MATEM. PER L'ECON. 12ª MA 11/11

- APPLICAZIONI DI E.D.O.

IN UN MERCATO L'ECCESSO DI DOMANDA DI UN BENE PRODOTTO VIENE RAPPRESENTATO COME DIFFERENZA FRA DOMANDA E OFFERTA, ENTRAMBI FUNZ. DEL PREZZO P

$$E(p) = D(p) - S(p)$$

$p \in [0, +\infty)$ $\quad t \in [0, +\infty)$

$p(t)$ DERIV. CON

$$p'(t) = K\, E(p)$$

a) $E(p) = b - ap \qquad a, b > 0$

$$\Longrightarrow p'(t) + Kap = Kb$$

$$p(t) = \frac{b}{a} + c\, e^{-Kat} \qquad c \in \mathbb{R}$$

con $p_e = \frac{b}{a}$ prezzo di equilibrio

COND. INIZ. $p(0)=p_0$

$$\Longrightarrow p(t)=p_e+(p_0-p_e)e^{-kat}$$

b) $E(p)=\dfrac{h}{p}-\ell \qquad h,\ell>0 \qquad p<c$

$$\Longrightarrow p'(t)=k\left(\frac{h}{p}-\ell\right)$$

RISOLVO

$$\frac{h}{\ell}\ln\left(\frac{h}{p}\right)-p=k\ell t+c$$

$c\in\mathbb{R}$

POSTO $\dfrac{h}{p}-\ell=0 \Longrightarrow p_e=\dfrac{h}{\ell}$

prezzo di equilibrio

• ES.

$w=w(t)>0$ VALORE DI UN INVEST. AL TEMPO t

$r(t)$ TASSO DI INTERESSE IN R.F.I.C.

$\Longrightarrow w'=r(t)\,w$ E.D. I LIN. OMOG.

$$\int \frac{dw}{w} = \int r(t)\,dt \implies$$

$$\ln w = R(t) + c$$

$$w(t) = c_1 e^{R(t)} \quad \text{dove } c_1 = e^c$$

COND. INIZ. $w(0) = w_0$

$$\implies w(t) = w_0 \, e^{R(t)-R(0)} =$$

$$= w_0 \, e^{\int_0^t r(t)\,dt}$$

- ES

SE IL CAPITALE COMPRENDE ANCHE UN DEPOSITO AL TASSO $y(t)$ E PRELIEVI AL TASSO $c(t)$

$$\implies w' = r(t)\,w + y(t) - c(t)$$

$$y' - Kh\frac{1}{\cancel{p}y} = -K$$

$$y' - Kh\frac{1}{y} = 0$$

$$y' \cdot y = Kh$$

$$\int y\,dy = \int Kh\,dx$$

$$\frac{y^2}{2} = Khx + c$$

$$\int p'(t) = \int \frac{Kh}{P} - K\ell$$

$$p(t) = Kh\log P - K\ell t + c$$

$$Kh\log P - P = K\ell t + c$$

$$E(P) = \frac{h}{P} - \ell = 0$$

$$P_e = \frac{h}{\ell} \quad \checkmark$$

⑨ $y'' + 4y' + 4y = 0$

$\lambda^2 + 4\lambda + 4 = 0$

$(\lambda + 2)^2 = 0$

$\lambda = -2 \quad m = 2$

$y^*(x) = c_1 e^{-2x} + c_2 x e^{-2x}$

$\begin{cases} y(0) = 3 \\ y'(1) = 0 \end{cases}$ COND. INIZIALI

$y(0) = c_1 + 0 = 3$

$y'(x) = -2c_1 e^{-2x} + c_2 \left(e^{-2x} + x e^{-2x}(-2)\right)$

$y'(1) = -2c_1 e^{-2} + c_2 (e^{-2} - 2e^{-2})$

$\begin{cases} c_1 = 3 \\ -6 e^{-2} + c_2 (-e^{-2}) = 0 \\ -6 - c_2 = 0 \end{cases} \quad c_2 = -6$

10) $\begin{cases} u'' - u = 1 \\ u'(0) = 1 \\ \lim\limits_{x \to +\infty} \dfrac{u(x)}{x} = 0 \end{cases}$

$$y'' - y = 0$$

$$\lambda^2 - 1 = 0 \quad \begin{matrix} \lambda_1 = 1 \\ \lambda_2 = -1 \end{matrix}$$

$$y^*_{OM}(x) = c_1 e^x + c_2 e^{-x}$$

$$\bar{y} = K \qquad -K = 1 \qquad K = -1$$

$$y^*(x) = c_1 e^x + c_2 e^{-x} - 1$$

$$y^*(0) = c_1 + c_2 - 1 = 3$$

$$y'(x) = c_1 e^x - c_2 e^{-x}$$

$$y'(0) = c_1 - c_2 = 1$$

$$\begin{cases} c_1 + c_2 = 4 \\ c_1 - c_2 = 1 \end{cases} \qquad \begin{cases} c_1 = \frac{5}{2} \\ c_2 = 4 - \frac{5}{2} = \\ = \frac{3}{2} \end{cases}$$

$$2c_1 = 5$$

5

ES. $u'' - u' = x$

$$\lambda^2 - \lambda = 0$$

$$\lambda(\lambda - 1) = 0$$

$$\lambda_1 = 0 \qquad \lambda_2 = 1$$

$$y^*_{OM}(x) = c_1 + c_2 e^x$$

$$\text{No}\begin{cases} \bar{y} = ax + b \\ \bar{y}' = a \qquad \bar{y}'' = 0 \end{cases}$$

$$\bar{y} = ax^2 + bx + c$$

$$\bar{y}' = 2ax + b$$

$$\bar{y}'' = 2a$$

$$2a - 2ax - b = x$$

$$\begin{cases} -2a = 1 \\ 2a - b = 0 \end{cases} \qquad \begin{cases} a = -\frac{1}{2} \\ -1 - b = 0 \\ b = -1 \end{cases}$$

6

$$y^*(x) = c_1 + c_2 e^x - \frac{1}{2}x^2 - x$$

ES. $y'' - y' = 5e^x$

$$y^*_{OM}(x) = c_1 + c_2 e^x$$

$$\bar{y} = (ax+b)e^x$$

$$\bar{y}' = a e^x + (ax+b)e^x$$

$$\bar{y}'' = a e^x + a e^x + (ax+b)e^x$$

$$2a e^x + (ax+b)e^x - a e^x - (ax+b)e^x = 5e^x$$

$$2a + \cancel{ax} + \cancel{b} - a - \cancel{ax} - \cancel{b} = 5$$

$$a = 5$$

$$y^*(x) = c_1 + c_2 e^x + 5x e^x$$

$\boxed{4}$

a) $\begin{cases} y'' - 3y' + 2y = 2x^3 + x \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

b) $\begin{cases} y'' - 3y' = 2x + 1 \\ y(0) = 1 \\ y'(0) = -5/9 \end{cases}$

c) $\begin{cases} y'' - y' - 6y = e^x \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

d) $\begin{cases} y'' - 3y' + 2y = x e^x \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

(d) $y'' - 3y' + 2y = 0$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} \left\langle \begin{matrix} 2 \\ 1 \end{matrix} \right.$$

$$y^*_{om}(x) = c_1 e^{2x} + c_2 e^{x}$$

$\bar{y} = Kxe^{x}$ No!

$$\bar{y}' = K(e^{x} + xe^{x})$$

$$\bar{y}'' = K(e^{x} + e^{x} + xe^{x})$$

$$K(2e^{x} + xe^{x}) - 3K(e^{x} + xe^{x}) + 2Kxe^{x} = xe^{x}$$

$$2K + \cancel{Kx} - 3K - \cancel{3Kx} + \cancel{2Kx} = \cancel{x}$$

$$\bar{y} = Kx^2 e^{x}$$

$$\bar{y}' = K(2xe^{x} + x^2 e^{x})$$

$$y'' = K(2e^x + 2xe^x + 2xe^x + x^2e^x)$$

SOSTITUISCO:

$$K(2e^x + 4xe^x + x^2e^x) - 3(K2xe^x + Kx^2e^x) + 2Kx^2e^x = xe^x$$

$$2K + 4Kx + \underline{Kx^2} - 6Kx \underline{-3Kx^2} + \underline{2Kx^2} = x$$

$$2K - 2Kx = x$$

$$\begin{cases} -2K = 1 \longrightarrow K = -\frac{1}{2} \\ 2K = 0 \longrightarrow K = 0 \end{cases}$$

No!

PROVO $\bar{y} = (ax^2 + bx + c)e^x$

$$\bar{y}' = (2ax + b)e^x + (ax^2 + bx + c)e^x$$

$$\bar{y}'' = 2ae^x + (2ax+b)e^x + (2ax+b)e^x + (ax^2+bx+c)e^x$$

$$2a + 2ax + b + 2ax + b + ax^2 + bx + c - 3(2ax + b + ax^2 + bx + c) + 2ax^2 + 2bx + 2c = x$$

1) $ax^2-3ax^2+2ax^2=0$

2) $\underline{2ax}+\underline{2ax}+\cancel{bx}\underline{-6ax}-3\cancel{bx}+2\cancel{bx}$

3) $2a+b+b+\cancel{c}-3b-3\cancel{c}+2\cancel{c}$

$$\begin{cases}-2a=1\\2a-b=0\end{cases}\qquad\begin{cases}a=-\frac{1}{2}\\-1-b=0\end{cases}\qquad\begin{cases}a=-\frac{1}{2}\\b=-1\end{cases}$$

$\forall c$ $\qquad \overline{y}=\left(-\frac{1}{2}x^2-x\right)e^x$ ok!