MATEMATICA PER L'ECONOMIA

5ª LEZ. (C.N. / C.S.) 20/10/'25

TEO 5.11 pag. 142 LIBRO

CONDIZ. (5.7) È

$$\det\begin{bmatrix} \frac{\partial f(\underline{x}_0)}{\partial x_1} & \frac{\partial f(\underline{x}_0)}{\partial x_2} \\ \frac{\partial g(\underline{x}_0)}{\partial x_1} & \frac{\partial g(\underline{x}_0)}{\partial x_2} \end{bmatrix} = 0$$

È EQUIVALENTE A

$$\begin{cases} \frac{\partial f}{\partial x_1}(\underline{x}_0) = \lambda \frac{\partial g}{\partial x_1}(\underline{x}_0) \\ \frac{\partial f}{\partial x_2}(\underline{x}_0) = \lambda \frac{\partial g}{\partial x_2}(\underline{x}_0) \end{cases}$$

DOVE $f(x_1, x_2)$ e $g(x_1, x_2) = b$

$\uparrow$ MAX/MIN

$$\begin{cases} \text{MAX } f(\underline{x}) \\ \underline{g}(\underline{x}) \leq \underline{b} \\ \underline{x} \geq \underline{0} \end{cases} \qquad \begin{cases} \text{MIN } f(\underline{x}) \\ \underline{g}(\underline{x}) \geq \underline{b} \\ \underline{x} \geq \underline{0} \end{cases}$$

ESTREMI VINCOLATI PER $f(\underline{x})$ CON VINCOLI DI DISUGUAGLIANZA

<u>ES.</u> DET. GLI ESTREMI ASS. DI

$$f(x_1,x_2)=x_1^3-x_1(x_2-1)^2$$

Con

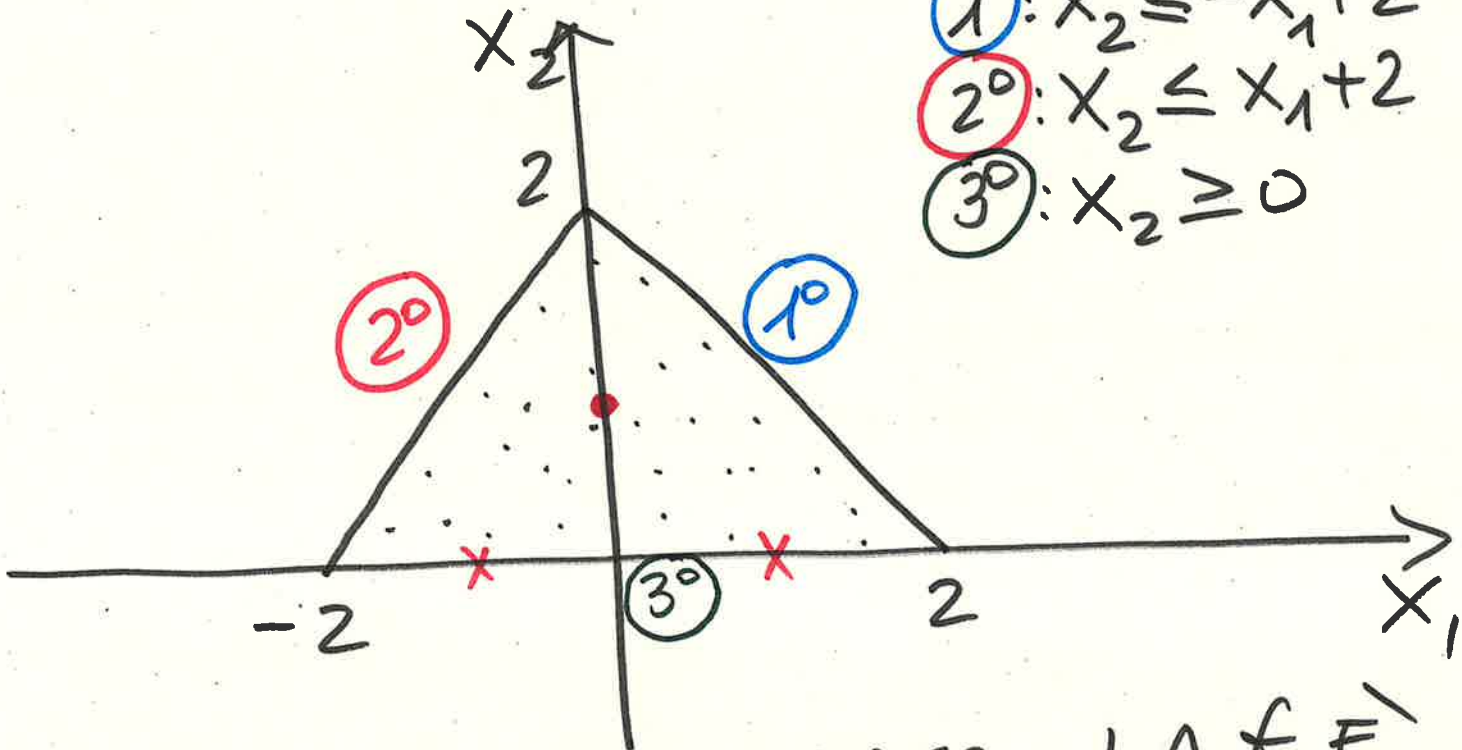
$$x_1+x_2-2\leq 0$$

$$-x_1+x_2-2\leq 0$$

$$x_2\geq 0$$

$n=2$ VAR.
$m=3$ VINC.

1°: $x_2\leq -x_1+2$
2°: $x_2\leq x_1+2$
3°: $x_2\geq 0$



<u>TEO DI WEIERSTRASS</u>: LA f È CONTINUA SU UNA R.A. CHIUSA E LIMITATA $\Longrightarrow$ f AMMETTE ALMENO UN P. DI MAX E 1 DI MIN ASSOLUTI!!!

RICERCO I MAX/MIN LIBERI

$$\begin{cases} \dfrac{\partial f}{\partial x_1} = 3x_1^2 - (x_2-1)^2 = 0 \\ \dfrac{\partial f}{\partial x_2} = -x_1 \cdot 2(x_2-1) = 0 \end{cases}$$

$$A \begin{cases} x_1 = 0 \\ x_2 = 1 \end{cases} \qquad B \begin{cases} x_2 = 1 \\ x_1 = 0 \end{cases}$$

$$(0,1) \qquad H = \begin{bmatrix} 6x_1 & -2(x_2-1) \\ -2(x_2-1) & -2x_1 \end{bmatrix}$$

$$H(0,1) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ NULLA!!

STUDIO LA FRONTIERA:

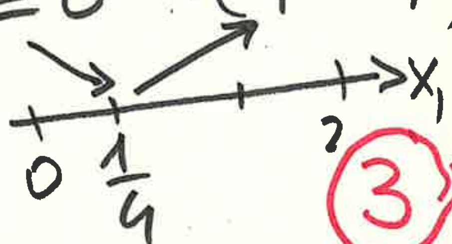
$\boxed{1^\circ}$ $x_2 = -x_1 + 2 \qquad 0 \leq x \leq 2$

$$f(x_1, -x_1+2) = x_1^3 - x_1(-x_1+2-1)^2 =$$
$$= x_1^3 - x_1(-x_1+1)^2$$

$(x_1^2 - 2x_1 + 1)$

$$f' = 3x_1^2 - 3x_1^2 + 4x_1 - 1 \geq 0$$

$$4x_1 \geq 1 \qquad x_1 \geq \frac{1}{4}$$

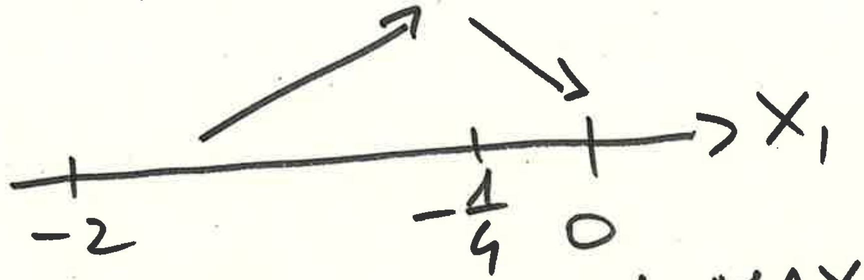


$x_1 = 0$ MAX $\quad x_1 = \frac{1}{4}$ MIN $\quad x_1 = 2$ MAX

$\boxed{2^\circ}$ $x_2 = x_1 + 2$

$$f(x_1, x_1+2) = x_1^3 - x_1(x_1+2-1)^2 =$$
$$= x_1^3 - x_1(x_1^2 + 2x_1 + 1) =$$
$$= \cancel{x_1^3} - \cancel{x_1^3} - 2x_1^2 - x_1$$

$$f' = -4x_1 - 1 \geq 0 \qquad x_1 \leq -\frac{1}{4}$$

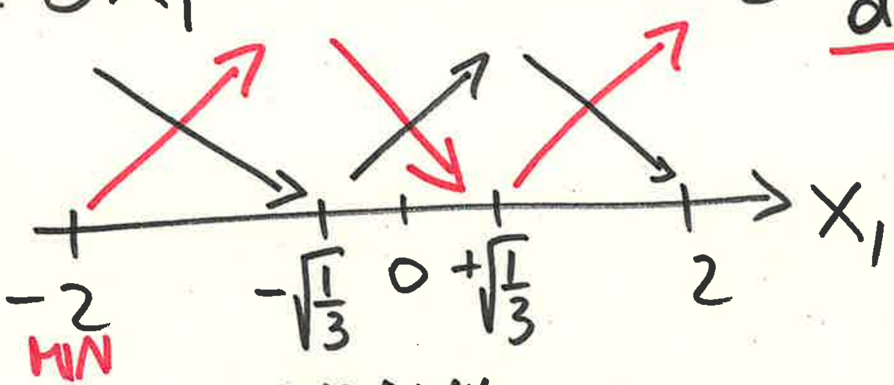


$x_1 = -2$ MIN, $x_1 = -\frac{1}{4}$ MAX, $x_1 = 0$ MIN

$\boxed{3^\circ}$ $x_2 = 0$ $\qquad f(x_1, 0) = x_1^3 - x_1$

$$f' = 3x_1^2 - 1 \geq 0 \qquad -\sqrt{\frac{1}{3}} \leq x_1 \leq +\sqrt{\frac{1}{3}}$$

decresce



$f(0,1) = 0$

$f\left(-\frac{1}{4}, \frac{7}{4}\right) = \frac{1}{8}$ MAX R.

$f\left(\frac{1}{4}, \frac{7}{4}\right) = -\frac{1}{8}$ MIN R.

$f\left(-\sqrt{\frac{1}{3}}, 0\right) = \frac{2}{3\sqrt{3}}$ R.

$f\left(\sqrt{\frac{1}{3}}, 0\right) = -\frac{2}{3\sqrt{3}}$ MIN R.

$f(-2,0) = -8+2(-1)^2$ MIN.R. $f=-6$ ASS.

$f(2,0) = 8-2(-1)^2$ MAX R. $f=6$ ASS.

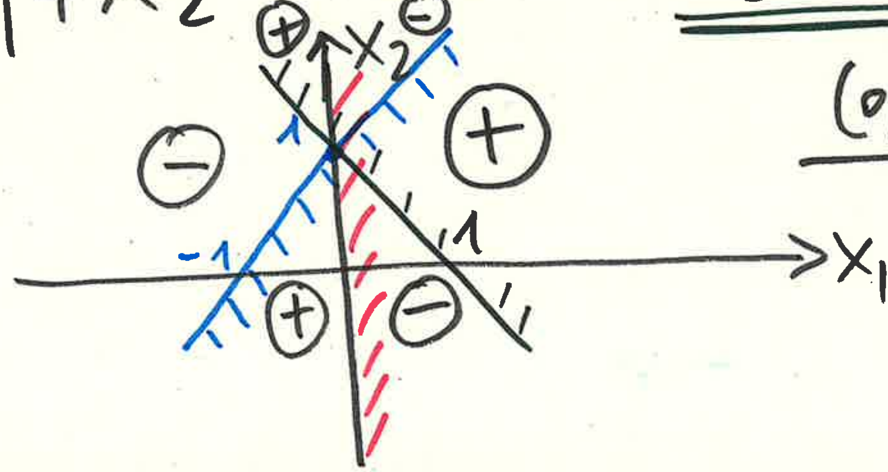
$$f(x_1,x_2) = x_1\left[x_1^2-(x_2-1)^2\right] \geq 0$$

1) $x_1 \geq 0$

2) $\left[x_1-(x_2-1)\right]\left[x_1+(x_2-1)\right] \geq 0$

2a) $x_1 - x_2 + 1 \geq 0$ $\quad x_2 \leq x_1+1$

2b) $x_1 + x_2 - 1 \geq 0$ $\quad x_2 \geq -x_1+1$



(0,1) SELLA

ES. SI CONSIDERINO I VINCOLI

$$g_1(\underline{x}) = x_1 = 0$$
$$g_2(\underline{x}) = x_2 = 0$$
$$g_3(\underline{x}) = x_1^2 + 2x_2^2 - x_2 - (x_3-1)^2 + 1 = 0$$

IL PUNTO $\underline{x}_0 = (0, 0, 2)$ È AMMISSIBILE : • $x_1 = 0$

$x_2 = 0$

$0 + 0 - 0 - 1 + 1 = 0$

$$\begin{bmatrix} \nabla g_1(\underline{x}_0) \\ \nabla g_2(\underline{x}_0) \\ \nabla g_3(\underline{x}_0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2x_1 & (+2x_2-1) & \cdot -2(x_3-1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & -2 \end{bmatrix} \quad r = 3$$

I VINCOLI SONO QUALIFICATI IN $\underline{x}_0$ POICHÈ LA MATRICE HA $r = 3$
$\Longrightarrow$ 3 VETT. LIN. INDIP.

ES.

$$\max f(x_1,x_2) = -2x_1 - \frac{1}{4}x_2^4 + 27x_2$$

$$x_1 \geq 0 \longrightarrow -x_1 \leq 0$$
$$x_2 \geq 0 \longrightarrow -x_2 \leq 0$$

$$L(x_1,x_2,\lambda_1,\lambda_2) = -2x_1 - \frac{1}{4}x_2^4 + 27x_2 + \lambda_1 x_1 + \lambda_2 x_2$$

$$\begin{cases} -2+\lambda_1 = 0 \\ -x_2^3 + 27 + \lambda_2 = 0 \\ -x_1 \leq 0 & \lambda_1 \cdot x_1 = 0 \\ -x_2 \leq 0 & \lambda_2 \cdot x_2 = 0 \\ \lambda_1, \lambda_2 \geq 0 \end{cases}$$

$$A \begin{cases} \lambda_1 = 2 \implies x_1 = 0 \\ \lambda_2 = 0 \\ x_2^3 = 27 \implies x_2 = 3 \end{cases} \qquad B \begin{cases} \lambda_1 = 2 \\ x_1 = 0 \\ x_2 = 0 \\ \lambda_2 = -27 \end{cases}$$

NO

$(0,3,2,0)$ UNICA SOL.

PUNTO MAX $(0,3)$

$$f(0,3) = -\frac{1}{4}\cdot 81 + 27\cdot 3 = \frac{-81+81\cdot 4}{4}$$

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__ES.__

$$f(x_1, x_2) = -5x_1^2 - 4x_2^2 - x_1 x_2 + 252x_1 + 180x_2 - 190$$

MAX

$$x_1 - 3x_2 \geq 0$$
$$x_1 \geq 0$$

vinc. 1

$$-x_1 + 3x_2 \leq 0$$
$$-x_1 \leq 0$$

$$L(x_1, x_2, \lambda_1, \lambda_2) = f(x_1, x_2) + \lambda_1(x_1 - 3x_2) + \lambda_2 x_1$$

$$\begin{cases} -10x_1 - x_2 + 252 + \lambda_1 + \lambda_2 = 0 \\ -8x_2 - x_1 + 180 - 3\lambda_1 = 0 \\ -x_1 + 3x_2 \leq 0 \\ -x_1 \leq 0 \\ \lambda_1 \geq 0 \\ \lambda_2 \geq 0 \end{cases}$$

- $\lambda_1(-x_1 + 3x_2) = 0$
- $\lambda_2 x_1 = 0$

1° CASO : $\lambda_1 = 0, \lambda_2 = 0$

2° CASO : $\lambda_1 = 0, \lambda_2 > 0$

3° CASO : $\lambda_1 > 0, \lambda_2 = 0$

4° CASO : $\lambda_1 > 0, \lambda_2 > 0$

UNICA SOLUZ.

$$\begin{cases} x_1 = 27 \\ x_2 = 9 \\ \lambda_1 = 27 \\ \lambda_2 = 0 \end{cases}$$

$$\overline{H} = \begin{bmatrix} 0 & -1 & 3 \\ -1 & -10 & -1 \\ 3 & -1 & -8 \end{bmatrix}$$

$$\det \overline{H} > 0 \implies \text{MAX R.}$$

$$\begin{bmatrix} -10 & -1 \\ -1 & -8 \end{bmatrix} = H_L$$

$$\text{MIN } \sigma_p^2 = \underline{x}^T V \underline{x}$$

$$\underline{x}^T \cdot \underline{1} = 1$$

$$\underline{x}^T \cdot \underline{m} = m^*$$

$$\underline{x} \geq \underline{0}$$