MAT. PER L'ECON. LEZ 8a

MA 28/10/25

1° TIPO E.D.O. I ORDINE

$$F(x, y(x), y'(x)) = 0$$

$$y'(x) = f(x, y(x))$$

ES. $y'(x) = x$ EDO I

$$\frac{dy}{dx} = x$$

$$dy = x\,dx$$

$$\int dy = \int x\,dx$$

$$y^*(x) = \frac{x^2}{2} + c \quad \text{1! c}$$

ES.

$$\begin{cases} y' = \ln x + 3 \\ y(1) = 3 \end{cases}$$

$$dy = (\ln x + 3)\,dx$$

$$y^* = x\ln x + x + 3x + c$$

$$y^*(x) = x\ln x + 4x + c$$

$$y(1) = 4 + c = 3$$

$$\boxed{c = -1}$$

$$y^*_{PART.}(x) = x\ln x + 4x - 1$$

2° TIPO E.D.O. A VAR. SEP.

$$y' = g(x)\cdot h(y)$$

$$\int \frac{1}{h(y)}\,dy = \int g(x)\,dx$$

Es. $y' = 2\sqrt{y}$ $\quad y \geq 0$ $\quad (y' \geq 0)$

1) $y = 0$ soluz. ✓

2) $y > 0$ $\quad \dfrac{dy}{\sqrt{y}} = 2\,dx$

$$\int y^{-\frac{1}{2}}\,dy = \int 2\,dx$$

$$y^{\frac{1}{2}} = 2x + c$$

$$y^*(x) = (x + c)^2$$

$x + c \geq 0$

$x > -c$

Es. $y' = \dfrac{y}{x}$ $\quad x \neq 0$

$$\frac{1}{y}\,dy = \frac{1}{x}\,dx \quad y \neq 0$$

1) $y = 0$ soluz. ✓

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\log|y| = \log|x| + c$$

$$e^{\log|y|} = e^{\log|x| + c}$$

$$|y| = |x| \cdot K$$

$$y^* = K \cdot x$$

FASCIO DI RETTE PASSANTI PER (0,0) ESCLUSA ASSE Y

$$y = 0$$

PART. $\bar{y}$

ES. $y' = \frac{2y}{x}$ $\quad x \neq 0$

ES. $y' = -\frac{x}{y}$ $\quad y \neq 0$

ES. $y' = -\frac{y}{x}$ $\quad x \neq 0$

ES. $y' = +\frac{x}{y}$ $\quad y \neq 0$

ES. $y' = ay(1-by)$ $a, b > 0$

EDO. I ORDINE A VAR. SEPAR.

EQUAZ. LOGISTICA

1) $y = 0$ ✓ $(K=0)$

2) $y = \frac{1}{b}$ ✓

3) $y \neq 0$ $y \neq \frac{1}{b}$

$$\int \frac{1}{y(1-by)}\, dy = \int a\, dx$$

$$\frac{1}{y(1-by)} = \frac{A}{y} + \frac{B}{1-by}$$

$$= \frac{A(1-by) + By}{y(1-by)}$$

$$1 = A - Aby + By$$

$$\begin{cases} B - Ab = 0 \\ A = 1 \end{cases} \qquad \begin{cases} A = 1 \\ B = b \end{cases}$$

$$\int \frac{1}{y}\,dy + \int \frac{b}{1-by}\,dy =$$

$$= \ln|y| + \cancel{b}\,\frac{\ln|1-by|}{\cancel{-b}} + \cancel{C}$$

$$= \ln \frac{|y|}{|1-by|}$$

$$\ln\left|\frac{y}{1-by}\right| = ax + C$$

$$\left|\frac{y}{1-by}\right| = e^{ax+C} = e^{ax} \cdot K$$

$$0 < y < \frac{1}{b} \qquad \frac{y}{1-by} = K e^{ax}$$

$$y = ke^{ax} - byke^{ax}$$

$$y + byke^{ax} = ke^{ax}$$

$$y(1 + bke^{ax}) = ke^{ax}$$

$$y^*(x) = \frac{ke^{ax}}{1 + bke^{ax}}$$

$$\frac{ke^{ax}}{1 + bke^{ax}} = \frac{1}{b}$$

$$bke^{ax} = 1 + bke^{ax}$$

$$\nexists k \in \mathbb{R}$$