MATEM. PER L'ECONOMIA LU 10/11

- LEZ. 6 EQ. DIFF. (VITALI)
- EQUAZ. LOGISTICA (WIKIPEDIA)

ESEMPI:

① $y' = ay$ LEGGE DI MALTHUS

E.D.O. LINEARE DEL 1° ORDINE A COEFF. COST. OMOG.

$\lambda = a$

$y^*(x) = c\, e^{ax}$

INFATTI

$y^{*\prime}(x) = c \cdot a \cdot e^{ax} = a\, y^*(x)$

② $y' = ay + b$

E.D.O. LIN. 1° A COEFF. COST. NON OMOG.

$y' = ay$ OMOG.

$y^*_{OM}(x) = c\, e^{ax}$

$\bar{y}(x) = K \quad \bar{y}' = 0$

$\Rightarrow 0 = a \cdot K + b \Rightarrow K = -\frac{b}{a}$

$y^*(x) = c\, e^{ax} - \frac{b}{a} \quad a \neq 0$ ①

③ $y' = a(x)\, y$

E.D.O. LIN. 1° A COEF. NON COST. OMOG.

$\Rightarrow$ SEPARO

$$\int \frac{dy}{y} = \int a(x)\, dx$$

$$\log|y| = \int a(x)\, dx$$

$$\Rightarrow y^* = c\, e^{\int a(x)\, dx}$$

④ $y' = a(x)\, y + b(x)$

E.D.O. LIN. 1° COEFF. NON COST. NON OMO GENEA

$$\Rightarrow y^* = e^{\int a(x)\, dx} \left[\int b(x) e^{-\int a(x) dx} dx + c \right]$$

⑤ $y' = y\,(a - by)$

LEGGE DI VERHULST

E.D.O. 1° A VAR. SEPARABILI

$$\int \frac{dy}{y(a-by)} = \int dx$$

2 CASI PARTICOLARI: $\bar{y} = 0$, $\bar{y} = \frac{a}{b}$ ②

SOLUZ. GENERALE:

RICERCA DELLE COST. A E B

$$\frac{1}{y(a-by)} = \frac{A}{y} + \frac{B}{a-by}$$

$$1 = A(a-by) + By$$

$$\begin{cases} B - Ab = 0 \\ Aa = 1 \end{cases} \qquad \begin{cases} B = \frac{b}{a} \\ A = \frac{1}{a} \end{cases}$$

$$\frac{1}{a}\log|y| + \frac{\cancel{b}}{a}\frac{\log(a-by)}{-\cancel{b}} = x + c$$

$$\frac{1}{a}\log\left|\frac{y}{a-by}\right| = x + c$$

$$\log\left|\frac{y}{a-by}\right| = ax + c$$

$$\frac{y}{a-by} = e^{ax}\cdot K$$

$$y = Ke^{ax}(a-by)$$

$$y + Ke^{ax}by = Kae^{ax}$$

$$y^* = \frac{Kae^{ax}}{1+Kbe^{ax}} = \frac{Ka}{e^{-ax}+Kb} \quad (3)$$

$$\frac{a}{\frac{1}{K}e^{-ax}+b}$$

COND. INIZIALE

$$y(0) = \frac{a}{b+K}$$

I CASO: $y(0) > \frac{a}{b} \Longrightarrow \frac{a}{b+k} > \frac{a}{b}$

$$\Longrightarrow \frac{1}{b+k} > \frac{1}{b} \Longrightarrow \boxed{K<0}$$

II CASO: $y(0) < \frac{a}{b} \Longrightarrow \frac{a}{b+k} < \frac{a}{b}$

$$\Longrightarrow \frac{1}{b+k} < \frac{1}{b} \Longrightarrow \boxed{K>0}$$

ES. $a = 2$, $b = 0,4$

$\frac{a}{b} = \frac{2}{0,4} = \frac{1}{0,2} = 5$ VAL. DI EQUIL.

$$y(0) = 7 \Longrightarrow \frac{2}{0,4+K} = 7 \Longrightarrow$$

$$2 = 2,8 + 7K \Longrightarrow -\frac{0,8}{7} = K < 0$$

$$\boxed{K \simeq -0,114}$$

ES. $a=2$, $b=0,4$, $\frac{a}{b}=5$ E $y(0)=1$

$$\Longrightarrow \frac{2}{0,4+K} = 1 \Longrightarrow 2 = 0,4+K \Longrightarrow \boxed{K = 1,6}$$

$$-\frac{u''}{u'} = \alpha \qquad \alpha = +3$$

$$-u'' = 3u'$$

$$+u'' + 3u' = 0 \qquad \text{E.D. } 2^\circ \text{ OMOG.}$$

$$u'' = -3u'$$

$$\int u''\,du = \int -3u'\,du$$

$$u' = -3u + c$$

$$u' + 3u = c$$

u
$u' > 0$
$u'' < 0$
CRESC.
CONC.

$$\lambda + 3 = 0$$

$$\lambda = -3$$

$$u^*_{om}(x) = K e^{-3x}$$

$$\bar{u}(x) = h \qquad \bar{u}' = 0$$

$$u^*(x) = K e^{-3x} + \frac{c}{3} \qquad K<0$$

$$x^2 y' + 2xy = 1$$

LIN. 1°
A COEFF. NON
COST, NON
OMOG.

$$x^2 y' + 2xy = 0$$

$$y' = -\frac{2\cancel{x}y}{x^{\cancel{2}}}$$

$$\frac{dy}{y} = \frac{-2}{x}\,dx$$

$$\int \frac{dy}{y} = \int -\frac{2}{x}\,dx$$

$$\log|y| = -2\ln|x| + c$$

$$y = e^{-2\ln|x|} \cdot e^c$$

$$y = |x|^{-2} K$$

$$y^*_{\text{он}} = \frac{K}{x^2}$$

$$\text{No}\begin{cases} \bar{y}(x) = q & \bar{y}' = 0 \\ x^2 \cdot 0 + 2x \cdot q = 1 \Rightarrow q = \frac{1}{x} \end{cases}$$

$$\bar{y}(x) = \frac{q}{x} \qquad \bar{y}' = -\frac{q}{x^2}$$

$$\cancel{x^2}\left(-\frac{q}{\cancel{x^2}}\right) + 2\cancel{x} \cdot \frac{q}{\cancel{x}} = 1$$

$$-q + 2q = 1$$

$$y^*(x) = \frac{K}{x^2} + \frac{1}{x}$$

$$x > 0 \qquad x \to +\infty \qquad \frac{K}{\infty} + \frac{1}{\infty} = 0$$

$$y(2) = 2y(1)$$

$$\frac{K}{4} + \frac{1}{2} = 2(K+1)$$

$$\frac{K}{4} - 2K = 2 - \frac{1}{2}$$

$$-\frac{7}{4}K = \frac{3}{2} \implies K = \frac{3}{\cancel{2}}\left(-\frac{\overset{2}{\cancel{4}}}{7}\right)$$

$$K = -\frac{6}{7}$$