MATEMATICA PER L'ECONOMIA 2ª LEZ. (7/10/'25)

DIFFERENZIALE

$X \subseteq \mathbb{R}^n \qquad f: X \to \mathbb{R}$

f è DIFFERENZIABILE IN $\underline{x}_0$

SE $\exists\, \underline{a} \in \mathbb{R}^n \mid$

$$f(\underline{x}_0 + \underline{h}) - f(\underline{x}_0) = \underline{a}^T \cdot \underline{h} + o(\|\underline{h}\|)$$

PER $\underline{h} \to \underline{0}$, $\forall \underline{h} \in \mathbb{R}^n$ t.c.

$\underline{x}_0 + \underline{h} \in X$

ERRORE

DIFFERENZIALE DI f IN $\underline{x}_0$ È L'APPLICAZIONE LINEARE

$$\underline{h} \longmapsto \underline{a}^T \cdot \underline{h}$$

SI INDICA CON $df(\underline{x}_0)$

SE $m=1$ $\quad f(x)=y$

$x_0 \in D$

$$f(x_0+h)-f(x_0)=ah+o(h)$$

$h \to 0$

SE $m=2$

$$f(\underline{x}_0+\underline{h})-f(\underline{x}_0)=(a_1, a_2)\cdot\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}+$$

SE $m=1$

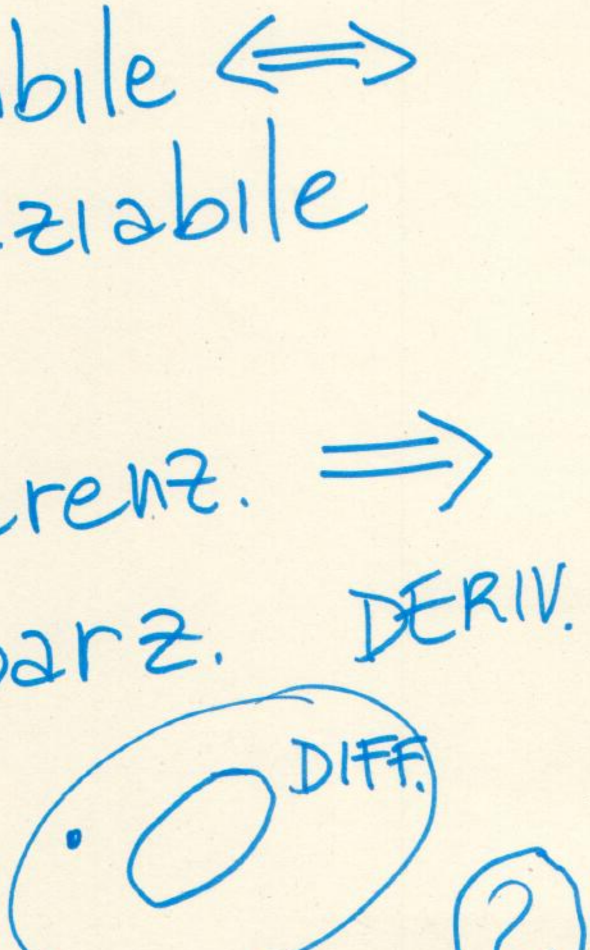
f è derivabile $\Longleftrightarrow$

f è differenziabile

SE $m=2$ $\Longrightarrow$

SE f è differenz. $\Longrightarrow$

f è deriv. parz.



$$\underline{a} = \nabla f(\underline{x}_0)$$ VETTORE GRAD.

$$\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y}$$

$$f(x_1, x_2) = (x_1 + x_2)^2 + x_2 + x_1 \cdot e^{3x_1}$$

$$\frac{\partial f}{\partial x_1} = 2(x_1 + x_2) + e^{3x_1} + x_1 \cdot e^{3x_1} \cdot 3$$

$$\frac{\partial f}{\partial x_2} = 2(x_1 + x_2) + 1$$

$$df(\underline{x}) = \frac{\partial f}{\partial x_1} \cdot dx_1 + \frac{\partial f}{\partial x_2} \cdot dx_2 =$$

$$= \underbrace{\nabla f(\underline{x})^T}_{\underline{a}} \underbrace{\begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix}}_{\underline{h}}$$

$$= \left[\quad 1^{a} \quad \right] dx_1 + \left[2^{a} \right] dx_2$$

ES. $f(x, y) = x^2 + xy$

$\underline{x}_0 = [1 \;\; 3] = (1, 3)$

$\frac{\partial f}{\partial x} = 2x + y$ $\qquad \frac{\partial f}{\partial x}(1,3) = 5$

$\frac{\partial f}{\partial y} = x$ $\qquad \frac{\partial f}{\partial y}(1,3) = 1$

$\nabla f(1,3) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$df(1,3) = 5 \cdot dx + 1 \cdot dy$

TEO DIFFER. TOTALE

$f : X \to \mathbb{R} \qquad X \subseteq \mathbb{R}^n$

X APERTO

SE in un $I(\underline{x}_0)$ f è deriv. parz. e le $\frac{\partial f}{\partial x_i}$ continue $\Longrightarrow$ f è diff. in $\underline{x}_0$

EQ. IPERPIANO TANG.

$$z - \cancel{}_{f(1,3)} = 5(x-1) + 1(y-3)$$

$$f(1,3) = 1 + 3 = 4$$

$$z - 4 = 5x - 5 + y - 3$$

$$z = 5x + y - 4 \quad \text{PIANO}$$

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$$f(x,y,z) = \ln(x+y+z)$$

$$\underline{x}_0 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x+y+z} \cdot 1 = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z}$$

$$\downarrow \quad \frac{1}{2} \qquad \frac{1}{2} \qquad \frac{1}{2}$$

$$df(1,2,-1) = \frac{1}{2}dx + \frac{1}{2}dy + \frac{1}{2}dz$$

$$f(1,2,-1) = \ln(2)$$

$$u - \ln(2) = \frac{1}{2}(x-1) + \frac{1}{2}(y-2) + \frac{1}{2}(z+1)$$

$$u = \frac{1}{2}x + \frac{1}{2}y + \frac{1}{2}z + \ln 2 - \cancel{\frac{1}{2}} - 1 + \cancel{\frac{1}{2}}$$

$$u = \frac{1}{2}x + \frac{1}{2}y + \frac{1}{2}z + \ln 2 - 1$$

SVILUPPO DI TAYLOR

$n = 2 \qquad f(x,y) \qquad (x_0,y_0) \in D$

$$p(x,y) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0) \cdot$$
$$\cdot (x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0) \cdot (y-y_0) +$$
$$+ \frac{1}{2} \frac{\partial^2 f}{\partial x^2}(x_0,y_0) (x-x_0)^2 +$$
$$+ \frac{1}{2} \frac{\partial^2 f}{\partial y^2}(x_0,y_0) (y-y_0)^2 +$$
$$+ \frac{\partial^2 f}{\partial x \partial y}(x_0,y_0) (x-x_0)(y-y_0)$$

$$f(x,y) = x^2 + xy$$

$\frac{\partial f}{\partial x}(1,3) = 5 \qquad \frac{\partial f}{\partial y}(1,3) = 1$

$f(1,3) = 1+3 = 4$

$\frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 1 \qquad \frac{\partial^2 f}{\partial y^2} = 0$

$$P(x,y) = 4 + 5(x-1) + 1\cdot(y-3) +$$
$$+ \frac{1}{2}\cdot 2\,(x-1)^2 + \frac{1}{2}\cdot 0\cdot (y-3)^2 +$$
$$+ 1\cdot (x-1)(y-3) =$$
$$= \cancel{4} + \cancel{5x} - \cancel{5} + \cancel{y} - \cancel{3} + x^2 - \cancel{2x} + \cancel{1} +$$
$$+ xy \cancel{-3x} - \cancel{y} + \cancel{3}$$
$$= x^2 + xy = f(x,y)$$

FOGLIO 3

$$\frac{\partial^2 f}{\partial x_1^2} = 2 + e^{3x_1}\cdot \overset{\nearrow 8}{3} + 3\cdot e^{3x_1} + 3x_1\cdot e^{3x_1}\cdot 3$$

$$f(0,0) = 0$$

$$\frac{\partial^2 f}{\partial x_2^2} = 2 \qquad \frac{\partial f}{\partial x_1} = 1 + 0$$

$$\frac{\partial^2 f}{\partial x \partial y} = 2 \qquad \frac{\partial f}{\partial x_2} = 1$$

$$p(x_1,x_2) = 0 + 1\cdot(x-0) + 1\cdot(y-0) +$$
$$+ \frac{1}{2}\cdot 8\,(x-0)^2 + \frac{1}{2}\cdot 2\,(y-0)^2 + 2xy$$

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$$p(x,y) = x + y + 4x^2 + y^2 + 2xy$$