Somme Poisson

X Poisson λ_A

Y Poisson λ_B

$S = X + Y$ Poisson $\lambda = \lambda_A + \lambda_B$

$$P(S=k) = P(X=0, Y=k) + P(X=1, Y=k-1) + \dots + P(X=j, Y=k-j) + \dots + P(X=k, Y=0)$$

$$P(S=k) = \sum_{j=0}^{k} \left(\frac{e^{-\lambda_A} \lambda_A^j}{j!} \right) \left(\frac{e^{-\lambda_B} \lambda_B^{k-j}}{(k-j)!} \right) =$$

$$= \frac{1}{\cancel{k}!} e^{-(\lambda_A + \lambda_B)} \sum_{j=0}^{k} \frac{\lambda_A^j \lambda_B^{k-j} \cdot \cancel{k}!}{j! \, (k-j)!} =$$

$$= \frac{1}{k!} e^{-(\lambda_A + \lambda_B)} \cdot \sum_{j=0}^{k} \binom{k}{j} \lambda_A^j \lambda_B^{k-j} =$$

$$= e^{-(\lambda_A+\lambda_B)} \cdot \frac{(\lambda_A+\lambda_B)^K}{K!}$$

che è Poisson $\lambda = \lambda_A + \lambda_B$

il risultato deriva da

$$(a+b)^K = \sum_{j=0}^{K} a^j \, b^{K-j} \cdot \binom{K}{j}$$

Esempio

$$(a+b)^2 = a^2 + b^2 + 2ab$$