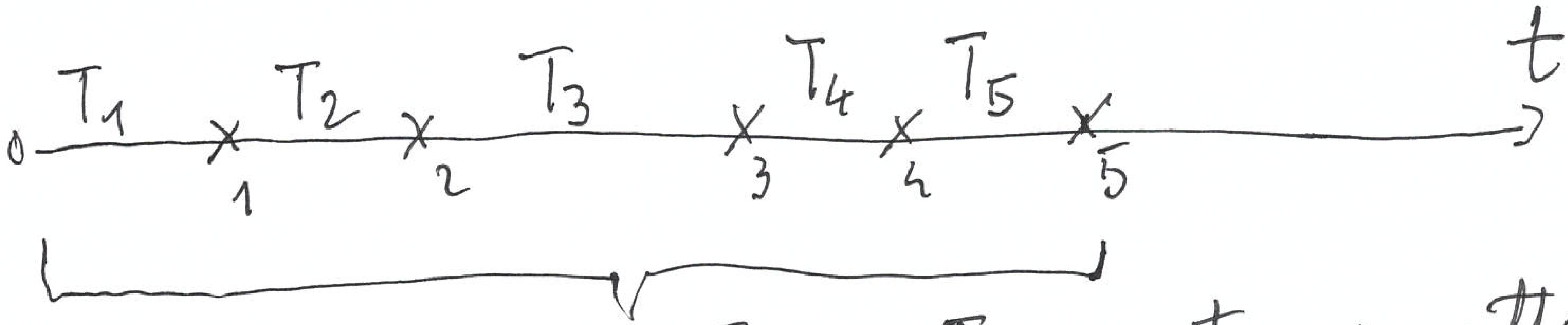
Relazione Poisson Gamma



$$S_5 = T_1 + T_2 + T_3 + T_4 + T_5$$ tempo atteso

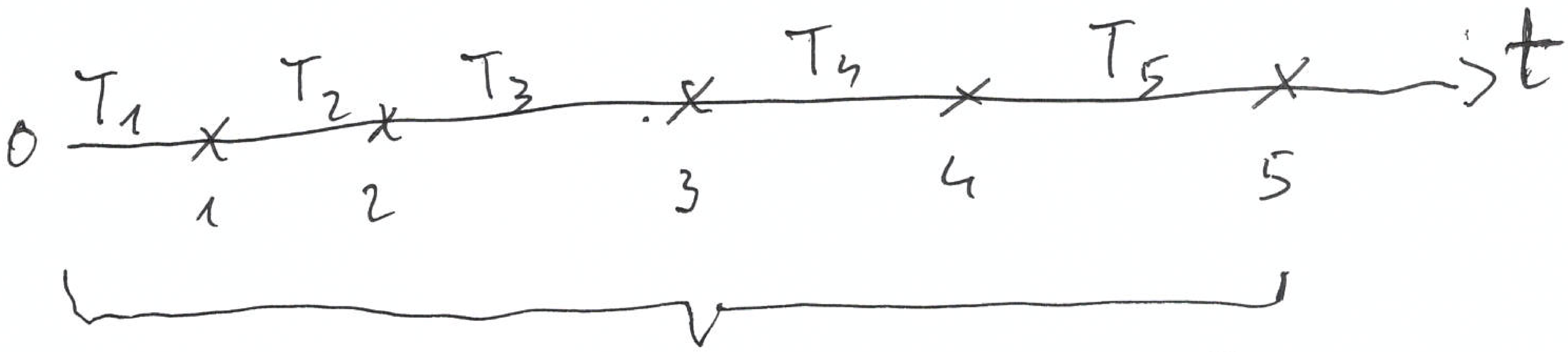
$$S_n = \sum_{i=1}^{n} T_i$$

RELAZIONE FONDAMENTALE

$$N(0,t) \geq n \iff S_n \leq t$$

$$N(0,t) \geq 5 \iff S_5 \leq t$$

Relazione Poisson Gamma



$$S_5 = T_1 + T_2 + T_3 + T_4 + T_5$$

$$S_1 = T_1$$

$$N(0,t) \geq 5 \iff S_5 \leq t$$

$$S_n = \sum_{i=1}^{n} T_i$$

$$S_n \leq t \iff N(0,t) \geq n$$

Funzione ripartizione S_n

$$F_{S_n}(t) = \sum_{j=n}^{\infty} e^{-\lambda t} \frac{(\lambda t)^j}{j!} = 1 - \sum_{j=0}^{n-1} e^{-\lambda t} \frac{(\lambda t)^j}{j!}$$

funzione di densità

$$f_{S_n}(t) = \frac{\partial F_{S_n}(t)}{\partial t} = \lambda^n t^{n-1} e^{-\lambda t} \cdot \frac{1}{(n-1)!} =$$

$$= \frac{\lambda^n}{\Gamma(n)} t^{n-1} e^{-\lambda t}$$

Gamma
$\alpha = n-1$
λ

$$E(S_n) = \frac{n}{\lambda} \qquad Var(S_n) = \frac{n}{\lambda^2}$$

Caso particolare $n = 1$ (esponenziale)

$S_1 = T_1$

$f_{S_1}(t) = \lambda e^{-\lambda t}$

$F_{S_1}(t) = 1 - e^{-\lambda t}$

$E(S_1) = \frac{1}{\lambda}$ $Var(S_1) = \frac{1}{\lambda^2}$

esercizi 2.10

arrivi a un pronto soccorso

Poisson $\lambda = 1$ (all'ora)

$$P(S_1 > 2) = 1 - (1 - e^{-2}) = e^{-2}$$

$$P(S_5 \leq 2) = 1 - \sum_{j=0}^{4} e^{-2} \frac{2^j}{j!} = 0.0526$$

$$E(S_{10}) = 10 \text{ ore}$$