MATEMATICA PER L'ECONOMIA

PARTE $\underline{\text{II}}$: FUNZ. A PIÙ VARIABILI ($n=2$, $n=3$)

LIBRO Vol. 6 CAP. 4

DEF. $X \subseteq \mathbb{R}^n$. UNA FUNZIONE f
$f : X \longrightarrow \mathbb{R}$ È UNA RELAZIONE CHE $\forall \underline{x} \in X \subseteq \mathbb{R}^n$
ASSOCIA UNO ED UNO SOLO (1!) NUMERO REALE z |

$$z = f(\underline{x}) = f(x_1, x_2, \dots, x_n)$$

$$\underline{x} \longrightarrow f(\underline{x})$$

X DOMINIO

$\mathbb{R}$ CODOMINIO

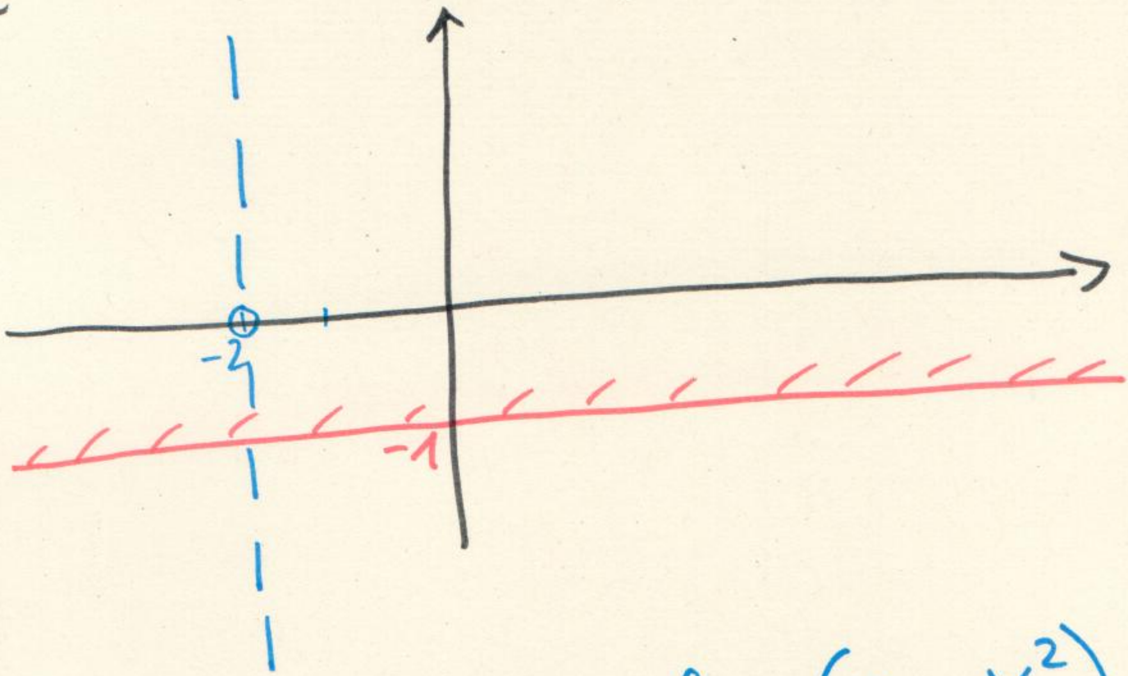
$\mathrm{Im}(f)$ INSIEME IMMAGINE $\subseteq \mathbb{R}$

$\boxed{1^a \text{ LEZ. LU } 6/10/2025}$

ES. 4.1 $\quad f(\underline{x}) = \frac{\sqrt{y+1}}{x+2}$

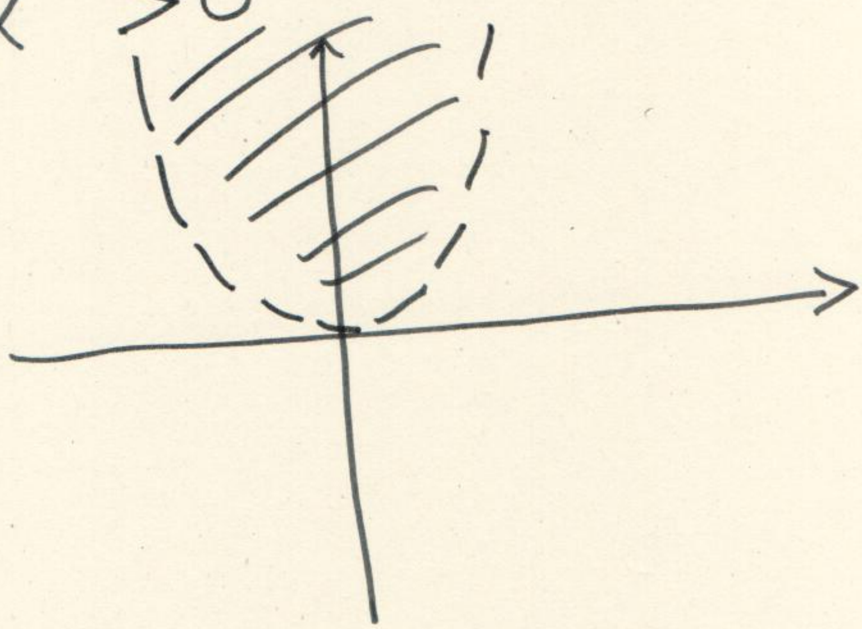
$x_1 = x$
$x_2 = y$

$$D: \begin{cases} y+1 \geq 0 \\ x+2 \neq 0 \end{cases} \quad \begin{cases} y \geq -1 \\ x \neq -2 \end{cases}$$



ES. 4.2 $\quad f(\underline{x}) = \log_3 (y - x^2)$

$D: \; y - x^2 > 0$



DEF. $f: X \to \mathbb{R}$, $X \subseteq \mathbb{R}^2$

$K \in \mathbb{R}$ COSTANTE, UNA CURVA DI LIVELLO È

$$C_K = \{ \underline{x} \in X \mid f(\underline{x}) = K \}$$

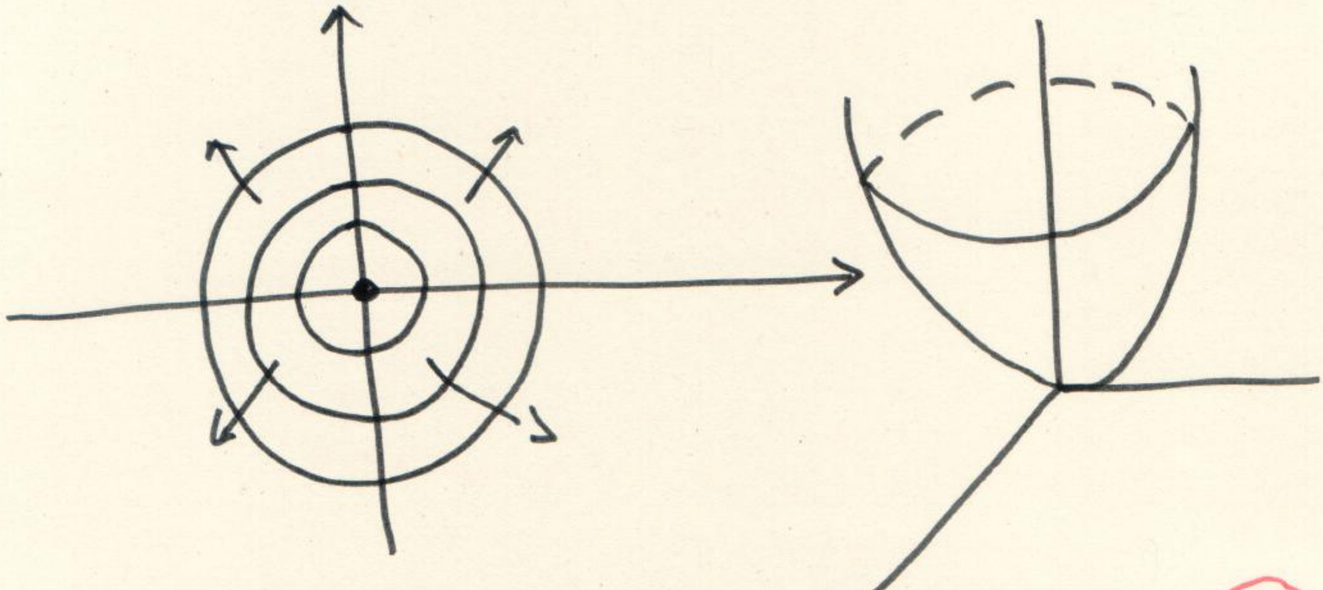
ES. $f(x,y) = x^2 + y^2$ PARABOLOIDE

$x^2 + y^2 = K$ 3 CASI

1) $K = 0 \Rightarrow x^2 + y^2 = 0 \Rightarrow (0,0)$

2) $K > 0 \Rightarrow x^2 + y^2 = K$ CIRCONF. DI CENTRO $(0,0)$ E RAGGIO $\sqrt{K}$

3) $K < 0 \Rightarrow \nexists$ c.d.l.



DEF. DI LIMITE IN 2 VARIABILI

$f: X \to \mathbb{R}$, $X \subseteq \mathbb{R}^n$

$\underline{x}_0$ p. di accumulazione per X

$\ell \in \mathbb{R}^* = \mathbb{R} \cup \{+\infty, -\infty\}$

$$\lim_{\underline{x} \to \underline{x}_0} f(\underline{x}) = \ell$$

SE $\forall\, I(\ell)\ \exists\, I(\underline{x}_0) \mid$

$\forall\, \underline{x} \in I(\underline{x}_0) \cap X$ CON $\underline{x} \neq \underline{x}_0$

$\implies f(\underline{x}) \in I(\ell)$

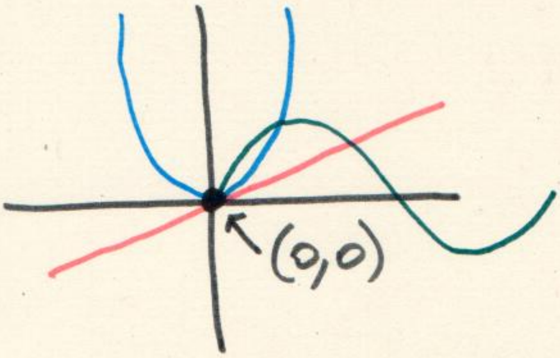
ES. 4.11 $\lim_{\underline{x} \to \underline{0}} \ln(1 + x^2 + y^2) = 0$

LA FUNZIONE È CONTINUA $\implies$

$x \to 0,\ y \to 0 \implies \ln(1) = 0$

IN QUANTI MODI, NEL PIANO, CI SI PUÒ AVVICINARE AL PUNTO (0,0)?

INFINITI



ES. 4.12 $\lim\limits_{\underline{x}\to\underline{0}} \frac{2xy}{x^2+y^2} = ?$

SE PRENDO I PUNTI DELL'ASSE Y CON IL $\lim\limits_{(x,y)\to(0,y)} \frac{2xy}{x^2+y^2} =$

CON $x=0$
$y\neq 0$

$$= \frac{2y\cdot 0}{y^2+0} = \frac{0}{y} = 0$$

SE PRENDO I PUNTI DELL'ASSE X CON $y=0$, $x\neq 0$ SI HA

$$\lim_{(x,y)\to(x,0)} \frac{2xy}{x^2+y^2} = \frac{2x\cdot 0}{x^2+0} = \frac{0}{x^2} = 0$$

MA SE PRENDO I PUNTI DELLA BISETTRICE $y=x$ SI HA

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim \frac{2x^2}{2x^2} = 1$$

DEF. DI CONTINUITÀ

$f: X \to \mathbb{R}$, $X \subseteq \mathbb{R}^n$

$\underline{x}_0$ punto di accumulazione di X

f è continua in $\underline{x}_0$ se

$$\lim_{\underline{x} \to \underline{x}_0} f(\underline{x}) = f(\underline{x}_0)$$

DEF. DI DERIVATE PARZIALI

$f: X \to \mathbb{R}$, $X \subseteq \mathbb{R}^n$, APERTO

$\underline{x}_0 \in X$, $n > 1$ (almeno 2 var.)

DERIVATE PARZIALI PRIME $(n=2)$

$$\frac{\partial f}{\partial x}(\underline{x}) = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$\frac{\partial f}{\partial y}(\underline{x}) = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

SE CI FOSSE LA 3ª VARIABILE:

$$\frac{\partial f}{\partial z}(\underline{x}) = \lim_{h \to 0} \frac{f(x, y, z+h) - f(x, y, z)}{h}$$

ESERCIZI

1) $f(x,y) = x + y$

$\frac{\partial f}{\partial x} = 1 \qquad \frac{\partial f}{\partial y} = 1$

2) $f(x,y) = x^2 + y^2$

$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = 2y$

3) $f(x,y) = x \cdot y$

$\frac{\partial f}{\partial x} = 1 \cdot y = y \qquad \frac{\partial f}{\partial y} = x \cdot 1 = x$

4) $f(x,y,z) = xz + 2y^2 z$

$\frac{\partial f}{\partial x} = z \qquad \frac{\partial f}{\partial y} = 4yz \qquad \frac{\partial f}{\partial z} = x + 2y^2$

5) $f(x,y) = (1+x)^y \leftarrow$ POTENZA RISP. X
ESPON. RISP. y

$\frac{\partial f}{\partial x} = y\,(1+x)^{y-1} \qquad \frac{\partial f}{\partial y} = (1+x)^y \ln(1+x)$

6) $f(x,y) = \sqrt{4 - x^2 - y^2}$

$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{4-x^2-y^2}} \cdot (-2x) \qquad \frac{\partial f}{\partial y} = \frac{1}{2\sqrt{}} (-2y)$

7) $f(x,y) = e^{\sqrt{x^2+y^2}}$

$$\frac{\partial f}{\partial x} = e^{\sqrt{x^2+y^2}} \cdot \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x$$

$$\frac{\partial f}{\partial y} = e^{\sqrt{}} \cdot \frac{1}{2\sqrt{}} \cdot 2y$$

E SE VOLESSIMO CALCOLARE TUTTE QUESTE DERIVATE IN UN PUNTO DEL DOMINIO (INTERNO)?

PER ESEMPIO LA 4) NEL PUNTO $(1,-2,3)$:

$$\frac{\partial f}{\partial x}(1,-2,3) = 3 \quad \frac{\partial f}{\partial y}(1,-2,3) = -24$$

$$\frac{\partial f}{\partial z}(1,-2,3) = 1+8 = 9$$

PER ES. LA 5) NEL PUNTO $(1,0)$

$$\frac{\partial f}{\partial x}(1,0) = 0 \cdot 2^{-1} = 0 \quad \frac{\partial f}{\partial y}(1,0) = 2^0 \ln 2 = \ln 2$$

E NEL PUNTO $(0,1)$

$$\frac{\partial f}{\partial x}(0,1) = 1 \cdot 1^0 = 1 \quad \frac{\partial f}{\partial y}(0,1) = 1^1 \ln 1 = 0$$

DERIVATE PARZIALI SECONDE:

$$z = f(x,y)$$

2 DER. PARZ. 1^e
4 DER. PARZ. 2^e

$$\begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$ VETTORE GRADIENTE

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$ 2X2 MATRICE HESSIANA

ES. RIFACCIAMO I 7 ES. CON LE DERIV. PARZ. 2^e

1) $\frac{\partial f}{\partial x} = 1$ $\quad \frac{\partial^2 f}{\partial x^2} = 0$ $\quad \frac{\partial^2 f}{\partial x \partial y} = 0$

$\frac{\partial f}{\partial y} = 1$ $\quad \frac{\partial^2 f}{\partial y^2} = 0$ $\quad \frac{\partial^2 f}{\partial y \partial x} = 0$

2) $\frac{\partial f}{\partial x} = 2x$ $\quad \frac{\partial^2 f}{\partial x^2} = 2$ $\quad \frac{\partial^2 f}{\partial x \partial y} = 0$

$\frac{\partial f}{\partial y} = 2y$ $\quad \frac{\partial^2 f}{\partial y^2} = 2$ $\quad \frac{\partial^2 f}{\partial y \partial x} = 0$

3) $\dfrac{\partial f}{\partial x} = y \qquad \dfrac{\partial^2 f}{\partial x^2} = 0 \qquad \dfrac{\partial^2 f}{\partial x \partial y} = 1$

$\dfrac{\partial f}{\partial y} = x \qquad \dfrac{\partial^2 f}{\partial y^2} = 0 \qquad \dfrac{\partial^2 f}{\partial y \partial x} = 1$

4) $\dfrac{\partial f}{\partial x} = z \qquad \dfrac{\partial f}{\partial y} = 4yz \qquad \dfrac{\partial f}{\partial z} = x + 2y^2$

$\dfrac{\partial^2 f}{\partial x^2} = 0 \qquad \dfrac{\partial^2 f}{\partial y^2} = 4z \qquad \dfrac{\partial^2 f}{\partial z^2} = 0$ PURE 3

$\dfrac{\partial^2 f}{\partial x \partial y} = 0 \qquad \dfrac{\partial^2 f}{\partial y \partial z} = 4y \qquad \dfrac{\partial^2 f}{\partial z \partial x} = 1$ MISTE 6

$$H = \begin{bmatrix} 0 & 0 & 4y \\ 0 & 4z & 1 \\ 4y & 1 & 0 \end{bmatrix} \quad 3 \times 3$$

5) $\dfrac{\partial^2 f}{\partial x^2} = y(y-1)(1+x)^{y-2} = (y^2 - y)(1+x)^{y-2}$

$\dfrac{\partial^2 f}{\partial y^2} = (1+x)^y \left[\ln(1+x)\right]^2$

$\dfrac{\partial^2 f}{\partial x \partial y} = (2y)(1+x)^{y-2} + (y^2 - y)(1+x)^{y-2} \cdot \ln(1+x)$

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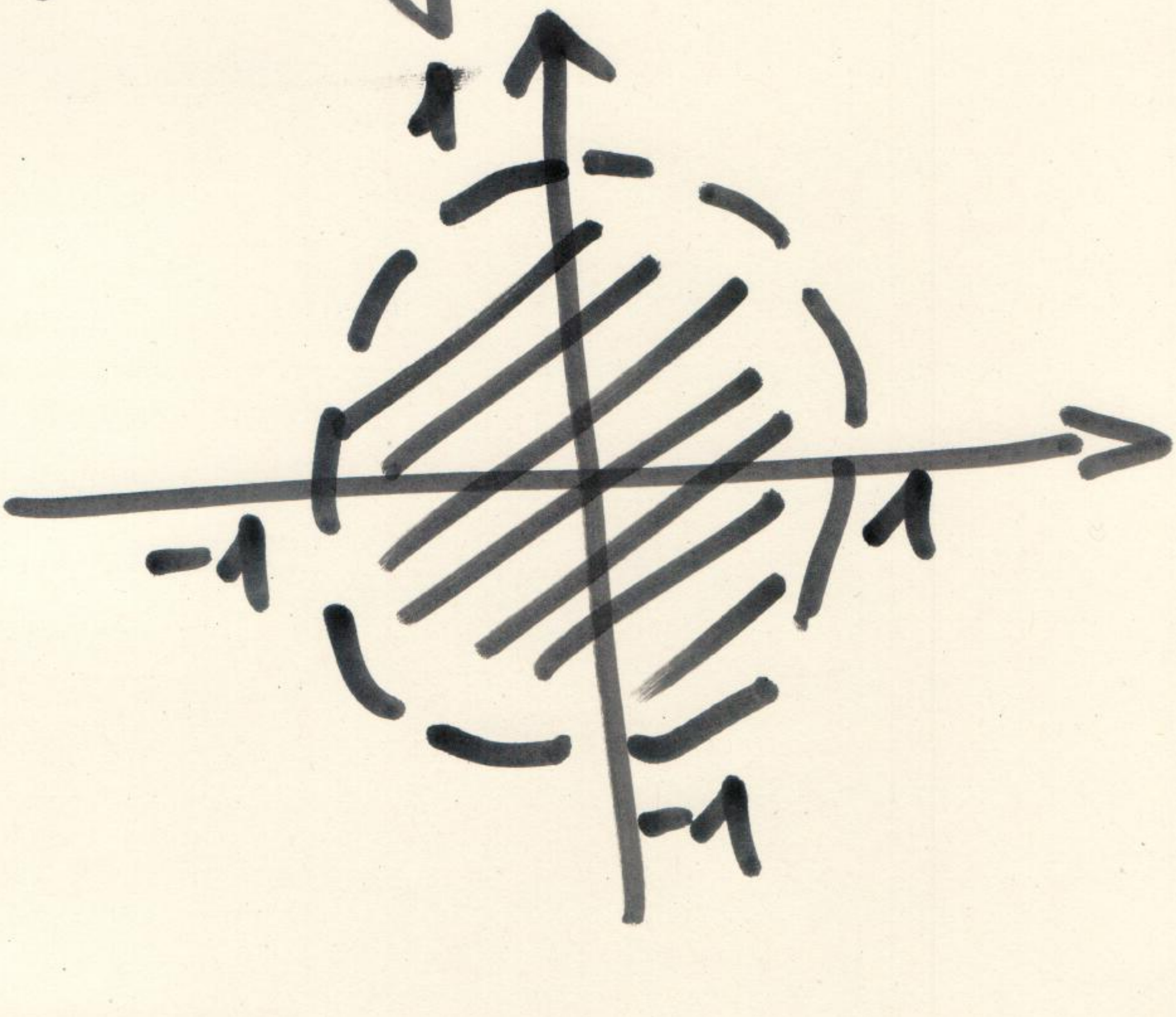
$$f(x,y)=\log_3(1-x^2-y^2)$$

$$1-x^2-y^2>0$$

$$x^2+y^2<1$$



$$\frac{\partial f}{\partial x} = \frac{1}{1-x^2-y^2}(-2x)\cdot \ln 3$$

$$\frac{\partial f}{\partial y} = \quad '' \quad (-2y)\cdot \ln 3$$

$$1^a = -2\ln 3\,\frac{x}{1-x^2-y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = -2\ln 3\,\frac{1-x^2-y^2+2x^2}{(\qquad)^2}$$

$$2^a = -2\ln 3\,\frac{y}{1-x^2-y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = -2\ln 3\,\frac{1-x^2-y^2+2y^2}{(\qquad)^2}$$

(9) $f(x,y) = e^{\frac{1}{x}} + e^{\frac{1}{y}}$

$$\frac{\partial f}{\partial x} = e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)$$

$$\frac{\partial f}{\partial y} = e^{\frac{1}{y}} \left(-\frac{1}{y^2}\right)$$

$$f(x,y) = e^{\frac{x}{y}}$$