OTTIMIZZAZIONE LIBERA

RICERCARE I MAX/MIN

SENZA VINCOLI

OTTIMIZZAZIONE — STATICA / DINAMICA (VARIABILE t)

$$\text{MAX/MIN } f(x_1, \dots, x_n)$$

FUNZIONE OBIETTIVO

LA FUNZ. NON DIPENDE DAL tempo t

$$f : \mathbb{R}^n \longrightarrow \mathbb{R} \qquad n \geq 2$$

$X \subseteq \mathbb{R}^n$ DOMINIO APERTO

$\Longrightarrow$ MAX/MIN INTERNI A X

DEF. $f: X \to \mathbb{R}$, $X \subseteq \mathbb{R}^n$

$\underline{x}_0 \in X$ SI DICE PUNTO DI MAX ASSOL. (GLOBALE) SE

$\forall \underline{x} \in X : f(\underline{x}) \leq f(\underline{x}_0)$

DEF. $\underline{x}_0 \in X$ SI DICE PUNTO DI MIN. ASS. (GLOBALE) SE

$\forall \underline{x} \in X : f(\underline{x}) \geq f(\underline{x}_0)$

(**N.B.** SE VALE $<$, o $>$, IL MAX/MIN SI DICONO STRETTI)

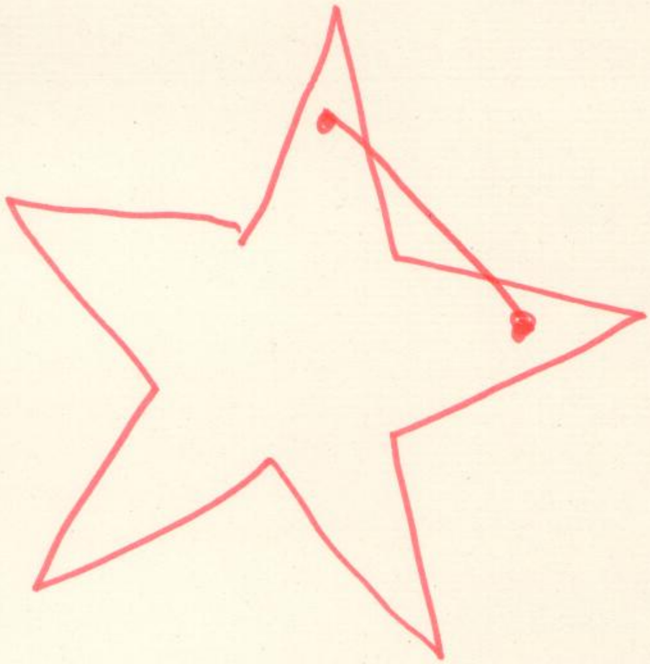
DEF. $\underline{x}_0$ SI DICE PUNTO DI MAX LOCALE O RELAT. SE

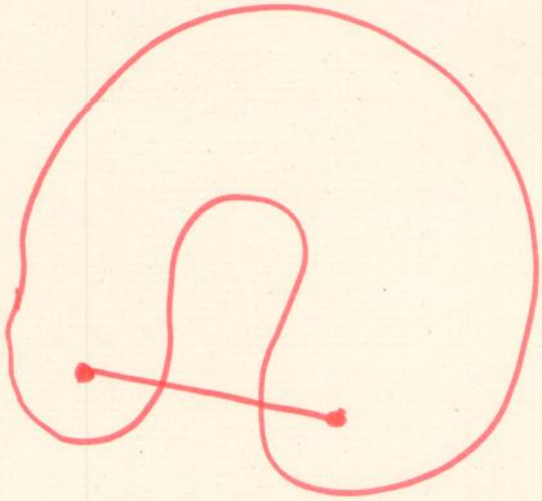
$\exists I(\underline{x}_0)\ \forall \underline{x} \in I(\underline{x}_0) \cap X :$

$f(\underline{x}) \leq f(\underline{x}_0)$

DEF. $\underline{x}_0$ SI DICE PUNTO DI MIN LOCALE O RELAT. SE.....

$f(\underline{x}) \geq f(\underline{x}_0)$





TEO. $f: X \to \mathbb{R}$, $X \subseteq \mathbb{R}^n$, APERTO.
SE f È DIFFERENZIABILE IN $\underline{x}_0$, PUNTO DI ESTREMO LOCALE,

$$\Longrightarrow \nabla f(\underline{x}_0) = \underline{0} \quad \text{(FERMAT)}$$

IL PUNTO $\underline{x}_0$ SI DICE CRITICO O STAZIONARIO

CRITERIO: PER INDIVIDUARE GLI EVENTUALI PUNTI DI MAX/MIN, SI CERCA TRA GLI EVENTUALI PUNTI STAZIONARI.

TEO. $f: X \to \mathbb{R}$, $X \subseteq \mathbb{R}^n$, APERTO E CONVESSO. SE f È DIFFERENZ. E CONVESSA* E $\underline{x}_0$ È UN PUNTO STAZIONARIO
$\Longrightarrow$ $\underline{x}_0$ È UN PUNTO DI MIN* ASSOLUTO.

*CONCAVA *MAX

DEF. $f : X_1 \times X_2 \longrightarrow \mathbb{R}$, $X_1 \subseteq \mathbb{R}^m$,

$X_2 \subseteq \mathbb{R}^{n-m}$

$\underline{x}_0 = \begin{bmatrix} \underline{y}_0 \\ \underline{z}_0 \end{bmatrix} \in \mathbb{R}^n$, $\underline{y}_0 \in \mathbb{R}^m$, $\underline{z}_0 \in \mathbb{R}^{n-m}$

SI DICE PUNTO DI SELLA o COLLE SE VALE

$$f\left(\begin{bmatrix} \underline{y} \\ \underline{z}_0 \end{bmatrix}\right) \leq f\left(\begin{bmatrix} \underline{y}_0 \\ \underline{z}_0 \end{bmatrix}\right) \leq f\left(\begin{bmatrix} \underline{y}_0 \\ \underline{z} \end{bmatrix}\right)$$

$\forall \underline{y} \in X_1$, $\forall \underline{z} \in X_2$

ES. $f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_1x_3$

CERCO I PUNTI CRITICI:

$$\nabla f(\underline{x}) = \underline{0}$$

$$\begin{cases} 2x_1 - 2x_2 - 2x_3 = 0 \\ 6x_2 - 2x_1 = 0 \\ 4x_3 - 2x_1 = 0 \end{cases}$$

$$\begin{cases} X_1 - X_2 - X_3 = 0 \\ X_2 = \frac{1}{3} X_1 \\ X_3 = \frac{1}{2} X_1 \end{cases} \qquad \begin{cases} X_1 - \frac{1}{3} X_1 - \frac{1}{2} X_1 = 0 \\ X_2 = \dots \\ X_3 = \dots \end{cases}$$

$$\begin{cases} \frac{6X_1 - 2X_1 - 3X_1}{6} = 0 \end{cases} \Longrightarrow \begin{matrix} X_1 = 0 \\ X_2 = 0 \\ X_3 = 0 \end{matrix}$$ P. CRITICO

$(0,0,0)$ CALCOLO H

$$H = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 6 & 0 \\ -2 & 0 & 4 \end{bmatrix} \quad 3 \times 3$$

DEF. : $1^\circ > 0$

$2^\circ > 0$

$3^\circ \; -2(12) + 4(8) = 32 - 24 > 0$

DEF. POS. $\Longrightarrow (0,0,0)$ MIN. REL.

N.B. $f(x_1, x_2, x_3) = (x_1 - x_2 - x_3)^2 +$

$+ (x_2 - x_3)^2 + x_2^2 \geq 0$

$\Longrightarrow$ MIN. ASSOL.

5

ES. $f(x_1, x_2) = x_1^2(3x_2^2 - x_1)$

$$\begin{cases} 2x_1(3x_2^2 - x_1) + x_1^2(-1) = 0 \\ x_1^2(6x_2) = 0 \end{cases}$$

$$\begin{cases} x_1\left[6x_2^2 - 2x_1 - x_1\right] = 0 \\ x_1^2 \cdot x_2 = 0 \end{cases}$$

① $\begin{cases} x_1 = 0 \\ 0 = 0 \end{cases}$

$\begin{cases} x_1 = 0 \\ \forall x_2 \end{cases}$

② $\begin{cases} x_2 = 0 \\ x_1(-3x_1) = 0 \end{cases}$

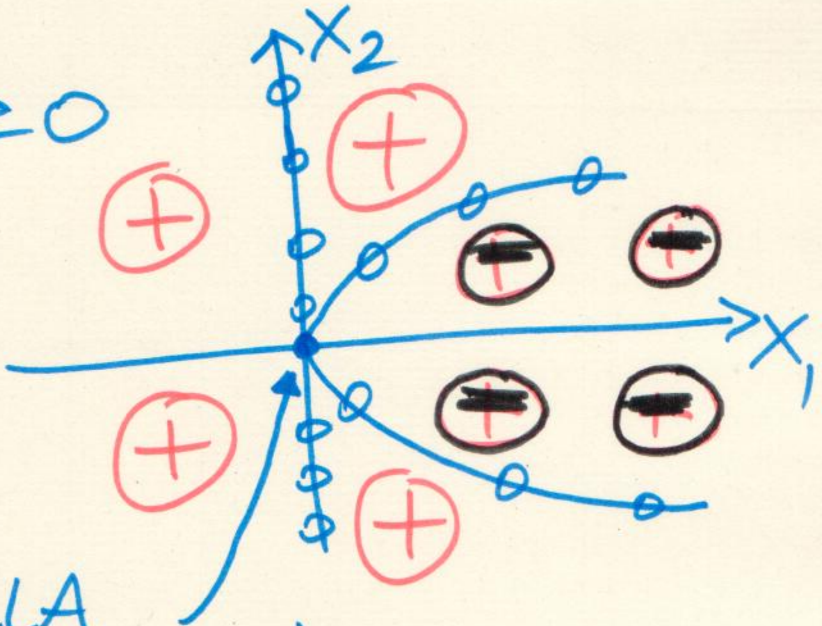
$\begin{cases} x_2 = 0 \\ x_1 = 0 \end{cases}$ ORIGINE

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & 12x_1x_2 \\ 12x_1x_2 & 6x_1^2 \end{bmatrix}$$

$$H(0, x_2) = \begin{bmatrix} \sqcup & 0 \\ 0 & 0 \end{bmatrix}$$ BOH!!!

$f(0, x_2) = 0$

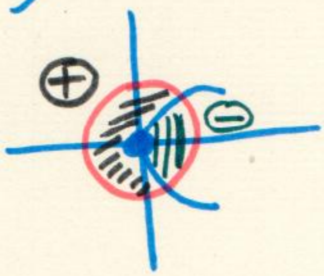
$x_1^2 (3x_2^2 - x_1) \geq 0$



$3x_2^2 \geq x_1$

$x_1 \leq 3x_2^2$

$(0,0)$ P. DI SELLA



$(0, x_2)$ $x_2 > 0$ MIN. R.

$(0, x_2)$ $x_2 < 0$ MIN. R.

ES. $f(x_1, x_2) = x_2^2 (x_2 - x_1^2 - 2x_1) =$

$= x_2^3 - x_1^2 x_2^2 - 2x_1 x_2^2$

$$\begin{cases} -2x_1 x_2^2 - 2x_2^2 = 0 \\ 3x_2^2 - 2x_1^2 x_2 - 4x_1 x_2 = 0 \end{cases}$$

$$\begin{cases} x_2^2 (-x_1 - 1) = 0 \\ x_2 (3x_2 - 2x_1^2 - 4x_1) = 0 \end{cases}$$

① $\begin{cases} x_2 = 0 \\ 0 = 0 \end{cases}$ $\begin{cases} x_2 = 0 \\ \forall x_1 \in \mathbb{R} \end{cases}$

② $\begin{cases} x_1 = -1 \\ x_2(3x_2 - 2 + 4) = 0 \end{cases}$

$\begin{cases} x_1 = -1 \\ x_2 = 0 \end{cases}$ $\begin{cases} x_1 = - \\ 3x_2 = - \end{cases}$

$x_2 = -$

$$H = \begin{bmatrix} -2x_2^2 & -4x_1x_2 - 4x_2 \\ -4x_1x_2 - 4x_2 & 6x_2 - 2x_1^2 - 4x_1 \end{bmatrix}$$

$\to -4(-1)\left(-\frac{2}{3}\right) + \frac{8}{3}$

$-\frac{8}{3}$

$\to 6\left(-\frac{2}{3}\right) - 2 +$

$-4 + 2 = -$

$$H(x_1, 0) = \begin{bmatrix} 0 & 0 \\ 0 & \sim \end{bmatrix}$$ BoH!

$$H\left(-1, -\frac{2}{3}\right) = \begin{bmatrix} -\frac{8}{9} & 0 \\ 0 & -2 \end{bmatrix}$$

$1° < 0$

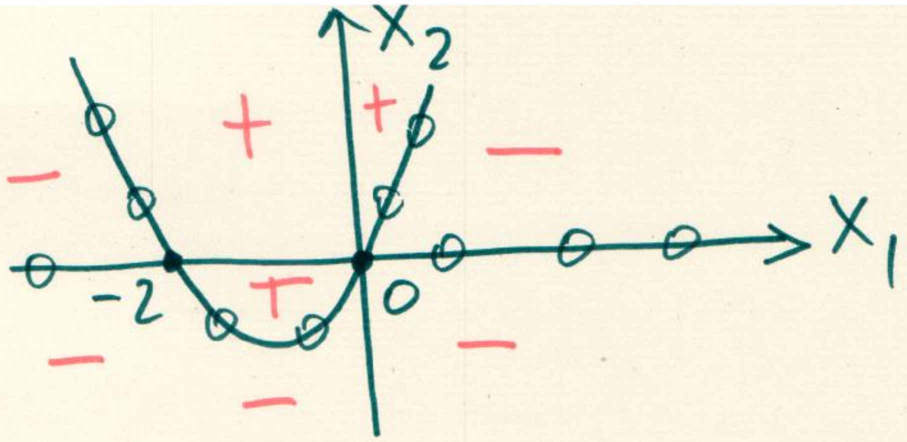
$2° > 0$

MAX REL.

$f(x_1, 0) = 0$

$f(x_1, x_2) \geq 0$ $\quad x_2 - x_1^2 - 2x_1 \geq 0$

$x_2 \geq x_1^2 + 2x_1$



$(-2,0)$ e $(0,0)$ P. DI SELLA

$(x_1,0)$ CON $-2<x_1<0$ MIN. R.

$(x_1,0)$ CON $x_1<-2 \vee x_1>0$ MAX. R.

ES. 2 pag 162

$$f(x,y)=e^{xy}$$

$$\begin{cases} e^{xy}\cdot y=0 \\ e^{xy}\cdot x=0 \end{cases} \qquad \begin{cases} x=0 \\ y=0 \end{cases}$$

$(0,0)$ SELLA

$$H=\begin{bmatrix} e^{xy}y^2 & e^{xy}xy+e^{xy}\cdot 1 \\ '' & e^{xy}x^2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

4) $f(x,y)=\sqrt{y^2-2xy-y}$

$D: y^2-2xy-y\geq 0$

$$\begin{cases} \frac{1}{\cancel{2}\sqrt{\ }}\cdot(-\cancel{2}y)=0 \\ \frac{1}{\cancel{2}\sqrt{\ }}(2y-2x-1)=0 \end{cases}$$

$$\begin{cases} y=0 \\ +2x+1=0 \end{cases} \qquad \begin{cases} y=0 \\ x=-\frac{1}{2} \end{cases}$$

1) $\underline{x}_0 \in D$
2) $\underline{x}_0$ INTERNO
3) $\exists \ \frac{\partial f}{\partial x} \ \frac{\partial f}{\partial y}$

FRONT. $y(y-2x-1)=0$

1) $y=0$ ASSEX MIN. ASS.
2) $y=2x+1$

10

(46) $f(x,y) = \log(y^2 - 2xy - \cancel{y})$

$y^2 - 2xy - y > 0$

$$\begin{cases} \frac{1}{(\ )} \cdot (-2y) = 0 \\ \frac{1}{(\ )} (2y - 2x - 1) = 0 \end{cases}$$

$$\begin{cases} y = 0 \\ +2x + 1 = 0 \end{cases} \qquad \begin{cases} x = -\frac{1}{2} \\ y = 0 \end{cases}$$

---

(298) $f(x,y) = \ln x^2 \cdot \ln y$

$$\begin{cases} x^2 > 0 \\ y > 0 \end{cases}$$

$$\begin{cases} \frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x} \cdot \ln y = 0 \\ \ln x^2 \cdot \frac{1}{y} = 0 \end{cases}$$

$$\begin{cases} y = 1 \\ x^2 = 1 \end{cases} \qquad \begin{cases} y = 1 \\ x = \pm 1 \end{cases}$$

11

$(1,1) \quad (-1,1)$ P. CRITICI

$$H=\begin{bmatrix} -\frac{1}{x^2}\,2\ln y & \frac{2}{x}\,\frac{1}{y} \\ \frac{2}{x}\,\frac{1}{y} & \ln x^2\left(-\frac{1}{y^2}\right) \end{bmatrix}$$

$$H(1,1)=\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$ P. di SELLA

$$H(-1,1)=\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$$ P. di SELLA

300 $f(x,y)=e^{1+x^2y^2}$

$$\begin{cases} e^{\sim}\cdot(2xy^2)=0 \\ e^{\sim}\cdot(2x^2y)=0 \end{cases}$$

$$\begin{cases} x y^2 = 0 \\ x^2 y = 0 \end{cases}$$

$$1)\ \begin{cases} x = 0 \\ x^2 = 0 \end{cases} \qquad 2)\ \begin{cases} x = 0 \\ y = 0 \end{cases} \qquad 3)\ \begin{cases} y^2 = 0 \\ x^2 = 0 \end{cases}$$

$$4)\ \begin{cases} y^2 = 0 \\ y = 0 \end{cases}$$

MIN R.
e ASS.

$(0, y)$ $(x, 0)$ CRITICI

$$H = \begin{bmatrix} e^{1+x^2y^2}(2xy^2)^2 + e^{\sim} 2y^2 & e^{\sim} 2x^2y \cdot 2xy^2 + e^{\sim} \cdot 4xy \\ & e^{1+x^2y^2}(2x^2y)^2 + e^{\sim} 2x^2 \end{bmatrix}$$

$$f(0, y) = e \qquad e^{1+x^2y^2} \geq e$$

$$f(x, 0) = e \qquad \cancel{1} + x^2y^2 \geq \cancel{1}$$