11 - Esercizi Integrali indefiniti-1

Indicheremo con $G(x,c)$ la famiglia delle primitive al variare di $c \in R$.

1) $\int\left(x+2\sqrt{x}-\frac{1}{x^3}\right)dx = \int x dx + 2\int\sqrt{x}dx - \int\frac{1}{x^3}dx = \frac{1}{2}x^2+\frac{4}{3}x^{3/2}+\frac{1}{2x^2}+c$

$G(1,c)=\frac{1}{2}+\frac{4}{3}+\frac{1}{2}+c=0 \to c=-\frac{7}{3}$

$F(x)=\frac{1}{2}x^2+\frac{4}{3}x^{3/2}+\frac{1}{2x^2}-\frac{7}{3}$ **C.E.: $x>0$**

2) $\int\frac{x^4-2x}{x^3}dx=\int\frac{x^3-2}{x^2}dx=\int x dx - 2\int\frac{1}{x^2}dx=\frac{1}{2}x^2+\frac{2}{x}+c$

$G(1,c)=\frac{1}{2}+2+c=0 \to c=-\frac{5}{2}$

$F(x)=\frac{1}{2}x^2+\frac{2}{x}-\frac{5}{2}$ **C.E.: $x>0$**

3) $\int\left(e^2+4^x+\frac{5}{x^4}\right)dx=\int e^2dx+\int 4^x dx+\int\frac{5}{x^4}dx = xe^2+\frac{4^x}{\ln 4}-\frac{5}{3x^3}+c$

$G(1,c)=e^2+\frac{4}{\ln 4}-\frac{5}{3}+c=0 \to c=-\left(e^2+\frac{4}{\ln 4}-\frac{5}{3}\right)$

$F(x)=xe^2+\frac{4^x}{\ln 4}-\frac{5}{3x^3}-\left(e^2+\frac{4}{\ln 4}-\frac{5}{3}\right)$ **C.E.: $x>0$**

4) $\int\frac{3x^2+1}{1+x+x^3}dx=\ln\left|1+x+x^3\right|+c$

$G(1,c)=\ln 3+c=0 \to c=-\ln 3$

$F(x)=\ln(x^3-x+1)-\ln 3$ **C.E.**: $x>\alpha$ dove $-1<\alpha<0$ è la soluzione dell'equazione

$x^3+x+1=0 \Leftrightarrow x^3=-x-1$

5) $\int\frac{2e^{2x}+1}{e^{2x}+x}dx=\ln\left|e^{2x}+x\right|+c$

$G(1,c)=\ln\left(e^2+1\right)+c=0 \to c=-\ln\left(e^2+1\right)$

$F(x)=\ln(e^{2x}+x)-\ln\left(e^2+1\right)$ **C.E.**: $x>\alpha$ dove $-1<\alpha<0$ è la soluzione dell'equazione

$e^{2x}+x=0 \Leftrightarrow e^{2x}=-x$

6) $\int\left[(2x-1)^3+2x-1\right]dx=\int(2x-1)^3dx+x^2-x$ con sostituzione: $t=2x-1 \to dt=2dx$

$\frac{1}{2}\int t^3dt+x^2-x=\frac{1}{2}\int t^3dt+x^2-x=\frac{t^4}{8}+x^2-x+c=\frac{(2x-1)^4}{8}+x^2-x+c$

$G(1,c)=\frac{1}{8}+c=0 \Rightarrow c=-\frac{1}{8}$

$$F(x)=\frac{(2x-1)^4}{8}+x^2-x-\frac{1}{8}$$ **C.E.**: **R**

7) $\int 2x\sqrt{1-x^2}\,dx$ sostituzione: $t=1-x^2 \rightarrow dt=-2xdx$

$$-\int\sqrt{t}\,dt=-\frac{2}{3}t^{\frac{3}{2}}+c=-\frac{2}{3}(1-x^2)^{\frac{3}{2}}+c=-\frac{2}{3}\sqrt{(1-x^2)^3}+c$$

$$G(1,c)=c=0$$

$$F(x)=-\frac{2}{3}\sqrt{(1-x^2)^3}$$ **C.E.**: $-1\le x\le 1$

8) $\int\frac{1}{\sqrt{3x+2}}dx=\frac{1}{3}\int 3(3x+2)^{-1/2}dx=\frac{2}{3}\sqrt{3x+2}+c$

$$G(1,c)=\frac{2}{3}\sqrt{5}+c=0\rightarrow c=-\frac{2}{3}\sqrt{5}$$

$$F(x)=\frac{2}{3}\sqrt{3x+2}-\frac{2}{3}\sqrt{5}$$ **C.E.**: $x>-\frac{2}{3}$

9) $\int\frac{1}{e^x+e^{-x}}dx=\int\frac{1}{e^x+\frac{1}{e^x}}dx=\int\frac{1}{\frac{e^{2x}+1}{e^x}}dx=\int\frac{e^x}{e^{2x}+1}dx=\arctan e^x+c$

$$G(1,c)=\arctan e+c=0\rightarrow c=-\arctan e$$

$$F(x)=\arctan e^x-\arctan e$$ **C.E.**: **R**

10. $\int\frac{x^3}{x^2+3x+2}dx=$ si esegue la divisione in modo da ottenere un quoziente e un resto:

$$=\int(x-3)dx+\int\frac{(7x+6)}{x^2+3x+2}$$

$$\frac{7x+6}{x^2+3x+2}=\frac{A}{x+2}+\frac{B}{x+1}=\frac{(A+B)x+A+2B}{(x+2)(x+1)}\ \forall x\Leftrightarrow\begin{cases}A+B=7\\A+2B=6\end{cases}\Leftrightarrow\begin{cases}A=8\\B=-1\end{cases}$$

$$\int(x-3)dx+8\int\frac{1}{x+2}dx-\int\frac{1}{x+1}dx=\frac{1}{2}x^2-3x+8\ln|x+2|-\ln|x+1|+c=$$

$$=\frac{1}{2}x^2-3x+\ln\frac{(x+2)^8}{|x+1|}+c$$

$$G(1,c)=\frac{1}{2}-3+8\ln 3-\ln 2+c=0\rightarrow c=\frac{5}{2}-8\ln 3+\ln 2$$

$$F(x)=\frac{1}{2}x^2-3x+\ln\frac{(x+2)^8}{|x+1|}+\frac{5}{2}-8\ln 3+\ln 2$$ **C.E.: *x*>-1**

11. $\int\frac{1}{2x^2+3}dx=\frac{1}{3}\sqrt{\frac{3}{2}}\int\frac{1}{1+\left(\sqrt{\frac{2}{3}}x\right)^2}\cdot\sqrt{\frac{2}{3}}dx=\frac{1}{\sqrt{6}}\arctan\left(\sqrt{\frac{2}{3}}x\right)+c$

$$G(,c1)=\frac{1}{\sqrt{6}}\arctan\left(\sqrt{\frac{2}{3}}\right)+c=0\rightarrow c=-\frac{1}{\sqrt{6}}\arctan\left(\sqrt{\frac{2}{3}}\right)$$

$$F(x) = \frac{1}{\sqrt{6}}\arctan\left(\sqrt{\frac{2}{3}}x\right) - \frac{1}{\sqrt{6}}\arctan\left(\sqrt{\frac{2}{3}}\right)$$ **C.E.: R**

12. $\int \frac{2x^2}{x^2-4x+4}dx =$ si esegue la divisione in modo da ottenere un quoziente e un resto:

$$\int 2dx + 8\int \frac{x-1}{x^2-4x+4} = 2x + 8\int \frac{x-2+1}{(x-2)^2}dx = 2x + 8\int \frac{1}{x-2}dx + 8\int \frac{1}{(x-2)^2}dx =$$

$$= 2x + 8\ln|x-2| - \frac{8}{x-2} + c$$

$$G(1,c) = 2 + 0 + 8 + c = 0 \rightarrow c = -10$$

$$F(x) = 2x + 8\ln|x-2| - \frac{8}{x-2} - 10$$ **C.E.: *x*<2.**